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B_s and B_d mixing in full lattice QCD using NRQCD b quarks

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We give a progress report on studies of B_s and B_d mixing with valence NRQCD b quarks and asqtad light quarks on the MILC configurations including the effect of 2+1 flavours of sea quarks. We explore methods for reducing statistical and systematic errors in the ratio $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$.

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1. Introduction

The precise determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can impose important constraints on physics beyond the Standard Model (SM). One combination of CKM matrix elements that plays a relevant role in this analysis is $\left|\frac{V_{td}}{V_{ts}}\right|$, which is related to $B_0 - \bar{B}_0$ mixing.

In particular, this combination of CKM matrix elements can be extracted from the precisely experimentally measured quantities ΔM_s and ΔM_d, which are the mass differences between the heavy and light mass eigenstates in the $B^0_s - \bar{B}^0_s$ and $B^0_d - \bar{B}^0_d$ systems respectively. The relation is given by

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}. \tag{1.1}$$

The masses M_{B_s} and M_{B_d}, and the corresponding mass differences are known experimentally with very high precision [1]. For the ratio $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$, however, an accurate and consistent lattice calculation that fully incorporates vacuum polarization effects is not yet available. Our goal is to perform such a calculation and reduce the theoretical errors in the ratio ξ to a few percent. This will provide us with a high precision determination of the CKM ratio in (1.1).

The products of B_0 decay constants and bag parameters in (1.1) are determined by matrix elements between B_0 and $\bar{B}_0$ of the four-fermion operators appearing in the effective hamiltonian that describes $\Delta B = 2$ processes. The non-perturbative inputs for the calculation of $\Delta\Gamma_s$ and $\Delta\Gamma_d$ (with $\Delta\Gamma$ the width difference between the light and heavy mass eigenstate) are also given by this kind of hadronic matrix elements. For completeness, we are studying all the matrix elements needed to make theoretical predictions for ΔM_s, ΔM_d, $\Delta\Gamma_s$ and $\Delta\Gamma_d$.

2. Simulation details and milestones in the calculation

The four-fermion operators whose matrix element between B_0 and $\bar{B}_0$ are needed to make a complete study of B^s_0 and B^d_0 mixing in the SM are

$$OL^q \equiv \left[\bar{b}^i q^i\right]_{V-A}\left[\bar{b}^j q^j\right]_{V-A}; \qquad OS^q \equiv \left[\bar{b}^i q^i\right]_{S-P}\left[\bar{b}^j q^j\right]_{S-P};$$
$$O3^q \equiv \left[\bar{b}^i q^j\right]_{S-P}\left[\bar{b}^j q^i\right]_{S-P};$$
$$OLj1^q \equiv \tfrac{1}{2M}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}q^i]_{V-A}[\overline{b^j}\,q^j]_{V-A} + [\overline{b^i}\,q^i]_{V-A}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}q^j]_{V-A}\right\}; \tag{2.1}$$

with q being a strange or a down quark, and i, j colour indices. The last operator, as well as similar $1/M$ corrections $OSj1^q$ and $O3j1^q$ for the OS^q and $O3^q$ operators, are required at $\mathscr{O}(\Lambda_{QCD}/M)$.

The continuum matrix elements $\langle OX\rangle(\mu)^{\overline{MS}} \equiv \langle\bar{B}^q_0|OX|B^q_0\rangle^{\overline{MS}}(\mu)$ of the operators $OX = OL^q$, OS^q, $O3^q$ entering in the SM formulae, are related to those evaluated via lattice simulations by a perturbative one-loop matching relation through $\mathscr{O}(\alpha_s)$, $\mathscr{O}(\Lambda_{QCD}/M)$ and $\mathscr{O}(\alpha_s/(aM))$. The matching relations mix, already in the continuum, the four-fermion operators in (2.1) -see [2, 3] for the explicit expressions.

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$m_{light}^{sea}/m_s^{phys.}$	Volume	N_{confs}	$a(fm)$	am_b	$m_q^{val.}/m_s^{phys.}$	$N_{sources}$
0.5	$20^3\times 64$	486	0.12	2.8	1	1
					0.5	2
0.25	$20^3\times 64$	568	0.12	2.8	1	1
					0.25	2
0.175	$20^3\times 64$	402	0.12	2.8	1, 0.175	4
0.125	$24^3\times 64$	678	0.12	2.8	1, 0.125	4
0.2	$28^3\times 96$	563	0.09	1.95	1, 0.2	4

Table 1: Simulation parameters for the coarse (first four sea masses) and fine lattices (last line) for B_0^s ($m_q^{val.}/m_s^{phys.}=1$) and B_0^d ($m_q^{val.}=m_d^{sea}$).

The bare hadronic matrix elements are obtained by numerically evaluating the three-point and two-point correlation functions

$$C^{(4f)}(t_1,t_2)=\sum_{\vec{x}_1,\vec{x}_2}\langle 0|\Phi_{\bar{B}_q}(\vec{x}_1,t_1)\left[\hat{Q}\right](0)\Phi_{\bar{B}_q}\dagger(\vec{x}_2,-t_2)|0\rangle$$
$$C^{(B)}(t)=\sum_{\vec{x}}\langle 0|\Phi_{\bar{B}_q}(\vec{x},t)\Phi^\dagger_{\bar{B}_q}(\vec{0},0)|0\rangle \qquad (2.2)$$

with $\Phi_{\bar{B}_q}(\vec{x},t)=\bar{b}(\vec{x},t)\gamma_5 q(\vec{x},t)$ and $\hat{O}$ any of the four-fermion operators in (2.1). The simulations are performed on MILC configurations with $N_f=2+1$ sea quarks. The valence b fields are described by the NRQCD action improved through $\mathscr{O}(1/M^2)$, $\mathscr{O}(a^2)$ and leading relativistic $\mathscr{O}(1/M^3)$ [4], while the light valence (and sea) quarks are staggered asqtad fields [5]. An improved gluon action is also used to further reduce discretization errors.

The action parameters are fixed via light and heavy-heavy simulations, in particular the valence b and s quark masses are tuned to give the physical values of the Υ and K mesons. The different parameters in the simulations are collected in Table 1.

2.1 Mixing parameter for B_0^s mixing

Our work published in [2] analyzes the B_s^0 mixing parameters for two ensembles of MILC configurations with $(m_u^{sea}=m_d^{sea})/m_s=0.25, 0.50$ and $a=0.12fm$ (coarse lattice). This corresponds to the first two entries in Table 1 with $m_q^{val.}/m_s^{phys.}=1$.

The results obtained for the mass and width differences when using these parameters in the SM expressions are

$$\Delta M_s = 20.3(3.0)(0.8)ps^{-1} \quad \text{and} \quad \Delta\Gamma_s=0.10(3)ps^{-1}\,, \qquad (2.3)$$

which agree with experimental results within errors. The first error in ΔM_s, which is the dominant one, is from the lattice determination of $f^2_{B_s}B_{B_s}$ through the definition

$$\langle OL\rangle^{\overline{MS}}_{(\mu)}\equiv\langle\overline{B}_s|OL|B_s\rangle^{\overline{MS}}_{(\mu)}\equiv\frac{8}{3}f^2_{B_s}B_{B_s}(\mu)M^2_{B_s}\,, \qquad (2.4)$$

and the second one is an estimate of the error from $|V^*_{ts}V_{tb}|$ and $\overline{m_t}$. This 15% lattice error is dominated by a 9% statistics+fitting error and a 9% uncertainty associated with higher order operator matching. The large statistical errors are due to the fact that the simultaneous fits of two-point and three-point functions are unstable and we need to constrain the two-point parameters using the values obtained in fits to only two-point correlators.

The stability of the fits can be improved by using smearing techniques that reduce the overlap with excited states. We checked that the statistical+fitting error can be reduced from 9% down to as low as 2% by smearing the heavy quark in the two-point functions and further improvement is achieved by smearing also in the three-point functions, as described in the next section.

3. New results for B_0^s and B_0^d mixing parameters

We have generated two-point functions with both local and smeared sources and sinks, using a smearing we call *1S* since it takes an exponential form. Our three-point functions are local at the source (the site of the 4-quark operator), and have both local and smeared sinks at either end, with the same smearing as in the two-point case. This reduces the statistical+fitting error in our analysis. The general definition of these two-point and three-point functions is given in (2.2).

In addition to the matrix elements relevant in the determination of B_0^s mixing parameters, we have also calculated those corresponding to B_0^d mixing in full QCD. The s and b valence quarks masses are the physical ones, while the d valence quark mass is the same as m_d^{sea} for any ensemble. Two different lattice spacings have been studied, the MILC coarse lattice ($a = 0.12$) and the MILC fine lattice (a=0.09). On the first one we have the correlation functions calculated for four different values of the light sea quark masses and on the second one, so far we have results only for one light sea quark mass. The parameters of the simulations, quark masses, number of configurations, number of time sources, etc, are shown in Table 1.

We have not analyzed yet the $1/M$ corrections for all the data collected in Table 1, so the results presented in these proceedings are only coming from the dominant contribution in the $1/M$ expansion. We are also still working on the fits with the lightest sea mass on the coarse ensemble, $m_q = 0.005$, and results for this point will be presented elsewhere [6].

3.1 Reduction of statistical+fitting errors

The use of several time sources and smearing greatly reduce the statistical errors as can be seen in Figure 1. In that Figure, as an example of that reduction, we compare the results in our previous paper [2] for $f_{B_s}\sqrt{\hat{B}_{B_s}}(GeV)$ with our new results incorporating smeared correlation functions in the fits and new data. $f_{B_s}\sqrt{\hat{B}_{B_s}}(GeV)$ is plotted as a function of the light sea quark mass over the physical strange quark mass, m_q/m_s, and the errors are only statistical.

With these new data we are able to get stable simultaneous fits for two-point and three-point correlation functions without any constraints in the two-point function parameters. With stable we mean that the central values, errors and $\chi^2/ndof$ do not change when we add more excited states in the functional forms to be fitted. The result is a reduction of statistical errors from 4.5% to 1-2% in $f_{B_s}\sqrt{\hat{B}_{B_s}}(GeV)$, and similarly for $f_{B_d}\sqrt{\hat{B}_{B_d}}$.

Another technique that could reduce further the size of statistical errors is the use of random wall sources for the light propagators. We have already checked that the statistical errors in the B_s^0 two-point parameters are improved by a factor of two, comparing results for the same heavy-light

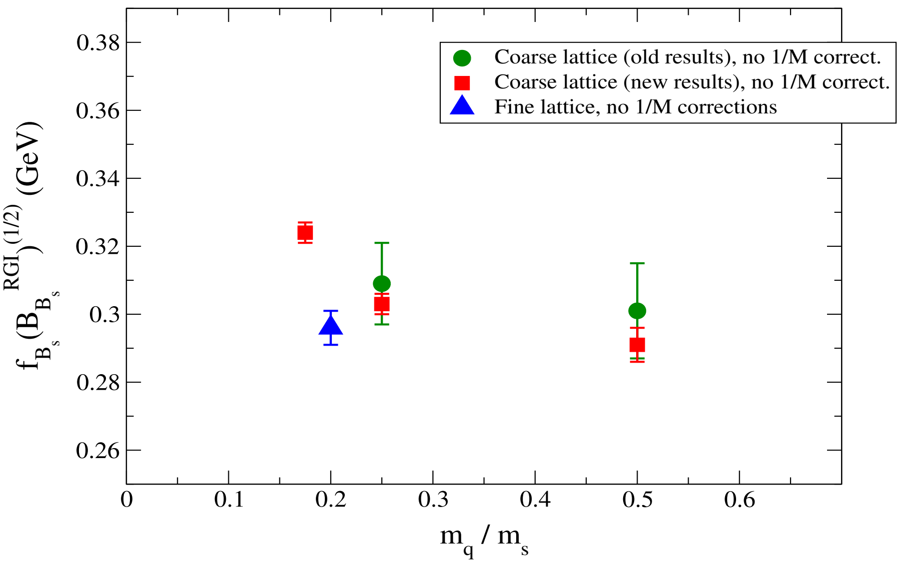

Figure 1: $f_{B_s}\sqrt{\hat{B}_{B_s}}(GeV)$. Errors are only statistical.

correlators we are using here but with HISQ [7] (Highly Improved Staggered Quarks) light valence quarks, with and without random wall sources -see [8] for more details about using random wall sources in heavy(NRQCD)-light(HISQ) correlators. Further study is needed to find how the use of this kind of source affect the three-point function parameters relevant for B^0 mixing.

3.2 Calculation of the ratio ξ

Some of the errors affecting the calculation of $f_{B_q}\sqrt{B_{B_q}}$ will cancel almost completely and others partially in the ratio $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$. In Figure 2 we show values for this ratio multiplied by the square root of the masses of the B_0^s and B_0^d mesons,

$$\frac{X_s}{X_q} = \frac{f_{B_s}\sqrt{B_{B_s}M_{B_s}}}{f_{B_q}\sqrt{B_{B_q}M_{B_q}}}, \tag{3.1}$$

together with the ratio $\Phi_s/\Phi_q = \frac{f_{B_s}\sqrt{M_{B_s}}}{f_{B_q}\sqrt{M_{B_q}}}$, without the bag parameters from [9]. The results are plotted as a function of $m_q/m_s^{phys.}$, where $m_q = m_d^{valence} = m_d^{sea}$.

The errors for X_s/X_q in Figure (2), which are only statistical, are larger than those for Φ_s/Φ_q because we have not yet taken into account the correlations between the data in the numerator and denominator in this ratio. We expect to reduce this error to less than 2% when these correlations are included (the current plotted values have 2.5% errors). Another error that should be significantly reduced is that for the fine lattice point since we have not yet included all of our data.

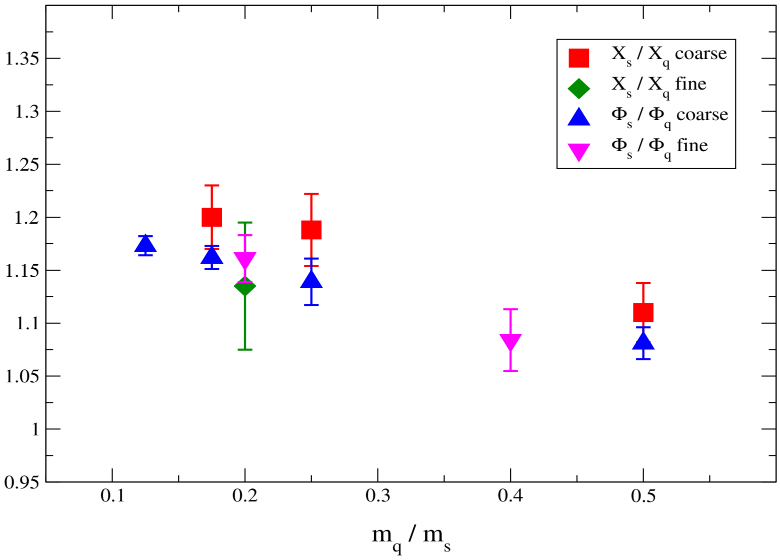

Figure 2: $\frac{X_s}{X_q} = \frac{f_{B_s}\sqrt{B_{B_s}M_{B_s}}}{f_{B_q}\sqrt{B_{B_q}M_{B_q}}}$ and $\Phi_s/\Phi_q = \frac{f_{B_s}\sqrt{M_{B_s}}}{f_{B_q}\sqrt{M_{B_q}}}$ as a function of the light valence quark mass in the denominator. Errors are only statistical in all the quantities plotted.

The statistical errors are not the only ones to be reduced by taking the ratio. Discretization, relativistic and higher order operator matching will affect $f_{B_s}\sqrt{B_{B_s}}$ and $f_{B_d}\sqrt{B_{B_d}}$ in the same way and largely will cancel in the ratio. One expects their effects to come in at the level of the corresponding error in $f_{B_q}\sqrt{B_{B_q}}$ times $a(m_s - m_d)$ or $(m_s - m_d)/\Lambda_{QCD}$. The results for f_{B_s}/f_{B_d} are nearly unchanged when adding one-loop and $1/M$ corrections [9] and we expect something similar here. We have already checked that the difference between tree level and one-loop results is less than 1%. The scale a^{-3} uncertainties, that lead to a 5% error in $f^2_{B_q}B_{B_q}$, do not affect the ratio ξ.

The next step in our calculation will be to carry out a chiral extrapolation of these results to the physical point including the effect of taste-changing errors, to account for the remaining systematic in the calculation and remove the dominant light discretization errors.

4. Summary and future work

We have calculated the mixing parameters in the B^0_s and B^0_d systems for two different lattice spacings and five different light quark masses. The statistical errors have been reduced from our previous work by a factor of 2-3, so statistics is no longer a dominant source of uncertainty in the calculation of $f^2_{B_q}B_{B_q}$. The largest error is now the uncertainty associated with the perturbative matching, that is also reduced from 9% to 6.5% by simulating on finer lattices. Further reduction of this source of error, as well as discretization errors, would also be possible by the use of MILC superfine lattices.

We also give preliminary results for the ratio ξ versus m_q/m_s, where many theoretical uncertainties are partially or completely cancelled between denominator and numerator.

The analysis of $1/M$ corrections and results for $m_d/m_s = 0.125$ and at least one other light quark mass on the fine lattice, will be presented in a forthcoming publication [6]. We are also exploring different smearings and better fitting approaches to further reduce the statistical errors. In particular, we are getting promising preliminary results using random wall sources for the light propagators.

Once other sources of errors have been reduced, we need to perform a chiral extrapolation of the $f_B\sqrt{B_B}$ and ξ results incorporating light discretization uncertainties (taste-changing errors) and perturbative errors. We will also be able to perform a continuum extrapolation, since we have results for two different values of the lattice spacing.

Other talks on unquenched calculations of B_0 mixing parameters in this conference can be found in [10].

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