Twisted mass QCD in the charm sector

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We present preliminary results for the charm quark mass m_c and the D and D_s mesons decay constants f_D and f_{D_s} from a lattice QCD calculation with $N_f = 2$ dynamical fermions. We use the twisted mass fermionic action defined at maximal twist so that physical quantities are automatically $\mathcal{O}(a)$ improved. Two lattice spacings are considered. The charm quark mass has been renormalised in the RI-MOM scheme. After a matching to the $\overline{\text{MS}}$ scheme, we obtain from the simulation at a fine lattice ($a \sim 0.09$ fm) $m_c^{\overline{\text{MS}}}(m_c) = 1.481 \pm 0.022 \pm 0.092$ GeV, $f_D = 205 \pm 13 \pm 17$ MeV, $f_{D_s} = 271 \pm 6 \pm 6$ MeV and from the simulation at the finer lattice ($a \sim 0.07$ fm) $m_c^{\overline{\text{MS}}}(m_c) = 1.474 \pm 0.041 \pm 0.132$ GeV, $f_D = 230 \pm 31 \pm 8$ MeV and $f_{D_s} = 264 \pm 5 \pm 8$ MeV. We chose three renormalisation conditions to determine m_c: the spread between the final results contributes to the systematic error. At both lattice spacings, particularly at the finer one, the error on m_c is dominated by present uncertainty on the renormalisation constant Z_P, which should be reduced before performing a reliable continuum limit.

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1. Introduction

The physics of charm bound states regained recently the attention of particle physicists with the discovery of the new resonances $X(3872)$, $X(3943)$, $Y(3940)$, $Y(4260)$ and $Z(3930)$ [1] and of a very narrow scalar state $D_s(2317)$ [2], whose composition is still an open question. The experimental evidence for oscillations in the $D^0 - \overline{D^0}$ system [3] might be the first signal for physics beyond the Standard Model in the charm sector. Even if it is expected that the long-distance physics is a dominant effect in that process, the $\Delta C = 2$ contribution to $x_D = \Delta M_D / \Gamma_D$ might be not negligible. It is given by a box diagram, as in the $B - \overline{B}$ system, and it is proportional to f_D^2. In the CKM matrix, V_{cs} is one of the elements having the largest uncertainty when one does not impose the 3×3 unitarity: $\frac{\Delta |V_{cs}|}{|V_{cs}|} = 9.82\%$ [4]: most of it comes from the theory. An appropriate way to extract V_{cs} is to measure the leptonic decay width $D_s \to l\bar{\nu}$, which however requires the estimation of the decay constant f_{D_s}. Eventually it is well established that V_{cb} can be constrained by analysing the inclusive semileptonic decay $\overline{B} \to X_c l\bar{\nu}$. An OPE is used in the Heavy Quark Expansion (HQE) and the total branching ratio reads [5] $\mathrm{BR}_{\mathrm{SL}}(\overline{B}) = \mathrm{BR}_{\mathrm{SL}}^{(0)}[1 + f(r)]$, $r = \left(\frac{m_c}{m_b}\right)^2$, where $\mathrm{BR}_{\mathrm{SL}}^{(0)}$ is the dominant term in the m_c/m_b expansion.

From those phenomenological aspects we conclude that it is important to determine as precisely as possible the decay constants f_D, f_{D_s} and the charm quark mass m_c. That quark is of course too heavy to make predictions by using Chiral Perturbation Theory (χPT) and too light to compute the amplitudes by using only the HQE: corrections of $\mathscr{O}(\Lambda_{QCD}/m_c)^n$ and $\mathscr{O}(m_c/m_b)^n$ might be larger than the precision of few % that we want to reach.

Lattice QCD is a good tool to study the charm sector. However the continuum limit can be difficult to reach because of large cut off effects (typically, $0.2 < am_c^0 < 0.4$). It is therefore crucial to improve the action and the currents regularised on the lattice. Several theories proposed in the literature so far have the common property that they require the tuning of a certain number of parameters to achieve improvement, by applying the Symanzik's program [6]. On the other side it has been shown [7] that Twisted mass QCD (TmQCD) [8, 9] requires the tuning of a single parameter (the untwisted mass m_0), so that hadronic quantities like the pseudoscalar meson masses and decay constants are automatically $\mathscr{O}(a)$ improved at maximal twist. Other nice properties of such an action are that the physical quark mass is related to the twisted mass parameter of the action, its renormalisation is only multiplicative and the pseudoscalar decay constant does not require the introduction of any renormalisation constant [9]: for two quark flavors 1 and 2 (for example a light flavor ℓ and a heavy flavor h) it is simply given by

$$f_{PS}(\mu_1, \mu_2) = \frac{\mu_1 + \mu_2}{m_{PS}^2(\mu_1, \mu_2)} |\langle 0|P^c(0)|P\rangle|, \quad P^c = \bar{\psi}_1(r)\gamma^5 \psi_2(-r), \tag{1.1}$$

where r is the Wilson parameter and we define the composite operator P^c in the physical basis.

We present a preliminary lattice QCD determination of the charm quark mass m_c and the decay constants f_D and f_{D_s}. We have performed full dynamical simulations for $N_f = 2$ light degenerate sea quarks; the strange and the charm quarks have been added in the valence sector. The calculation is based on the analysis of the gauge configurations ensembles $B_1 - B_5$ and $C_1 - C_4$ [10] (240 and 130 configurations of $B_1 - B_5$ and $C_1 - C_4$, respectively, have been analysed) which have been generated with the TlSym gauge action at $\beta = 3.9$ ($a = 0.0855(5)(31)$ fm) and $\beta = 4.05$ ($a =$
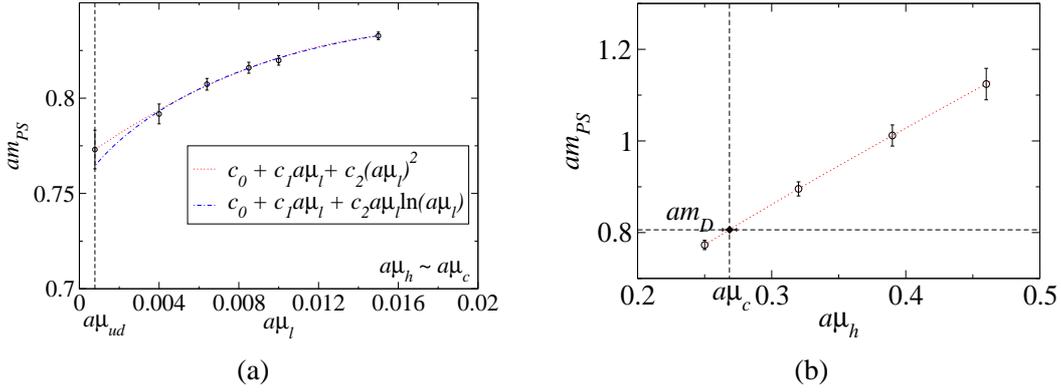
Figure 1: (a) Chiral extrapolation of the pseudoscalar heavy-light meson masses ($\beta = 3.9$). (b) Pseudoscalar heavy-light meson mass in function of the heavy quark mass ($\beta = 3.9$).

0.0667(5)(24) fm) respectively and the twisted mass fermionic action defined at maximal twist. The light quark masses are in the range $[m_s/6, 2m_s/3]$, to perform the chiral extrapolation, and we used masses around the strange mass and the charm mass to do the appropriate interpolations.

At each sea quark mass we have computed the two-point correlation functions of pseudoscalar mesons. Each measurement has been separated by 20 HMC trajectories, which is enough to avoid autocorrelation time effects. The statistical accuracy has been improved by using all to all stochastic propagators. Statistical errors on the meson masses and decay constants are evaluated at a given sea quark mass by using a jacknife procedure with 10 measurements discarded in each bin. The error obtained after a combination of data coming from simulations with different sea quark masses (i.e. statistically independent) is computed using a bootstrap method.

2. Charm quark mass

To estimate the charm quark mass we use the following strategy: we compute the pseudoscalar meson mass $m_{PS}(\mu_{\text{sea}}, \mu_\ell, \mu_h)$ (where ℓ and h are valence light and heavy quarks respectively) at the points $\mu_\ell = \mu_{\text{sea}}$, which allows us to extrapolate down to the physical light quark mass $\mu_{ud} \equiv \frac{\mu_u + \mu_d}{2}$, previously determined in [11]. Once the dependence on the light quark mass has been taken into account, one studies the dependence of m_{PS} on the heavy mass μ_h. The bare charm quark mass μ_c is determined by using the following condition: $m_{PS}(\mu_{ud}, \mu_c) = m_D$.

We have performed a quadratic extrapolation of $m_{PS}(\mu_\ell = \mu_{\text{sea}}, \mu_h)$ in μ_ℓ down to $\mu_\ell = \mu_{ud}$. This is illustrated in Figure 1 (left). We have also introduced a logarithmic dependence on μ_ℓ: $m_{PS}(\mu_\ell, \mu_h) = c_0(\mu_h) + a\mu_\ell[c_1(\mu_h) + c_2(\mu_h)\ln(a\mu_\ell)]$. As a third possibility we have done a simple linear extrapolation. The spread between these different fits is included in the systematic error at the end of the computation. We then performed a fit of m_{PS} as a function of μ_h: we used either a quadratic polynomial in μ_h, or a quadratic polynomial in $\frac{1}{\mu_h}$, or also, as a third ansatz, $m_{PS} = d_0 + d_1\mu_h + \frac{d_2}{\mu_h}$. We show in Figure 1 (right) the quality of the latter fit function, that appears to be the most appropriate to describe the data. Once the bare charm quark mass μ_c is extracted, we renormalise it in the RI-MOM scheme: $m_c^{\text{RI}-\text{MOM}} = Z_\mu^{\text{RI}-\text{MOM}} \mu_c$ where $Z_\mu^{\text{RI}-\text{MOM}} = 1/Z_P^{\text{RI}-\text{MOM}}$ in TmQCD [7]. Finally we perform a matching onto the $\overline{\text{MS}}$ scheme. We indicate in Table 1 the value of $m_c^{\overline{\text{MS}}}(m_c)$ for the two lattice spacings, knowing that $Z_P^{\text{RI}-\text{MOM}}(\beta = 3.9, 1/a) = 0.39(1)(2)$
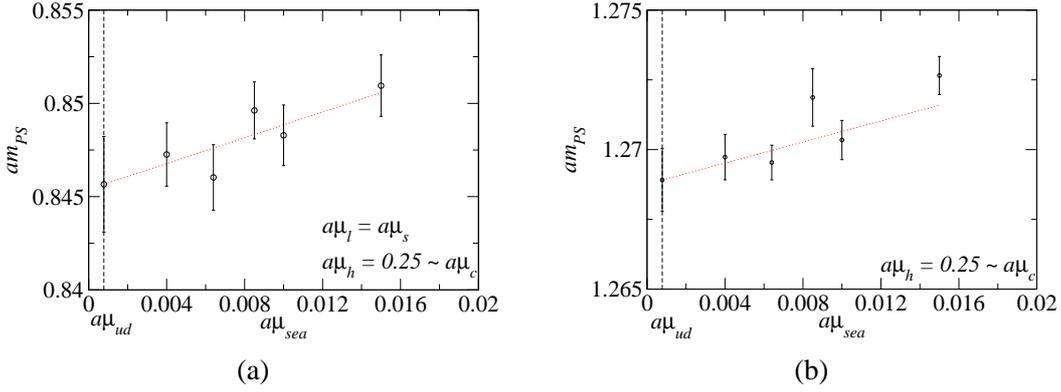
Figure 2: (a) Chiral extrapolation of $m_{PS}(\mu_{\text{sea}}, \mu_\ell = m_s, \mu_h \sim m_c)$ ($\beta = 3.9$). (b) Chiral extrapolation of $m_{PS}(\mu_{\text{sea}}, \mu_h, \mu_h)$ ($\beta = 3.9$).

and the *preliminary* value of $Z_P^{\text{RI}-\text{MOM}}(\beta = 4.05, 1/a) = 0.40(1)(4)^1$ [12]. The first error on m_c is statistical, the second is the systematic error coming from Z_P, the third comes from the uncertainty on a and the last one is the systematic error from the chiral extrapolation. In that table we have also collected the result of two other determinations of m_c, by using the following renormalisation conditions:

1) $m_{PS}(\mu_{\text{sea}} = \mu_{ud}, \mu_\ell = \mu_s, \mu_c) = m_{D_s}$ and 2) $m_{PS}(\mu_{\text{sea}} = \mu_{ud}, \mu_c, \mu_c) = m_{\eta_c}$.

μ_s is the bare strange quark mass which has been determined in [14]. The heavy-heavy pseudoscalar meson correlator has been computed by using the interpolating field $\bar{\psi}_h(r)\gamma^5\psi_h(r)$.

The dependence of those two observables on the sea quark mass is very weak, as shown in Figure 2. We used a linear fit in μ_ℓ and μ_{sea} to interpolate to μ_s and to extrapolate down to μ_{ud} respectively:

$$m_{PS}(\mu_{\text{sea}}, \mu_\ell, \mu_h) = p_0(\mu_h) + a\mu_\ell p_1(\mu_h) + a\mu_{\text{sea}}[p_2(\mu_h) + a\mu_\ell p_3(\mu_h)],$$

$$m_{PS}(\mu_{\text{sea}}, \mu_h, \mu_h) = q_0(\mu_h) + a\mu_{\text{sea}} q_1(\mu_h).$$

The uncertainty coming from the chiral extrapolation is quite reduced compared to $m_{PS}(\mu_{\text{sea}} = \mu_\ell, \mu_h)$. However we introduce a small uncertainty from μ_s on the first observable (third error on $m_c^{\overline{\text{MS}}}(m_c, m_{D_s})$) and from the disconnected diagram which contributes to the second but that we did not compute. The last error on $m_c^{\overline{\text{MS}}}(m_c, m_{D_s})$ and $m_c^{\overline{\text{MS}}}(m_c, m_{\eta_c})$ comes from the uncertainty on a. We have not performed a continuum limit extrapolation yet because only 2 lattice spacings have been considered so far and the uncertainty on Z_P is still rather large, especially at $\beta = 4.05$. Concerning cut off effects, it is remarkable that they appear to be rather weak on the unrenormalised charm mass at $\beta = 4.05$: indeed, the value of μ_c extracted from the 3 observables are much closer at this β than at $\beta = 3.9$.

To conclude this section we note that our values of m_c are large with respect to most of the recent lattice estimations [15]: however the currently large uncertainty on Z_P at $\beta = 4.05$ implies that *any conclusion about the continuum limit result would be untimely.*

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[1] At this β a chiral extrapolation has been performed in the valence sector, at $a\mu_{\text{sea}} = 0.003$, but not yet in the sea sector. However it was found at $\beta = 3.9$ that Z_P depends only weakly on the sea quark mass. Thus as a first step of the analysis we will include the sea effects in the systematic uncertainty. Moreover, an alternative estimate of Z_P at $\beta = 4.05$ may come from scaling as described in [13], which brings our final estimate of the systematic error to 0.04.
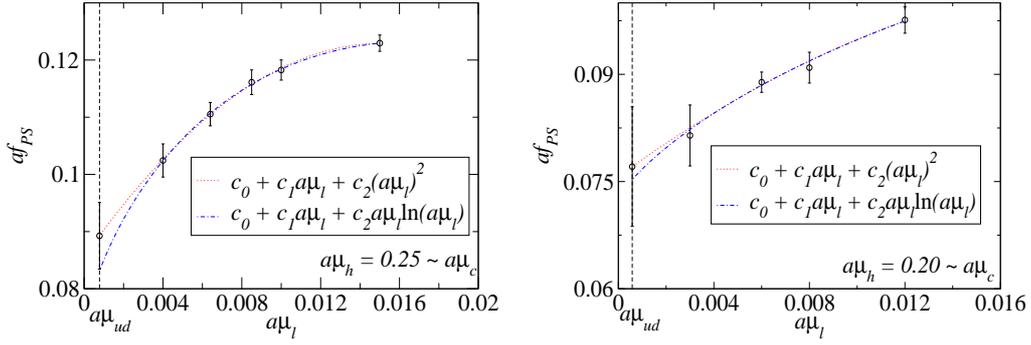
Figure 3: Chiral extrapolation of $f_{PS}(\mu_\ell = \mu_{\text{sea}}, \mu_h \sim \mu_c)$ at $\beta = 3.9$ (left) and $\beta = 4.05$ (right).

3. Heavy-light meson decay constants

To determine f_D and f_{D_s} we employ the same strategy as in the previous section, using eq. (1.1). We show in Figure 3 the chiral extrapolation of $f_{PS}(\mu_{\text{sea}} = \mu_\ell, \mu_h)$ at $\mu_h \sim \mu_c$ down to the physical light quark mass. We found that introducing a quadratic term in the extrapolation improves the fit, particularly for the coarse lattice. As before, we also introduced also a logarithmic dependence on μ_ℓ in the fit. We include the spread between the different chiral extrapolations in the systematic error.

At $\mu_\ell \sim \mu_s$, $f_{PS}(\mu_{\text{sea}}, \mu_\ell, \mu_h)$ has a similar linear dependence on μ_ℓ to the one of $m_{PS}(\mu_{\text{sea}}, \mu_\ell, \mu_h)$. Moreover for both the dependence on μ_{sea} is weak and very well described by a linear fit as well. We performed a fit of $f_{PS}\sqrt{m_{PS}}$ with a quadratic polynomial in $\frac{1}{m_{PS}}$ (we can not isolate any logarithmic dependence on $\alpha_s(m_{PS})$ in our range of heavy masses): $f_{PS}\sqrt{m_{PS}} = f_0 + \frac{f_1}{m_{PS}} + \frac{f_2}{m_{PS}^2}$. We show in Figure 4 the quality of the fit for $f_{PS}\sqrt{m_{PS}}(\mu_{ud}, \mu_h)$.

We give in Table 2 our values of f_D, f_{D_s} and f_{D_s}/f_D. The first error is a statistical error, the second error on f_{D_s} and on f_{D_s}/f_D comes from the uncertainty on the bare strange quark mass, the second error on f_D and the third error on f_{D_s} come from the uncertainty on the lattice spacing. The last error on f_D and f_{D_s}/f_D comes from the spread between different chiral fits. We have collected the most recent lattice estimations of those quantities in Figure 5 [16]. On the experimental side

β	$m_c^{\overline{\text{MS}}}(m_c, m_D)$	$m_c^{\overline{\text{MS}}}(m_c, m_{D_s})$	$m_c^{\overline{\text{MS}}}(m_c, m_{\eta_c})$
3.9	1.481(22)(63)(8)(27) GeV	1.450(12)(61)(10)(15) GeV	1.420(5)(60)(6) GeV
4.05	1.474(41)(129)(15)(5) GeV	1.498(6)(130)(12)(18) GeV	1.479(2)(129)(8) GeV

Table 1: Charm quark mass fixed by using different observables.

β	3.9	4.05
f_D	205(13)(3)(17) MeV	230(31)(6)(6) MeV
f_{D_s}	271(6)(4)(5) MeV	264(5)(4)(7)
$\frac{f_{D_s}}{f_D}$	1.35(4)(1)(7)	1.13(28)(2)(2)

Table 2: Decay constants f_D and f_{D_s} and f_{D_s}/f_D from our simulation

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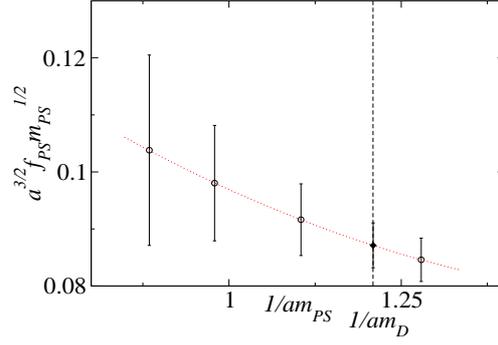
Figure 4: Scaling law of $f_{PS}\sqrt{m_{PS}}(\mu_{ud},\mu_h)$ as a function of $1/m_{PS}(\mu_{ud},\mu_h)$ at $\beta = 3.9$.
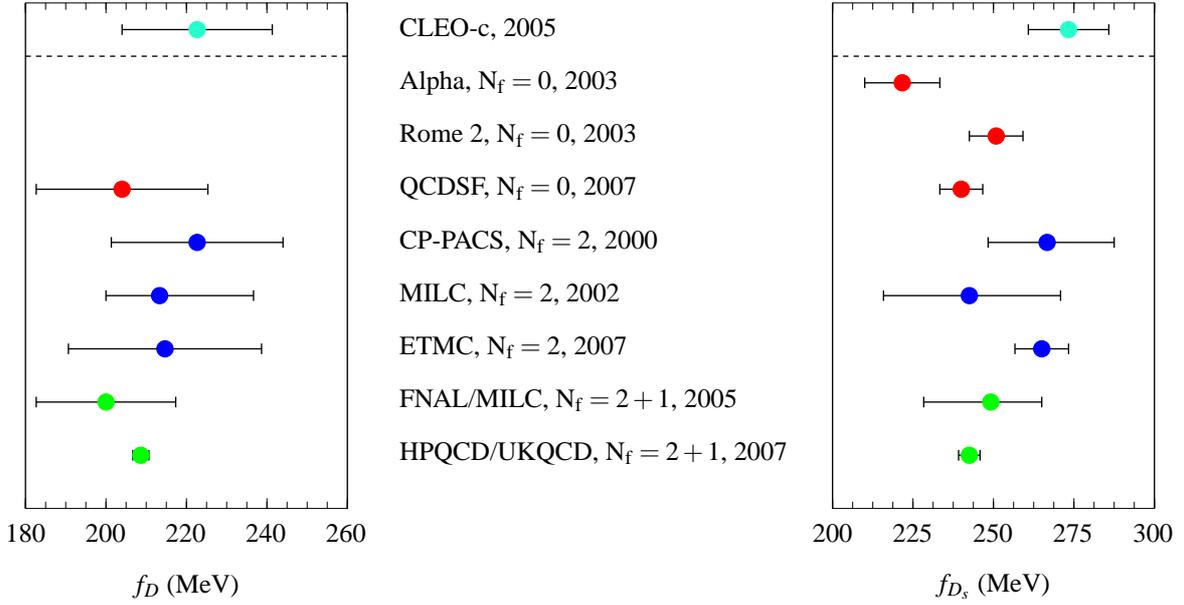
Figure 5: Recent lattice computations and experimental measurements of the decay constants f_D (left panel) and f_{D_s} (right panel). The ETMC values that we indicate are obtained by doing an average of the data at $\beta = 3.9$ and $\beta = 4.05$.

CLEO-c measured $f_D = 222.6 \pm 16.7^{+2.8}_{-3.4}$ MeV [17], $f_{D_s} = 274 \pm 13 \pm 7$ MeV [18], which is a combination of analysis of $D_s \to \mu$ and $D_s \to \tau$ leptonic decays. Note that BABAR measured $f_{D_s} = 283 \pm 17 \pm 7 \pm 14$ MeV [19].

4. Summary

We have presented preliminary results of a lattice computation of the charm quark mass m_c and the D and D_s mesons decay constants f_D and f_{D_s} by using the $N_f = 2$ TmQCD action defined at maximal twist. Encouraging results are found concerning cut off effects. Indeed, the bare charm quark mass extracted from 3 different observables looks consistent at the finer lattice. However, before performing the continuum limit on m_c, f_D and f_{D_s}, we still have to reduce as much as possible the uncertainty on the renormalisation constant Z_P and to increase the statistics at $\beta = 4.05$. A more detailed study of the light quark dependence of the D mesons masses and decay constants,

based on heavy-light chiral perturbation theory, is still missing at this stage. Finally a better control on the continuum limit extrapolation will come from the on-going analysis of the data at a coarser lattice ($a \sim 0.1$ fm).

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