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Confining vacua and Q-state Potts models with Q<1

Ferdinando Gliozzi*
Dipartimento di Fisica Teorica, Universitá di Torino and
INFN, Sezione di Torino, Italy
E-mail: gliozzi@to.infn.it

In most Yang-Mills models the vacuum where magnetic monopoles condense coincides with that where center vortices percolate, thus it is not clear which of these two properties is most directly involved in producing confinement. It is pointed out that there is a class of 3D gauge models, which can be thought of as duals of Q-state Potts models with Q < 1, where the magnetic monopole condensation is a necessary but not sufficient condition for percolation of center vortices. A set of numerical tests at $Q = \frac{1}{10}$ shows that there is a vacuum in which the magnetic monopole condensate does not yield confinement, in the sense that large Wilson loops obey a perimeter law. In such a vacuum the center vortices form a dilute gas of loops. At stronger coupling there is also a truly confining vacuum where both confining mechanisms are present.

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1. Introduction

Magnetic monopoles [1] and center vortices [2] are widely believed to be the most important degrees of freedom for confinement in Yang Mills theories.

Plausibility arguments suggest that magnetic monopole condensation implies a dual Meissner effect which pushes out of the vacuum the colour field and gives the well-known physical picture of confinement in terms of dual Abrikosov vortices describing the confining strings joining the quark sources. Considerable evidence for this dual Meissner effect has been accumulated on the lattice [3], including the definition of a disorder parameter demonstrating the condensation of magnetic monopole below the deconfining temperature [4]

Center vortices are string-like excitations formed out of the center of the gauge group which are expected to encode all the infrared physics of confinement [5, 6]. When they percolate [7], produce a very efficient disordering mechanism which could lead to the area law decay of large Wilson loops.

In most YM models the phase with magnetic monopole condensation coincides with that where center vortices percolate, thus it is not clear which of these two properties is most directly involved in producing confinement, or, to be more precise, the area law decay of large Wilson loops. There are many open, intertwined questions about the validity of these confinement mechanisms. In particular, is it possible to derive monopole condensation from percolation of center vortices or vice versa? Are both mechanisms necessary for confinement? are they also sufficient?

In this talk I try to answer these questions by studying a particularly simple class of 3D gauge models, which can be thought of as duals of Q-state Potts models. In these models the confining mechanisms can be easily identified in some specific geometric properties of the random graphs associated to the configurations of the Q-state Potts models. In particular it will be evident that the percolation of center vortices implies the condensation of magnetic monopoles. On the contrary, it is pointed out that when Q < 1 the magnetic monopole condensation is not necessarily associated to the percolation of center vortices: it is demonstrated through a numerical experiment that there is a vacuum state in which, although the magnetic monopoles condense, quark sources are not confined, because large Wilson loops decay exponentially with the perimeter instead of the area. It is also shown that in this theory there is a confining vacuum only when the magnetic monopole condensation is associated to the percolation of center vortices.

The contents of this contribution are as follows. In the next Section the main properties of the Q-state Potts model and of its gauge dual are described with a particular emphasis on the definition of Wilson loop, which plays an essential role in the studies of confinement. In the following Section the nature of the vacua of these gauge duals in the range $0 < Q < 1$ and the salient features of the phase diagram are discussed. In Section 4 a local Monte Carlo algorithm for simulating Potts models in the range $0 \leq Q < 1$ is presented. Finally, in the last Section, some numerical results generated with this algorithm are reported.

2. Q state Potts model and its dual

The Hamiltonian of the (ferromagnetic) Q-state Potts model is, for Q integer, $H = -\sum_{\langle i\,j\rangle} \delta_{\sigma_i\,\sigma_j}$, where the site variable σ_i takes the values $\sigma_i = 1, 2, \ldots, Q$, with $\langle i\,j\rangle$ ranging over the links of an

arbitrary lattice or graph Λ. This model is symmetric under S_Q, the group of permutations of Q elements. The canonical partition function $Z = \sum_{\{\sigma\}} e^{-\beta H}$ can be rewritten in the Fortuin Kasteleyn (FK) random cluster representation:

$$Z = \sum_{G \subseteq \Lambda} w_G = \sum_{b,c} \Omega(b,c)\, v^b Q^c \quad , \tag{2.1}$$

where $v = e^\beta - 1$ and the summation is over all spanning subgraphs of Λ, $w_G = v^b Q^c$ is their weight expressed in terms of the number b of edges of G, called bonds, and the number c of connected components, called FK clusters; $\Omega(b,c)$ is their multiplicity. This representation now defines a model for any real or complex Q, which acts as the fugacity controlling the number of FK clusters.

When Λ is a three-dimensional lattice, the FK random cluster representation is also useful to define a gauge dual of this spin model (for any complex Q) . The gauge dual lives in the dual lattice $\tilde{\Lambda}$. The most basic observables of any gauge model are the Wilson loops. In the present case these are associated to the closed paths $\gamma \in \tilde{\Lambda}$. For any spanning subgraph $G \subseteq \Lambda$ the Wilson loop W_γ measures the topological entanglement between γ and G. More precisely we attribute to the Wilson loop $W_\gamma(G)$ the value 1 if no FK cluster of G is linked to γ, otherwise we set $W_\gamma(G) = 0$. The vacuum expectation value of W_γ is defined accordingly:

$$\langle W_\gamma \rangle = \sum_{b,c} W_\gamma(G) \Omega(b,c)\, v^b Q^c / Z \; . \tag{2.2}$$

In the special cases where $Q = 2, 3, \ldots$ this definition coincides with the one obtained by applying the usual Kramers-Wannier duality, provided one defines the topological linking as a winding modulo Q [8]. The gauge theory dual to $Q = 1$ Potts model, corresponding to random percolation, has been studied in detail in [9]. In particular it has been shown that, although the gauge group is trivial, it behaves like a full-fledged gauge theory with a confining vacuum (corresponding to the percolating phase), a string tension having a well-behaved continuum limit, a non trivial glueball spectrum [10] and a deconfinement transition at a well determined temperature. In this talk I describe some new features of this kind of gauge models in the range $0 < Q < 1$.

3. Confining vacua

Wilson loops provide us with a fundamental tool for a precise definition of confinement in a pure gauge theory. A confining phase is expected to show up in an area law decay for the vacuum expectation value of large Wilson loops $\langle W_\gamma \rangle$. This exactly means that if γ is scaled up keeping its shape fixed and increasing the area A of the encircled minimal surface, then $\langle W_\gamma \rangle \propto e^{-\sigma A}$, where σ defines the string tension.

For a generic $Q > 0$ one expects two kinds of vacua, depending on the value of β. When β is small enough, the system is in a symmetric vacuum, characterised by the formation of FK clusters of finite size. If one probes this vacuum with Wilson loops of size much larger than the typical dimension of these loops, one finds a perimeter law decay $\langle W_\gamma \rangle \propto e^{-p|\gamma|}$ ($|\gamma|$ is the perimeter of the loop), the reason being that the only clusters that can be felt by W_γ are those near the closed path $\gamma \in \widetilde{\Lambda}$. When β is larger than a threshold value β_t which depends on the kind of lattice Λ, the S_Q symmetry of the model is spontaneously broken and the corresponding vacuum is characterised

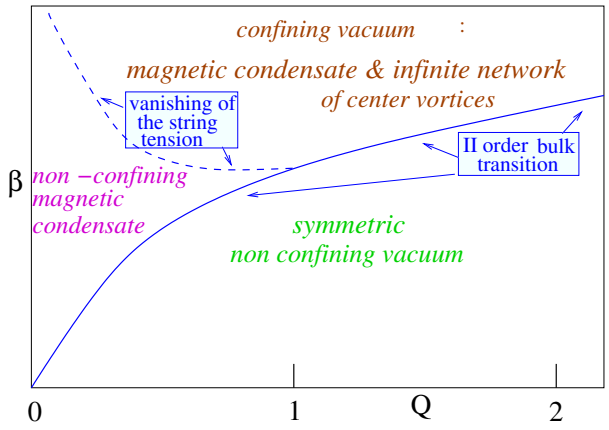

Figure 1: A schematic view of the phase diagram of gauge Q-state Potts model. The solid line denotes the bulk transition corresponding to the condensation of magnetic monopoles. The dashed line, corresponding to the vanishing of the string tension, does not imply any bulk transitions.

by the formation of an infinite, percolating, FK cluster $G_\infty \subset G$. The spin field σ_i associated to the sites of the lattice Λ is , in all respects, the disorder parameter of the dual gauge theory and the formation of an infinite, percolating, cluster is a direct sign of the condensation of magnetic monopoles, i.e.$\langle\sigma_i\rangle \neq 0$. It is clear that in this case the number of paths of G_∞ piercing the minimal surface encircled by γ grows with its area A, therefore one is tempted to argue that large Wilson loops obey an area law. Note however that the probe W_γ feels only those piercing paths which are closed, thus in order to conclude for a confining vacuum one has to assume that also the subgraph $\mathscr{C}$ composed by the circuits of G has got an infinite component $\mathscr{C}_\infty \subset \mathscr{C}$ for $\beta \geq \beta_t$. This has been demonstrated through numerical simulations only when $Q \geq 1$ [11, 9]. For lesser values this is not necessarily true.

Actually there is a very simple argument showing that, keeping constant the mean number of bonds $\langle b\rangle$, the size of $\mathscr{C}$ or, more precisely, the number $b_{\mathscr{C}}$ of bonds belonging to $\mathscr{C}$ is a decreasing function of Q and vanishes in the limit $Q \to 0$. In fact Q is the cluster fugacity of the system: when Q decreases, so does the number of clusters c. The only way to reduce c is to add *bridges*, i.e. bonds that join otherwise disconnected clusters. Now the total number of bonds b is the sum of $b_{\mathscr{C}}$ bonds belonging to $\mathscr{C}$ (i.e. to circuits) and of $b_{\mathscr{B}}$ bridges, i.e. $b = b_{\mathscr{C}} + b_{\mathscr{B}}$, where $G = \mathscr{C} \cup \mathscr{B}$; therefore a growth of the bridges keeping b constant implies decreasing of $b_{\mathscr{C}}$, q.e.d.

We note, as a side remark, that there is a general relationship between the bonds of the two kinds:

$$\langle b_{\mathscr{B}}\rangle \frac{v+Q}{v} + \langle b_{\mathscr{C}}\rangle \frac{v+1}{v} = N\,; \tag{3.1}$$

this is true for any Q-state Potts model on an arbitrary graph with N links [12]. In the special case of two-dimensional, infinite square lattice at the transition point ($v = \sqrt{Q}$) the self-duality of the model requires $\langle b\rangle = \frac{N}{2}$, thus at criticality we have $b_{\mathscr{C}} = N\frac{\sqrt{Q}}{2(\sqrt{Q}+1)}$. This exact result can be used as a check of the Monte Carlo algorithm described in the next Section.

The reduction of closed paths as Q decreases suggests, for Q and $\beta - \beta_t$ small enough, that the subset $\mathscr{C}$ does not longer percolate even in the phase where the symmetry is spontaneously broken. For instance, in the limit $Q, v \to 0$ with the ratio $w = v/Q$ held fixed [13] the surviving

configurations are spanning subgraphs not containing any circuits, i.e. $\mathscr{C} = \emptyset$, hence in a 3D lattice $W_\gamma(G) = 1\,, \forall\ \gamma$ and G and the $Q = 0$ dual gauge theory is trivial.

The above remarks suggest that when $Q < 1$ is small enough, the standard non-confining vacuum, corresponding to the symmetric phase of the Potts model, is separated from a truly confining vacuum, where $\mathscr{C}$, the subgraph of circuits, holds an infinite component, by an intermediate vacuum characterised by the condensation of magnetic monopoles (hence by the formation of an infinite, percolating FK cluster) which however is not confining, because the closed paths, corresponding to center vortices, form a dilute gas of loops embedded in the infinite cluster rather than a connected skein. A sketch of the expected phase diagram of the 3D Potts model in the small Q region is drawn in Fig.1.

4. A Monte Carlo algorithm for Q<1 Potts models

The non-local cluster algorithm of Swendsen and Wang [14] and its generalisation to non integer Q [15] is applicable only for $Q \geq 1$. In order to study the region $0 < Q < 1$ we are interested in we have to resort to some local Monte Carlo algorithm [16, 17]. I describe here a variant of the method described in [17] which can be implemented in an efficient way and works only when $0 \leq Q \leq 1$.

First, divide the interval $[0,1]$ in three parts $a\,, b-a\,, 1-b$, where a and $b \geq a \geq 0$ are suitable functions of β and Q, to be determined later. Then apply the following recursive procedure that generates a Markov sequence of spanning subgraphs $\cdots \to G^{(n)} \to G^{(n+1)} \to \ldots$ of an arbitrary lattice Λ:

i) Pick a link $\ell \in \Lambda$ and draw a uniformly distributed random number $0 \leq r_\ell \leq 1$;

ii) create or erase a bond on the link ℓ according to the following rules: (a) put a bond if $r_\ell \leq a$; (b) erase any bond if $r_\ell \geq b$; (c) in the remaining case $(a < r_\ell < b)$ put a bond only if it is a bridge, i. e. only if it connects two otherwise disjoint clusters.

The Markov chain generated in this way forms an ergodic trajectory in the space of configurations of the Q-state Potts model. Requiring detailed balance with respect the equilibrium distribution yields $a = 1 - e^{-\beta} \equiv p$ and $b = \frac{p}{Q(1-p)+p}$. The inequality $a \leq b$ implies $Q \leq 1$[1].

This Monte Carlo method with its variants was already used to locate the marginal value of $Q > 2$ in three dimensions [17] and in the study of the backbone exponent of critical Q-state Potts models in two dimensions [18].

Since this kind of algorithms implies a random sequence of disordering moves of type (a) and (b), randomly distributed over the lattice, it led to conjecture that they do not suffer of critical slowing down [16, 17]. A subsequent numerical analysis for some integer values of Q in two and three dimensions showed that this conjecture is false [19]: it reduces critical slowing down, but does not completely eliminate it, in the sense that its dynamical critical exponent is smaller than Swendsen-Wang, but in general does not vanish, at least for $Q > 2$ in $2D$ and $Q \geq 2$ in $3D$ [19].

Of course, for a practical use of viable Monte Carlo algorithms it matters not only the intrinsic dynamics of the transition rates, but also the efficiency of the numerical implementation. In the

[1] If one replaces the rule (c) with the new rule (c'): put a bond only if the number c of clusters is kept constant (this was the rule chosen in ref.[17] in the case $Q > 1$) then one finds $a = p/Q$ and $b = p$.

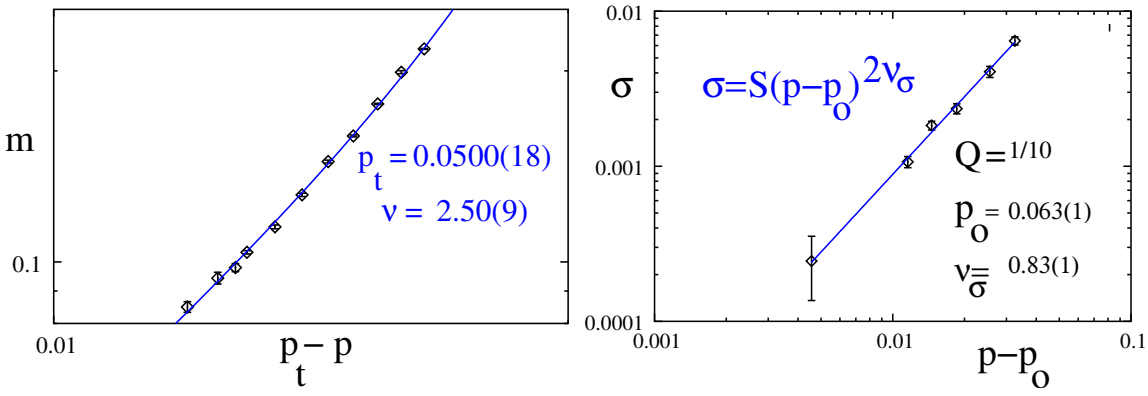

Figure 2: The mass of the lowest physical state in the symmetric vacuum (left) and the string tension in the confining vacuum(right). The data refer to $Q = 1/10$ Potts model in a lattice of size 32^3.

present case we succeeded in simulating lattice sizes of the order of those used in current gauge simulations.

5. Numerical results

A simple observable which can be used to locate the threshold $p_t = 1 - e^{-\beta_t}$ where an infinite FK cluster forms is the connectivity correlator $\mathscr{G}(x,y) = \langle \phi_{xy} \rangle$, where $\phi_{xy} = 1$ only if x and y belong to the same cluster, otherwise is set to zero. Clearly for $p < p_t$ one observes an asymptotic exponential decay $\mathscr{G}(x,y) \sim e^{-m|x-y|}$ with increasing separation $|x-y|$, where m is the mass of the lowest physical state. The correlation length $\xi = 1/m$ is of the order of the mean linear size of the FK clusters. As a consequence, m vanishes at p_t, and it is expected to obey a critical power law

$$m \simeq a\,(p_t - p)^{\nu} + b\,(p_t - p)^{\nu'} + \ldots \tag{5.1}$$

where $\nu = \nu(Q)$ is the thermal exponent. Unfortunately it appears that this critical exponent has not yet been calculated in 3D Q-state Potts models in the range $0 < Q < 1$, apart the special $Q \to 0$ limit (see the second paper of [13]).

We performed our simulations on a 32^3 cubic lattice at $Q = \frac{1}{10}$. We extracted the mass of the lowest physical state using the zero momentum projection and fitted the data to (5.1) as shown in Fig.2 (left). As a result the threshold value for the formation of a percolating FK cluster and the thermal critical exponent turn out to be $p_t = 0.0500(18)$ and $\nu = 2.50(9)$. Note that this value of ν is much larger than the corresponding value at $Q = 1$ $\nu_{Q=1} = 0.874(2)$. This agree with the fact that in two space dimensions the presumed exact value of ν increases as Q decreases.

On the same lattice at the same value of $Q = \frac{1}{10}$, but at larger values of p, where one can easily observe percolation of the sub-clusters made with the circuits of the FK clusters, we measured the vacuum expectation value of a set of square Wilson loops, in order to evaluate the string tension. They perfectly fitted the expected asymptotic functional form for the confining phase (including the log term due to the quantum fluctuations of the underlying confining string). The extracted string tension as a function of p is nicely described by a power law (see Fig.2 (right): the fitting

curve is a straight line in the log log scale). However the vanishing point of the string tension, $p_o = 0.063(1)$, where the deconfining phase starts, does not coincide with the threshold p_t. In the range $p_t \leq p \leq p_o$ the vacuum of this theory is characterised by a non-vanishing magnetic monopole condensate which is not confining, being $\sigma = 0$ there.

Note that, contrarily to what happens in the gauge duals of $Q \geq 1$ Potts models, the critical index ν_σ associated to the vanishing of the string tension is totally different from the thermal critical index ν. It is worth observing that there is no local order parameter that can signal when an infinite cluster of circuits forms , being a phenomenon of topological nature that can be detected only by large Wilson loops. As a consequence we do not expect that in these gauge models the vanishing of the string tension is associated to any sort of bulk transition.

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