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Heavy-light matrix elements in static limit with domain wall fermions

RBC and UKQCD Collaborations

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A progress report on the calculation of $B^0 - \overline{B^0}$ mixing with $2+1$ flavor dynamical domain wall fermions for the light quarks and static approximation for b quark to explore the chiral regime is presented.

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1. Introduction

Experimental efforts made it possible to determine the oscillation frequencies of $B_d^0 - \overline{B_d^0}$ and $B_s^0 - \overline{B_s^0}$ precisely [1]. The Kobayashi-Maskawa matrix elements V_{td} and V_{ts} can be obtained through these results, once the hadronic matrix elements are calculated. In the standard model, the oscillation frequency reads

$$\Delta m_q = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} |V_{tq}^* V_{tb}|^2 S_0(m_t^2/m_W^2)\eta_B \mathscr{M}_q, \tag{1.1}$$

where q is either d or s, S_0 is the Inami-Lim function, η_B is the short range QCD correction, $\mathscr{M}_q$ is the B_q^0–$\overline{B_q^0}$ mixing matrix element

$$\mathscr{M}_q = \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1-\gamma_5)q][\bar{b}\gamma_\mu(1-\gamma_5)q] | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}, \tag{1.2}$$

which needs to be calculated in QCD. Lattice QCD provides an ideal framework to calculate these low energy matrix elements.

In lattice calculation the $SU(3)$ breaking ratio $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} \propto \sqrt{\mathscr{M}_s/\mathscr{M}_d}$ can be obtained more precisely than the each matrix element, since the large fraction of the statistical and systematic errors cancel in the ratio. Through ξ and $\Delta m_s/\Delta m_d$, the ratio $|V_{td}/V_{ts}|$ is determined. $|V_{td}/V_{ts}|$ provides an important constraint on the unitarity triangle (see for the recent CKM fit in [2]). As the error of the ratio from the experiment is small (sub-percent) [3], the error of the lattice calculation of ξ dominates the width the allowed range from $|V_{td}/V_{ts}|$. Recent development in calculating the $B^0 - \overline{B^0}$ mixing on lattice should refer to the recent proceedings of plenary talks in the lattice conferences [4, 5, 6]. The full $2+1$ flavor estimate of ξ is indispensable for the reliable estimate of $|V_{ts}/V_{td}|$.

The RBC/UKQCD collaborations have been trying to calculate the mixing employing static approximation in the heavy quark effective theory (HQET) with the $2+1$ flavor domain wall fermions for the light flavors. The first results has been reported in [7]. Much lighter mass than those used there will be needed to have better control of the chiral extrapolation. This paper gives a progress report towards this direction.

2. Method

2.1 Set up of numerical calculation

We use domain wall fermions for the valence light quarks and improved static heavy quark for the b quark. The gauge configurations used here have been generated with Iwasaki gauge and $2+1$ flavor domain wall fermion action with $24^3 \times 64$ volume, same as those used for the other RBC/UKQCD projects (see [8, 9]). These ensembles are useful not only as the large volume $(2.7\text{ fm})^3$, but more importantly to explore the lighter quark mass (see Table 1). To get better signal/noise [10] of the heavy-light correlators, one step HYP smearing [11] with parameters $\vec{\alpha} = (0.75, 0.6, 0.3)$ is used for the link in the static quark action.

As the spatial volume is large, wall source will not have good overlap with the ground state. We have tested various different heavy and light quark sources. We use gauge invariant Gaussian

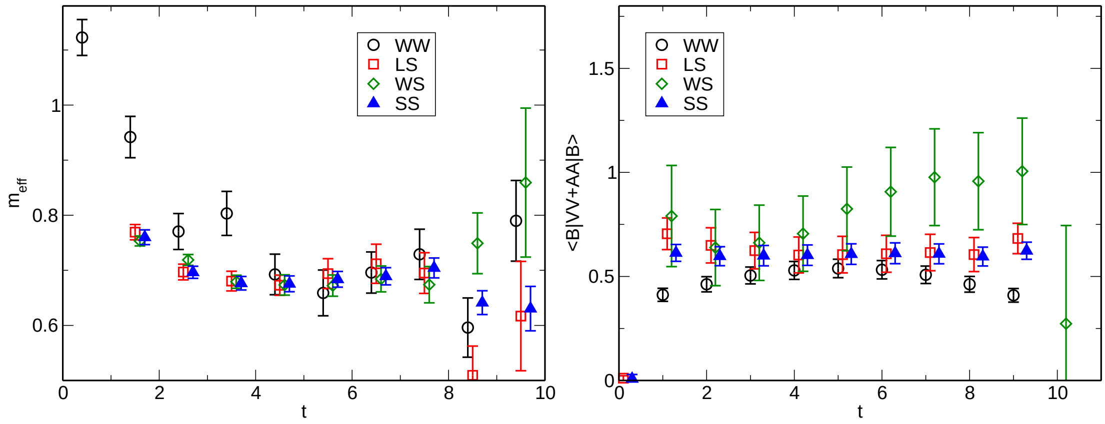

Figure 1: Test of the sources in the quenched approximation: left is the effective mass of heavy-light two point functions, where sink operator is same as the source operator with appropriate zero momentum projection. Right shows the HQET matrix element of $VV + AA$ operator obtained from three point function with $(t_{src}, t_{sink}) = (0, 10)$ divided by the appropriate two point function with $(t_{src}, t) = (0, 10)$.

smearing for the light quarks, where gauge links in the Gaussian kernel is smoothed by spatial APE smearing.

A meson interpolating operator is made of local heavy quark and smeared light quark, which is denotes as LS operator. The smearing parameters are tuned to optimize the overlap with the heavy-light meson ground state. A problem of this operator is that it can sample just single point of the whole spatial volume, thus noisy, because the heavy quark does not propagate to the spatial direction at all. We can have better sampling by making both the local and smeared sources spread over whole volume without gauge fixing, so to say smeared-wall source for the light quark and Wall source for the heavy quark, which we denote WS meson operator. The WS operator becomes a spatial sum of the LS operator after the gauge average. We also test the SS operator, where both heavy and light quarks are smeared from a single point. Smearing for the heavy quark is just to increase the spatial support of the heavy quark field to have increased sampling of the spatial volume compared to LS. Simple Coulomb-gauge-fixed wall sources for both heavy and light quarks are denoted as WW.

Figs. 1 show results from a test using quenched Wilson gauge configurations at $\beta = 5.7$ with $8^3 \times 16$ (about $1.6^3 \times 3.2$ [fm^4]) volume and valence domain wall fermion with $M_5 = 1.8$ (domain-wall height), $L_s = 4$ (fifth dimension size). The left figure is for the effective mass of the heavy-light state, which shows that WS operator works well for two point functions. However WS is worst of all for the mixing matrix element (right), where stochastic gauge noise comes from both source and sink of the meson operator in the three point function. In the following we will use SS operator, that appeared to be the best for both two and three[1] point functions in this test. We note that the primary quantity extracted from the three point function with a compact meson operator like SS is

[1]It looks WW is best for the S/N ratio for the matrix element. However, it has not reached the plateau. It seems the source-sink separation should be increased at least by 2, that makes the relative error of the denominator two-point function increase by a factor of 2. Thus, at least the error will enlarge by 2, that makes it equal as LS. If we use same physical volume as our dynamical $2+1$ flavor lattices, the WW operator will be far worse than LS.

the matrix element $\mathscr{M}_q$, but not the bag parameter, due to the number of volume degeneracy of the heavy quark state [12].

2.2 Renormalization and chiral symmetry

The relevant QCD matrix element takes the parity even part of the operator in Eq. (1.2), which we shall denote as $O_{VV+AA} = (\bar{b}\gamma^\mu q)(\bar{b}\gamma^\mu q) + (\bar{b}\gamma^\mu\gamma^5 q)(\bar{b}\gamma^\mu\gamma^5 q)$. In the HQET to QCD matching to NLO in continuum perturbation theory, $\tilde{O}_{SS+PP}$ HQET operator mixes to O_{VV+AA}

$$O_{VV+AA}(m_b) = Z_1(m_b,\mu)\tilde{O}_{VV+AA}(\mu) + Z_2(m_b,\mu)\tilde{O}_{SS+PP}(\mu), \tag{2.1}$$

where Z_2 is zero at LO. To complete the NLO matching of lattice HQET operator to QCD, thus, one needs to calculate the one-loop renormalization constant relating $\tilde{O}_{VV+AA}(\mu)$ and lattice operators $\tilde{O}^{latt}(a)$ only, while tree level $\tilde{O}_{SS+PP} = \tilde{O}^{latt}_{SS+PP}$ will suffice. We use mean field (MF) improved lattice perturbation theory with the fourth root of plaquette for the matching factors. Detailed description should refer to [13, 12, 14].

Perturbative renormalization factors are available only for the $L_s \to \infty$ limit, while we work with finite L_s which brings a small chiral symmetry breaking. The symmetry breaking gives rise to mixing with wrong chirality operators, which have been neglected in the perturbation. We can quantify this effect using $m_{\rm res}$, the induced mass (in lattice units) due to the breaking, by following the similar steps taken for the neutral kaon mixing [15]. All the relevant parity even operators in HQET written in the chiral basis read:

$$\begin{aligned}
\tilde{O}_{VV+AA} &= -(b^\dagger\overline{\sigma^\mu}q_R)(\tilde{b}\overline{\sigma^\mu}q_R) + (b^\dagger\sigma^\mu q_L)(\tilde{b}\sigma^\mu q_L), && (2.2)\\
\tilde{O}_{SS+PP} &= (b^\dagger\cdot q_R)(\tilde{b}\cdot q_R) - (b^\dagger\cdot q_L)(\tilde{b}\cdot q_L), && (2.3)\\
\tilde{O}_{VV-AA} &= (b^\dagger\overline{\sigma^\mu}q_R)(\tilde{b}\sigma^\mu q_L) - (b^\dagger\sigma^\mu q_L)(\tilde{b}\overline{\sigma^\mu}q_R), && (2.4)\\
\tilde{O}_{SS-PP} &= -(b^\dagger\cdot q_R)(\tilde{b}\cdot q_L) + (b^\dagger\cdot q_L)(\tilde{b}\cdot q_R), && (2.5)
\end{aligned}$$

where q_R and q_L (q for d or s) are the right and left handed Weyl spinor ($q = (q_R, q_L)^t$) for the light quarks, $b^\dagger$ and $\tilde{b}$ are the two-component spinors, creating the b quark or annihilating anti-b quark ($\bar{b} = \frac{1}{\sqrt{2}}(b^\dagger + \tilde{b}, b^\dagger - \tilde{b})$). $\tilde{O}_{SS+PP}$ does not mix to renormalized $\tilde{O}_{VV+AA}$ by heavy quark symmetry [16, 17], while the other mixings are in general possible [16]. As it is apparent from the above expression, however, $\tilde{O}_{VV-AA}$ and $\tilde{O}_{SS-PP}$ cannot mix to $\tilde{O}_{VV+AA}$ when chiral symmetry is exact. When chiral symmetry is broken owing to finite L_s, chirality flip $q_R \leftrightarrow q_L$ occurs by rate $m_{\rm res}$. Since at least one chirality flip is necessary for $\tilde{O}_{VV-AA}$ or $\tilde{O}_{SS-PP}$ to become $\tilde{O}_{VV+AA}$, these mixings are suppressed by a factor of $m_{\rm res}$. Numerically the matrix elements of $\tilde{O}_{VV-AA}$ and $\tilde{O}_{SS-PP}$ take similar value as $\tilde{O}_{VV+AA}$, and $m_{\rm res} = 0.0031$ for the results shown below. Thus these mixings are collections to sub-percent, hence negligible in our present accuracy.

3. Results

We give here a snapshot of our ongoing calculation of the $B^0 - \overline{B^0}$ mixing for larger volume and lighter masses than the those reported in [7]. We quote only statistical errors estimated by the jackknife method.

Table 1: *ud* quark masses used in this study. m_s/m_{ud} shows the approximate ratio of simulated *ud* mass and the physical estimate strange mass [8]. Statistics shows the number of configuration analyzed for each *ud* mass.

$m_f^{(ud)}$	0.005	0.01	0.02
m_s/m_{ud}	4.6	2.9	1.6
statistics	91	179	100

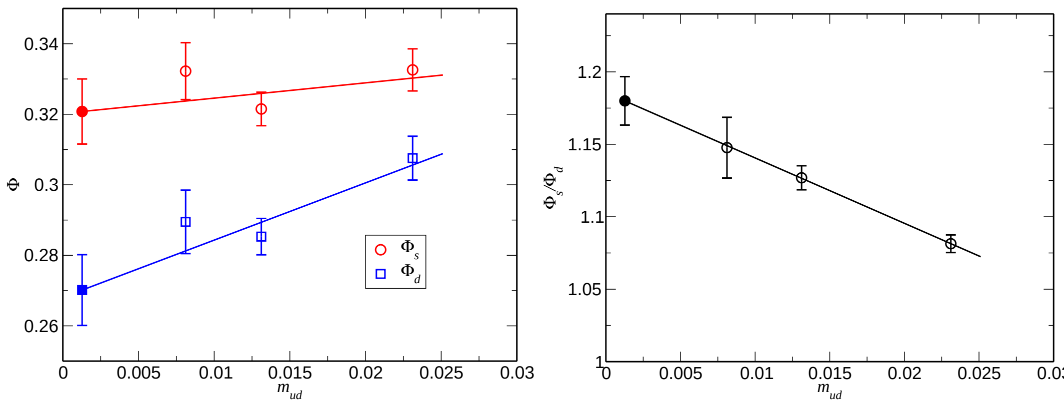

Figure 2: Bare decay constants $\Phi_q = f_{B_q}/\sqrt{m_{B_q}}$ for $q = d$, s (left) and the ratio Φ_s/Φ_d (right) as functions of m_{ud}. Solid symbols show the values extrapolated to the physical point by linear fit.

We use $2+1$ flavor domain wall fermion gauge configurations generated by RBC/UKQCD collaborations with $24^3 \times 64$ volume. The lattice spacing from the m_π, m_K, m_Ω input is $a^{-1} = 1.729(28)$ GeV, residual mass in the lattice unit is $m_{\rm res} = 0.00315(2)$. More detail of the parameter, u, d and s quark masses are presented in Ref. [8]. We use bare light quark mass $m_q = m_f^{(q)} + m_{\rm res}$ in the following figures, where q is *ud* or s, $m_f^{(q)}$ are the bare masses that appear in the domain wall fermion Lagrangian. $m_q \to 0$ is the chiral limit. All *ud* masses used in this study are the unitary points (valence quark mass is equal to sea quark mass). Used *ud* masses are listed in Table 1. Simulated strange mass is $m_f = 0.04$. For the B_s meson, we use $m_f^{(s)} = 0.04$ as well as the partially quenched point $m_f^{(s)} = 0.03$ to interpolate the physical s quark point.

Figure 2 left shows the b meson decay constants $\Phi_q = f_{B_q}/\sqrt{m_{B_q}}$ for $q = d$, s as functions of m_{ud}. The ratio Φ_s/Φ_d is shown in Fig. 2 right. Chiral perturbation theory [18] suggests the downward (upward) curvature towards the chiral limit for Φ_d (Φ_s/Φ_d) from the chiral log. In our present accuracy, no clear chiral log effect is observed.

Figure 3 shows the QCD matrix elements $\mathscr{M}_q m_{B_q}$ in lattice units renormalized at m_b with $\overline{\rm MS}$, NDR as functions of m_{ud}. The $SU(3)_f$ breaking ratio $(\mathscr{M}_d/\mathscr{M}_s)\cdot(m_{B_d}/m_{B_s})$ is shown in Fig. 3 right. The expected downward (upward) curvature towards the chiral limit for $\mathscr{M}_d$ (ratio) from chiral perturbation theory [18] has not been observed here either. Linearly extrapolating to the physical point, $(\mathscr{M}_s/\mathscr{M}_d)\cdot(m_{B_s}/m_{B_d}) = 1.36(9)$ is obtained. Inputting the experimental masses $m_{B_d} = 5279$ MeV and $m_{B_s} = 5368$ MeV, one obtains $\xi = \sqrt{\mathscr{M}_s/\mathscr{M}_d}(m_{B_s}/m_{B_d}) = 1.16(4)$, where

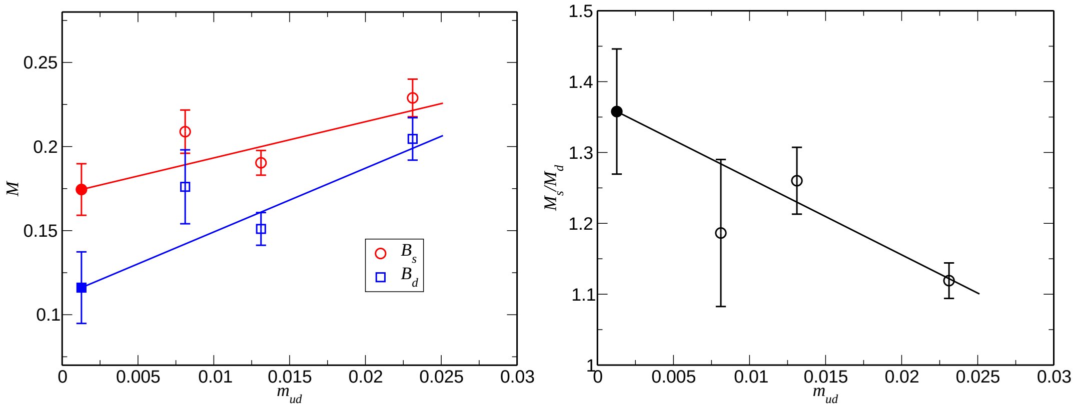

Figure 3: Left figure is for $B^0 - \overline{B^0}$ mixing matrix elements $M_q = \mathscr{M}_q m_{B_q}$ in lattice units renormalized at m_b with $\overline{\mathrm{MS}}$, NDR as functions of m_{ud}. Right shows the ratio M_s/M_d.

error is statistical only. This is consistent with our results [7] with larger masses and smaller volume: $1.11(7)$ for APE and $1.14(8)$ for HYP2 smearing for the links in the static quark action.

4. Summary and Outlook

We discussed on the on-going calculation of $B^0 - \overline{B^0}$ mixing for B_d and B_s with $2+1$ flavor dynamical domain wall fermions and static approximation for the b quark. The interpolation operator of B^0 meson with gauge invariant Gaussian smearing for both light and heavy quarks appeared to be optimal among the various operators tested. We argued that the operator mixing from explicit breaking of the chiral symmetry due to finite L_s is negligible to sub-percent level. Under the current statistical errors, the predicted chiral log is not visible for Φ_b and $\mathscr{M}_d$. Our lightest *ud* mass point about $m_{ud} \simeq m_s/4.6$ so far has poor statistics. Reducing the error of this point and also performing the calculation at partially quenched points would help to reduce the yet-to-be estimated systematic error of the chiral extrapolation.

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