An update in monopole condensation in two-flavour Adjoint QCD

Giuseppe Lacagnina[*]
Dipartimento di Fisica and INFN, Pisa, Italy
E-mail: lacagnina@df.unipi.it

Guido Cossu
Scuola Normale Superiore and INFN, Pisa, Italy
E-mail: g.cossu@sns.it

Massimo D'Elia
Dipartimento di Fisica and INFN, Genova, Italy
E-mail: Massimo.Delia@ge.infn.it

Adriano Di Giacomo
Dipartimento di Fisica and INFN, Pisa, Italy
E-mail: digiaco@df.unipi.it

Claudio Pica
Brookhaven National Laboratory, Physics Department, Upton, NY 11973-5000, USA
E-mail: pica@bnl.gov

QCD with fermions in the adjoint representation (aQCD) is a model for which a deconfinement and a chiral phase transition take place at different temperatures. In this work, we present a study of the deconfinement transition in the dual superconductor picture based on the evaluation of an operator which carries magnetic charge. The expectation value of this operator signals monopole condensation and is an order parameter for deconfinement as in the case of fermions in the fundamental representation. We find a sharp first order deconfinement transition. We also study the effects of the chiral transition on the monopole order parameter and find them negligible.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

---

[*]Speaker.

1. Introduction

Ordinary QCD, for which quarks are in the fundamental representation of $SU(3)$, shows two phase transitions, deconfinement and chiral symmetry restoration, which according to numerical simulations take place at the same temperature. The coincidence of the two transitions makes it very difficult to study the degrees of freedom which are relevant for each transition separately. On the other hand, QCD with fermions in the adjoint representation of $SU(3)$ (aQCD), is a model for which the two phase transitions seemingly happen at distinct temperatures [1, 2]. Furthermore, contrary to the case of standard QCD, quarks in the adjoint representation do not explicitly break the $Z(3)$ symmetry of the action, and the Polyakov loop is a good order parameter for deconfinement.

In this work, we study the deconfinement phase transition in the Dual Superconductor Picture (DSP), in which confinement follows from dual superconductivity of the QCD vacuum, realized as condensation of magnetic charges [3]. To study monopole condensation, we construct a magnetically charged operator whose expectation value vanishes exactly in the deconfined phase and becomes nonzero in the confined phase [4, 5, 6, 7], thus defining an order parameter. Our main goal is to take advantage of the features of aQCD to investigate the relation between dual superconductivity and the dynamics of chiral symmetry breaking. In particular, we study the behaviour of the magnetic order parameter in the proximity of the chiral phase transition.

The authors of [1, 2] have performed simulations of aQCD with $N_f = 2$ staggered quarks, and found a deconfinement and a chiral phase transition at different temperatures, with $\beta_{\text{dec}} < \beta_{\text{chiral}}$. The chiral transition has been further investigated in [2], where the authors made an extended analysis to determine its order. Their results from the magnetic equation of state indicated a second order chiral transition in the $3d$ $O(2)$ universality class in the zero quark mass limit.

The outline is as follows. In Section 2 we briefly describe aQCD and the monopole condensation order parameter. We summarize simulation details in Section 3. We discuss our results in Section 4 and draw conclusions Section 5.

2. aQCD and monopole condensation

2.1 aQCD

Quark fields in the adjoint representation of $SU(3)$ can be written as 3×3 hermitian traceless matrices

$$Q(x) = Q^a(x)\lambda_a \tag{2.1}$$

in terms of Gell-Mann's λ matrices. For the fermionic sector of the action one therefore needs to use the 8-dimensional, real representation of the link variables:

$$U_{(8)}^{ab} = \frac{1}{2}\text{Tr}\left(\lambda^a U_{(3)}\lambda^b U_{(3)}^\dagger\right) \tag{2.2}$$

Including the pure gauge sector, in which links are in the usual 3-dimensional representation, the full action reads:

$$S = S_G[U_{(3)}] + \sum_{x,y}\bar{Q}(x)M\left(U_{(8)}\right)_{x,y}Q(y) \tag{2.3}$$

where M is the fermionic matrix. The Polyakov loop is defined by

$$L_3 \equiv \langle \frac{1}{3L_s^3} | \sum_{\vec{x}} \text{Tr} \prod_{x_0=1}^{L_t} U_0^{(3)}(x_0,\vec{x}) | \rangle \qquad (2.4)$$

and is an order parameter for the spontaneous breaking of the center symmetry. The Polyakov loop is related to the free energy of an isolated static quark in a gluonic bath at temperature T: $L_3 \propto e^{-F/T}$ [8]. This result justifies its use as an order parameter for the deconfinement transition: the free energy is infinite in the confined phase implying $L_3 = 0$, while it is finite in the deconfined phase ($L_3 \neq 0$).

2.2 Monopole condensation

A possible order parameter for the deconfinement phase transition is given by the vacuum expectation value of a magnetically charged operator [4, 5, 6, 7]. This operator adds a magnetic monopole to a given gauge field configuration: a non vanishing expectation value is the signature of monopole condensation and of the Higgs breaking of the underlying magnetic symmetry. On the other hand, in the deconfined phase, where the symmetry is restored, the *vev* of the magnetic operator drops to zero. The evaluation of this order parameter involves a few steps. One starts by fixing the gauge with a procedure known as Abelian Projection [9]. However, as shown in [5], the particular gauge choice does not affect the behaviour of the order parameter. In practice, we can update the system without an Abelian Projection, which is equivalent to choosing a different random gauge at each step [6]. Next, for each configuration, the values of the action are evaluated in presence and in absence of a monopole field insertion in the temporal plaquettes of a given time slice [4]. Then, the expression for the order parameter is:

$$\langle \mu \rangle = \frac{1}{Z} \int [dU] e^{-\widetilde{S}} = \frac{\widetilde{Z}}{Z} \qquad (2.5)$$

where $\widetilde{S}$ is modified by the presence of the monopole field. A much easier quantity to evaluate is however

$$\rho = \frac{\partial}{\partial \beta} \ln \langle \mu \rangle = \langle S \rangle_S - \langle \widetilde{S} \rangle_{\widetilde{S}} \qquad (2.6)$$

which is expected to have a large negative drop at the transition point. Close to the transition ($\beta \simeq \beta_{\text{dec}}$), a scaling behaviour of the type

$$\rho \simeq L^{1/\nu} f(L^{1/\nu}(\beta_c - \beta)) \qquad (2.7)$$

is expected (for some function f), with $\nu = 1/3$ for a first order transition (scaling with spatial volume).

3. Simulation details

We simulate two flavours of staggered quarks on two lattices, with sizes of $L_s^3 \times L_t = 12^3 \times 4, 16^3 \times 4$, and bare quark masses of $am_q = 0.01, 0.04$ for several values of β in the range $3.0 - 7.0$. Since the evaluation of ρ requires two simulations for each value of β, we use the exact RHMC algorithm [10] in presence of the monopole insertion and the Φ algorithm [11] otherwise. Typical MD
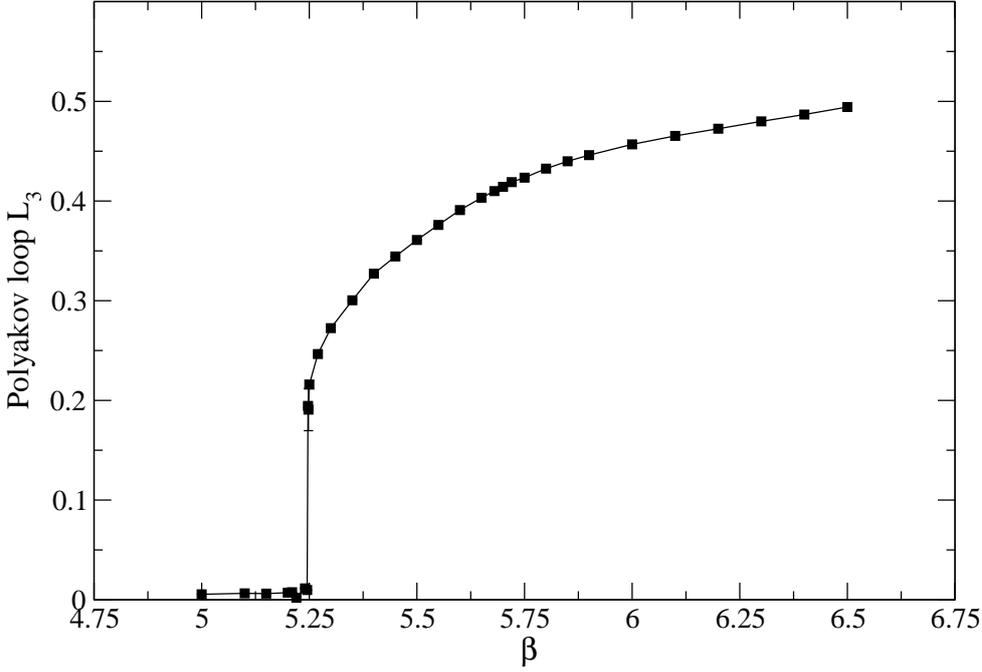
Figure 1: The Polyakov loop, with $am = 0.01$ and $16^3 \times 4$ lattice. Lines are left to guide the eye.

trajectories have a length of $N_{MD}\delta t = 0.5$, and an integration step $\delta t = 0.02 - 0.005$ depending on the mass. We employ the Conjugate Gradient to invert the fermion matrix. We also implement C^* boundary conditions as they become necessary when simulating a monopole insertion [12]. The simulations run on the ApeMille machine in Pisa and the ApeNEXT facility in Rome.

4. Results

For each configuration, we evaluate the plaquettes, the modified plaquette — in a different simulation — to calculate the ρ parameter, the Polyakov loop, the chiral condensate and its susceptibility. The Polyakov Loop, has the behaviour of a sharp first order transition for $\beta \simeq 5.25$ (see Fig. 1). We take this value of the pseudocritical β as an estimate for β_{dec} in our finite size scaling analysis. We evaluate the ρ parameter and study its scaling behaviour. The expected negative peak is found at values of β which are compatible with the discontinuity of the Polyakov loop (Fig. 2). By finite size scaling analysis, we find that ρ has the scaling properties of an order parameter for a first order transition for both values of the quark mass (see Fig. 3 and [13] for $am = 0.04$). Our main interest is that of finding possible effects of the chiral transition on the magnetic order parameter. To do so, we first perform a rough localization of the chiral transition by inspection of the chiral condensate and its susceptibility (Figs. 4, 5). Our results for the light mass $am = 0.01$ are compatible with with a chiral transition around $\beta_{\text{chiral}} = 5.8$, in agreement with [1, 2]. For both the simulated quark masses, we find that ρ does not change significantly in the vicinity of the chiral transition. We conclude that condensation of monopoles, associated to confinement, is a property only of the gauge sector of the theory and is not affected by the chiral transition.
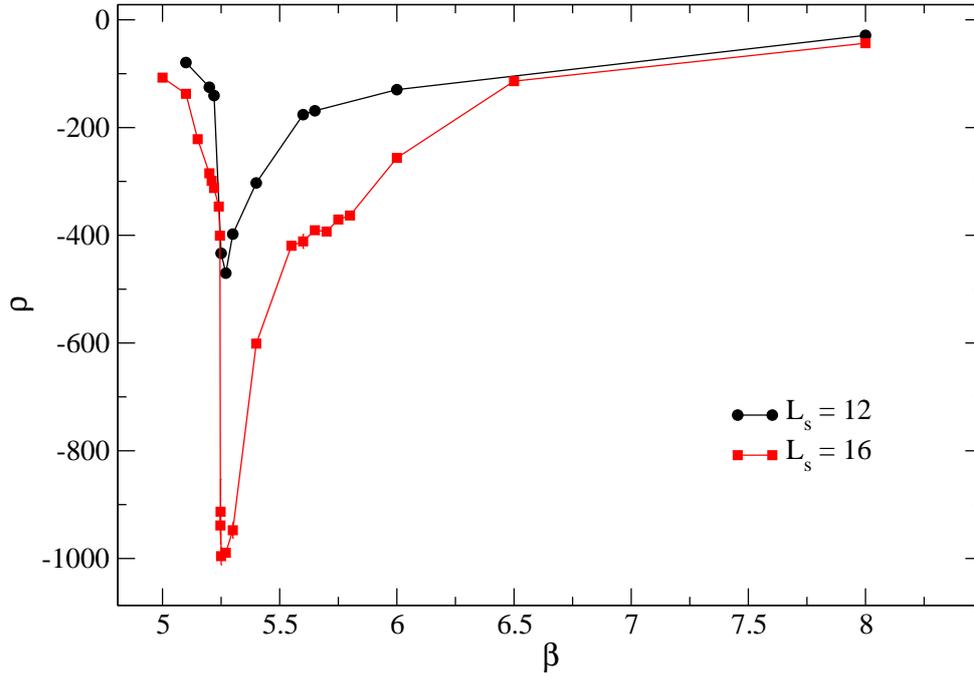
Figure 2: The ρ parameter, with $am = 0.01$, $L_t = 4$, for two different spatial volumes.
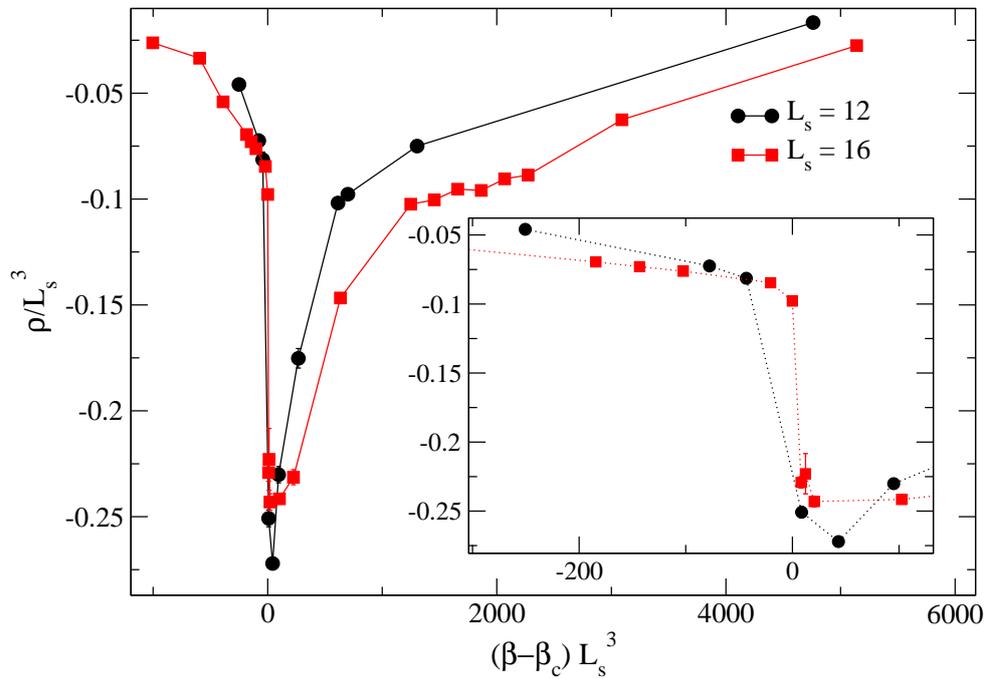
Figure 3: Scaling of the ρ parameter, $am = 0.01$, $L_t = 4$. $\beta_c = 5.25$, estimated from the Polyakov loop at $L_s = 16$.
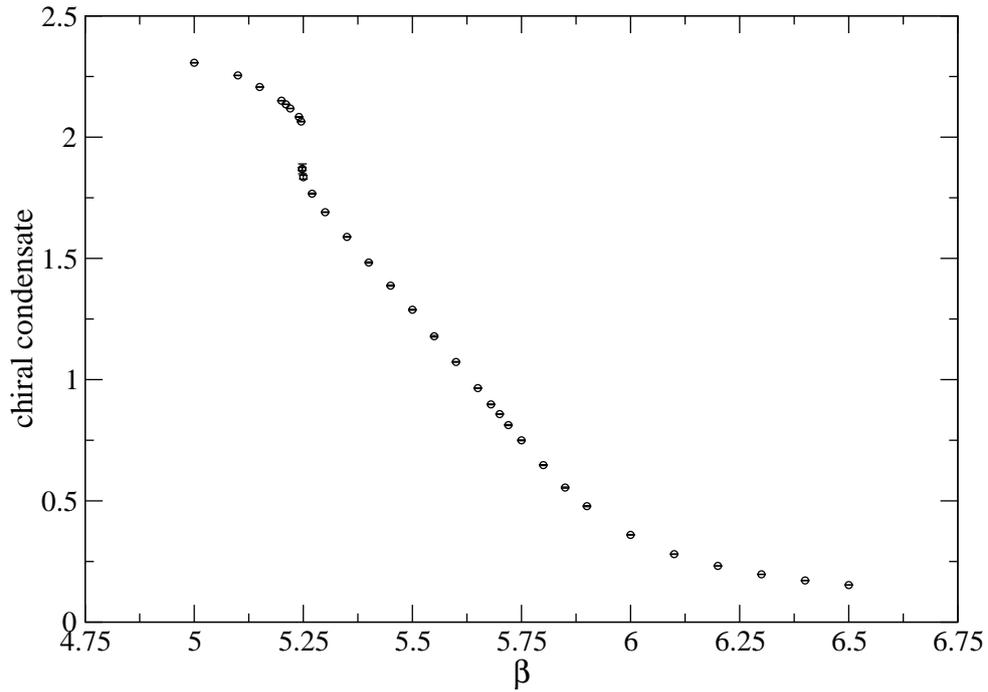
Figure 4: Chiral condensate, with $am = 0.01$, $16^3 \times 4$ lattice. A clear jump is also visible at the the deconfinement phase transition.
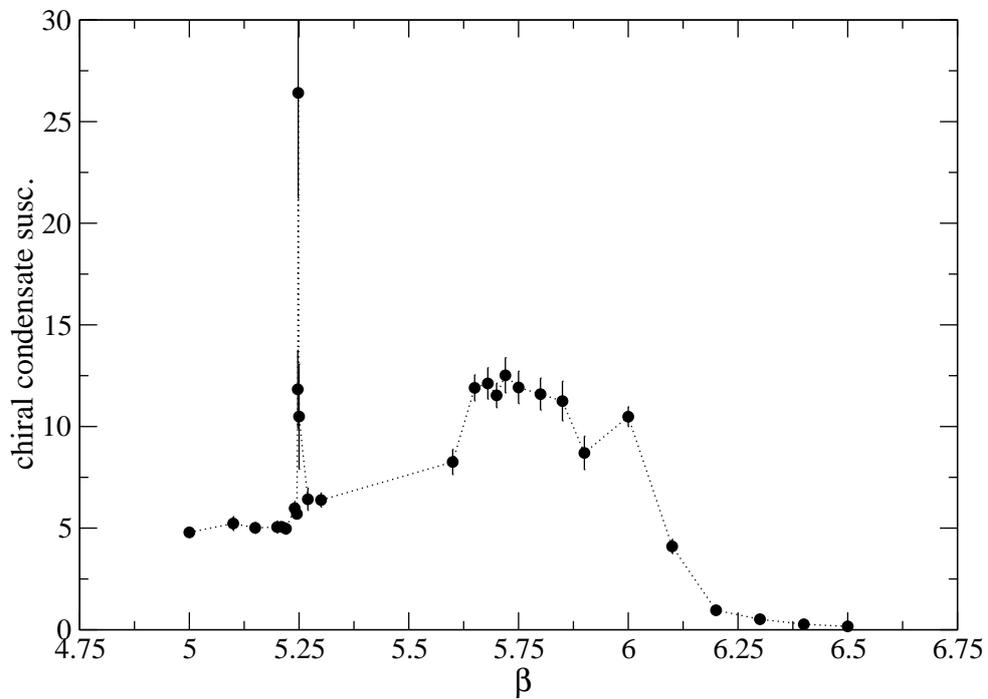
Figure 5: Susceptibility of the chiral condensate, with $am = 0.01$, $16^3 \times 4$ lattice.

5. Conclusions

We presented the results of our study on the effects of the chiral transition on monopole condensation for QCD with two flavours of staggered fermions in the adjoint representation. The appeal of this model is in the fact that its chiral and deconfinement phase transitions happen at distinct temperatures, making it possible to study the effects of one transition on the order parameter of the other. Within the framework of the Dual Superconductor Picture, we study the *vev* of a magnetically charged operator which signals monopole condensation and is expected to be an order parameter for deconfinement. Our analysis indicates a first order deconfinement transition, and the magnetic order parameter is found to be unaffected by the chiral transition. This result gives further evidence to the idea that the DSP mechanism of confinement is independent of the presence of fermions [7].

References

[1] F. Karsch, M. Lutgemeier, Nucl. Phys. B **550**, 449 (1999), hep-lat/9812023

[2] J. Engels, S. Holtmann, T. Schulze, Nucl. Phys. B **724**, 357 (2005), hep-lat/0509010

[3] G. 't Hooft, Nucl. Phys. B **138** (1978), 1

[4] A. Di Giacomo, B. Lucini, L. Montesi, G. Paffuti, Phys. Rev. D **61**, 034503 (2000), hep-lat/9906024

[5] A. Di Giacomo, B. Lucini, L. Montesi, G. Paffuti, Phys. Rev. D**61**, 034504 (2000), hep-lat/9906025

[6] J. M. Carmona, M. D'Elia, A. Di Giacomo, B. Lucini, G. Paffuti, Phys. Rev. D64, 114507 (2001), hep-lat/0103005

[7] M. D'Elia, A. Di Giacomo, B. Lucini, G. Paffuti, C. Pica, Phys. Rev. D**71**, 114502 (2005), hep-lat/0503035

[8] L. D. McLerran, B. Svetitsky, Phys. Rev. D **24**, 450 (1981)

[9] L. Del Debbio, A. Di Giacomo, B. Lucini and G. Paffuti, hep-lat/0203023

[10] M. A. Clark, A. D. Kennedy, Nucl. Phys. Proc. Suppl. **129**, 850 (2004), hep-lat/0309084

[11] S. Duane, A. D. Kennedy, B. J. Pendleton and D. Roweth, Phys. Lett. B **195**, 216 (1987)

[12] J. M. Carmona, M. D'Elia, A. Di Giacomo, B. Lucini, Int. J. Mod. Phys. C**11**, 637 (2000), hep-lat/0003002

[13] G. Lacagnina, G. Cossu, M. D'Elia, A. Di Giacomo and C. Pica, PoS **LAT2006** 069 (2006), hep-lat/0609049