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The Static Approximation to B Meson Mixing using Light Domain-Wall Fermions: Perturbative Renormalization and Ground State Degeneracies

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We discuss the theoretical input into the current RBC-UKQCD calculation of $f_{B_{d,s}}$ and $B_{B_{d,s}}$ using a smeared static heavy quark propagator, light domain-wall quarks and the Iwasaki gauge action. We present the complete one-loop, mean-field improved matching of heavy-light current and four-fermion lattice operators onto the static continuum theory renormalized in $\overline{\mathrm{MS}}$(NDR). The large degeneracies present in a static calculation are addressed, and a method for extracting f_B and B_B using only box sources is described; implications for future calculations are discussed.

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1. Introduction

Precision measurements of the CKM matrix put the Standard Model to a stringent test and constrain possible physics beyond it. Using the measured frequency of $B_q - \overline{B}_q$, $q \in \{d,s\}$ oscillations to determine the CKM matrix elements $|V_{tq}|$ requires a reliable lattice calculation of the non-perturbative $B_q - \overline{B}_q$ mixing matrix elements $\frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$. A $2+1$ flavor, unquenched calculation of f_{B_q} and B_{B_q} has been carried out by the RBC-UKQCD collaboration in the infinite heavy quark mass limit using light domain-wall fermions on a $(2\text{ fm})^3$ spatial volume [1, 2]; this is currently being extended to a $(3\text{ fm})^3$ spatial volume and towards physical light quark masses [3]. In the following, we discuss the perturbative lattice-continuum matching of the operators relevant for the RBC-UKQCD calculation, following in part the detailed discussion in Refs. [2, 4]. We also point out the subtle degeneracy of heavy-light meson ground states, and discuss its implications for the extraction of f_B and B_B from lattice correlation functions.

2. Action and Feynman Rules

The heavy b quark is described by an improved lattice version of the static limit of heavy quark effective theory with smeared, SU(3)-projected gauge links $\overline{V}_0(\vec{x},t)$ to reduce noise:

$$S_{\text{static}} = \sum_{\vec{x},t} \overline{h}(\vec{x},t+a) \left[h(\vec{x},t+a) - \overline{V}_0^\dagger(\vec{x},t) h(\vec{x},t) \right]. \tag{2.1}$$

The SU(3) projection (discussed in Ref. [2]) simplifies perturbative calculations by allowing the smeared gauge links to be expanded in terms of an effective gauge field $B_0^a(\vec{x},t)$; in momentum space $B_0^a(q) = h_\mu(q) A_\mu^a(q)$, where $A_\mu^a(q)$ is the physical gauge field and $h_\mu(q)$ is a form factor depending on the smearing scheme. We focus on one of the two schemes used in the RBC-UKQCD calculation (one-level APE blocking with parameter $\alpha = 1$), resulting in a heavy quark gluon vertex

$$Y_\mu^a(k,k') = -ig_0 T^a \delta_{\mu 0} e^{-i(k_0+k_0')/2} \longrightarrow \overline{Y}_\mu^a(k,k') = -ig_0 T^a h_\mu(q) e^{-i(k_0+k_0')/2}, \tag{2.2}$$

where g_0 is the bare lattice coupling, q is the gluon momentum, and $h_\mu(q)$ is given by

$$h_\mu(q) = (h_0(q), h_j(q)) = \left(1 - \frac{2}{3} \sum_{l=1}^{3} \sin^2\left(\frac{q_l}{2}\right), \frac{2}{3} \sin\left(\frac{q_0}{2}\right) \sin\left(\frac{q_j}{2}\right) \right). \tag{2.3}$$

The heavy quark two-gluon vertex and the heavy quark propagator are given in Ref. [4].

The light quarks are described by the domain-wall fermion action. Each light flavor is represented by a $(4+1)$-dimensional Wilson-style fermion field $\psi_s(\vec{x},t)$ where $1 \le s \le N$ labels the coordinate in the fifth dimension. The physical quark field $q(\vec{x},t)$ is constructed from chiral surface states at $s = 1$ and $s = N$ via $q(\vec{x},t) = P_R \psi_1(\vec{x},t) + P_L \psi_N(\vec{x},t)$. The domain-wall height M_5 is a fixed parameter of the theory; we set $M_5 = 1.8$ to match the RBC-UKQCD calculation. A detailed description of domain-wall fermions and their perturbative treatment for our choice of gauge action is given in Ref. [4] and references therein, especially Ref. [5]. In the perturbative calculation the light quark masses were set to zero and the size N of the fifth dimension was taken to be large, resulting in an exact chiral symmetry as $N \to \infty$. The gluons were described by the Iwasaki gauge action, whose Feynman rules are given in Ref. [4].

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3. Perturbative Lattice-Continuum Matching at One-Loop

The full QCD operators relevant for the extraction of f_B and B_B, defined in $\overline{\text{MS}}$(NDR) at the scale $\mu_b = m_b$ of the b quark mass, are the axial vector current $A_\rho = \bar{b}\gamma_\rho\gamma_5 q$ and the parity-even part of the $\Delta B = 2$ vector-axial four-quark operator:

$$\left[\bar{b}\gamma^\rho(1-\gamma_5)q\right]\left[\bar{b}\gamma_\rho(1-\gamma_5)q\right] \to O_{VV+AA} = \left(\bar{b}\gamma^\rho q\right)\left(\bar{b}\gamma_\rho q\right) + \left(\bar{b}\gamma^\rho\gamma_5 q\right)\left(\bar{b}\gamma_\rho\gamma_5 q\right). \tag{3.1}$$

We match these operators at the scale μ_b to lattice operators in the static effective theory (described in Sec. 2) at the lattice scale a^{-1} via the continuum version of the static effective theory renormalized at a scale μ. Throughout our one-loop calculation we choose to set $\mu = a^{-1}$; in the RBC-UKQCD calculation, the lattice scale is given by $a^{-1} = 1.62$ GeV. The full QCD operators are related to continuum static operators by

$$A_\rho(\mu_b) = C_A(\mu_b,\mu)\widetilde{A}_\rho(\mu) + \mathscr{O}(\Lambda_{\text{QCD}}/\mu_b), \tag{3.2}$$

$$O_{VV+AA}(\mu_b) = Z_1(\mu_b,\mu)\widetilde{O}_{VV+AA}(\mu) + Z_2(\mu_b,\mu)\widetilde{O}_{SS+PP}(\mu) + \mathscr{O}(\Lambda_{\text{QCD}}/\mu_b). \tag{3.3}$$

In terms of the static quark and antiquark fields $h^{(\pm)}(x) = e^{\pm i m_b v\cdot x}(1 \pm \not{v})b(x)/2$ and for $m_b \to \infty$,

$$\widetilde{A}_\rho = \bar{h}^{(+)}\gamma_\rho\gamma_5 q, \tag{3.4}$$

$$\widetilde{O}_{VV+AA} = 2\left(\bar{h}^{(+)}\gamma^\rho q\right)\left(\bar{h}^{(-)}\gamma_\rho q\right) + 2\left(\bar{h}^{(+)}\gamma^\rho\gamma_5 q\right)\left(\bar{h}^{(-)}\gamma_\rho\gamma_5 q\right), \tag{3.5}$$

$$\widetilde{O}_{SS+PP} = 2\left(\bar{h}^{(+)} q\right)\left(\bar{h}^{(-)} q\right) + 2\left(\bar{h}^{(+)}\gamma_5 q\right)\left(\bar{h}^{(-)}\gamma_5 q\right). \tag{3.6}$$

The static effective action discussed in Sec. 2 describes $h^{(+)}$ with $v = (1,\vec{0})$, corresponding to a stationary meson. The constants $C_A(\mu_b,\mu)$ and $Z_{1,2}(\mu_b,\mu)$ are known at one-loop; they are summarized in Ref. [2]. Using the latest PDG values for $\alpha_s^{\overline{\text{MS}}}(m_Z)$ and m_b, and running the coupling down at four-loops with the physical number of flavors to determine $\alpha_s^{\overline{\text{MS}}}(\mu_b)$ and $\alpha_s^{\overline{\text{MS}}}(\mu)$ we obtain $C_A = 1.057$, $Z_1 = 0.934$, $Z_2 = -0.151$.

We now describe the matching $\widetilde{A}_\rho(\mu) = \widetilde{C}_A(\mu, a^{-1})a^{-3}A_\rho^{\text{lat}}$ of the heavy-light axial currents $\widetilde{A}_\rho(\mu)$ and A_ρ^{lat} (which is dimensionless) in the continuum and lattice versions of the static effective theory. Results for the four-fermion operators are summarized at the end of this section. We compare the correlation function $\langle(\bar{h}(x)\Gamma q(x))h(y)\bar{q}(z)\rangle$ in both theories; in this discussion only one heavy quark field $h^{(+)} \equiv h$ enters. For the axial current $\Gamma = \gamma_\rho\gamma_5$, but the light quark chiral symmetry and the heavy quark spin symmetry $h \to e^{-i\phi_j\varepsilon_{jkl}\sigma_{kl}}h$ of both the continuum and the lattice theory render the matching Γ-independent. At one-loop and for small external quark momenta $p \simeq 0$ the continuum and lattice correlation functions are

$$\langle(\bar{h}\Gamma q)h\bar{q}\rangle = \frac{Z_h}{ip_0}\Gamma(1+\delta V)\frac{Z_2}{i\not{p}}, \quad \langle(\bar{h}\Gamma q)h\bar{q}\rangle_{\text{lat}} = \frac{Z_h^{\text{lat}}}{ip_0}\Gamma(1+\delta V^{\text{lat}})\frac{(1-w_0^2)Z_w Z_2^{\text{lat}}}{i\not{p}}, \tag{3.7}$$

where the Feynman diagrams contributing at one-loop are shown in Fig. 1. All Z-factors have values $1 + \mathscr{O}(\alpha_s)$, and the vertex corrections $\delta V, \delta V^{\text{lat}}$ are $\mathscr{O}(\alpha_s)$ and Γ-independent as noted above.

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The continuum quantities are known [4]; we focus on the lattice correlation function: $w_0 = 1 - M_5$ is a domain-wall fermion specific constant, and an overlap factor $1 - w_0^2$ connecting the five-dimensional and physical quark fields is present even at tree level. The light quark wavefunction renormalization $Z_w Z_2^{\mathrm{lat}}$ due to Fig. 1 (a) and (b) was calculated in Ref. [5]. Z_2^{lat} can be viewed as the four-dimensional wavefunction renormalization, while Z_w renormalizes the overlap factor $1 - w_0^2$. Due to tadpoles, the one-loop correction to Z_w is enormous. As described in Ref. [5], this is remedied by reorganizing the perturbation series according to the mean-field approach, resulting in the prescriptions $M_5 \to \widetilde{M}_5 = M_5 - 4(1-u)$, $w_0 \to w_0^{\mathrm{MF}} = 1 - \widetilde{M}_5$ and $q^{\mathrm{lat}} \to q^{\mathrm{lat,\,MF}} = u^{-1/2} q^{\mathrm{lat}}$ to be made throughout the calculation; here $u = P^{1/4}$ where P is the measured average plaquette (for the RBC-UKQCD calculation $u = 0.8757$) and the superscript 'MF' identifies mean-field improved quantities. We calculate the matching factor $\widetilde{C}_A(\mu, a^{-1})$ using both the usual continuum $\overline{\mathrm{MS}}$ coupling and a mean-field improved version, enabling an estimate of $\mathscr{O}(\alpha_s^2)$ corrections. $\alpha_s^{\overline{\mathrm{MS}}}(\mu)$ was obtained by running down to the c quark mass with the physical number of flavors and back up to μ using only three dynamical flavors to match the RBC-UKQCD 2+1 flavor calculation: $\alpha_s^{\overline{\mathrm{MS}}}(\mu) = 0.326$ and $\alpha_s^{\mathrm{MF}}(\mu) = 0.177$. The calculation of the vertex correction δV^{lat} in Fig. 1 (c) and the heavy quark wavefunction renormalization Z_h^{lat} in Fig. 1 (d) and (e) is straightforward [2, 4]. Infrared divergences only occur in QED-like diagrams and are regulated by a gluon mass λ which cancels from the matching factor. Furthermore, only the unsmeared $\delta_{\mu 0}$ part of $h_\mu(q)$ in Eq. (2.3) gives rise to infrared divergences; the sine functions in the smeared part of $h_\mu(q)$ cancel all infrared divergent loop propagators. A generic feature of domain-wall fermion perturbation theory is the appearance of correlation functions $\langle q(-p)\overline{\psi}_s(p)\rangle$, $\langle \psi_s(-p)\overline{q}(p)\rangle$ connecting external four-dimensional quarks to five-dimensional quarks propagating in loops, as shown in Fig. 2. A subtlety pointed out in Ref. [6] is that the correct renormalization prescription for $Z_h^{\mathrm{lat,\,MF}}$ includes the linearly divergent heavy quark mass renormalization:

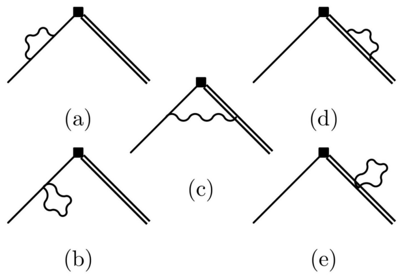

Figure 1: One-Loop Corrections to the Heavy-Light Axial Current

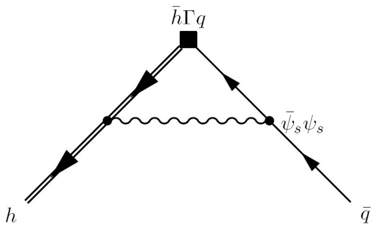

Figure 2: One-Loop Vertex Correction to the Heavy-Light Axial Current

$$Z_h^{\mathrm{lat,\,MF}} = 1 - i \frac{\partial \Sigma(p_0)}{\partial p_0}\bigg|_{p_0=0} + \Sigma(p_0 = 0), \tag{3.8}$$

where the heavy quark self energy $\Sigma(p_0)$ itself is not affected by mean-field improvement. Comparing the correlation functions in Eq. (3.7) after mean-field improvement gives a matching factor

$$\widetilde{C}_A(\mu, a^{-1}) = \frac{\sqrt{u}}{\sqrt{(1-(w_0^{\mathrm{MF}})^2) Z_w^{\mathrm{MF}}}} Z_A^{\mathrm{MF}}(\mu, a^{-1}), \quad Z_A^{\mathrm{MF}}(\mu, a^{-1}) = 1 + \frac{\alpha_s}{3\pi}(-1.584). \tag{3.9}$$

The overall factor $Z_\Phi(\mu_b,a^{-1}) = C_A(\mu_b,\mu)\widetilde{C}_A(\mu,a^{-1})$ relating the axial currents in full QCD and the lattice static effective theory, computed using both $\alpha_s^{\overline{\rm MS}}(\mu)$ and $\alpha_s^{\rm MF}(\mu)$, is $Z_\Phi^{\overline{\rm MS}}(\mu_b,a^{-1}) = 0.902$, $Z_\Phi^{\rm MF}(\mu_b,a^{-1}) = 0.961$. While the one-loop result is small and reliable, the large difference between $\alpha_s^{\overline{\rm MS}}(\mu)$ and $\alpha_s^{\rm MF}(\mu)$ induces a $\sim 7\%$ systematic error ultimately warranting nonperturbative renormalization.

For completeness, we quote the lattice-continuum matching constants (calculated in Refs. [2, 4]) for the four-fermion operators in Eqs. (3.5) and (3.6). For $i \in \{VV+AA, SS+PP\}$ and at one-loop

$$\widetilde{O}_i(\mu) = \frac{u}{(1-(w_0^{\rm MF})^2)Z_w^{\rm MF}} Z_i^{\rm MF}(\mu,a^{-1})a^{-6}O_i^{\rm lat}, \quad Z_{VV+AA}^{\rm MF} = 1+\frac{\alpha_s}{4\pi}(-4.462), \quad Z_{SS+PP}^{\rm MF} = 1, \tag{3.10}$$

where the $O_i^{\rm lat}$ are dimensionless. Since the coefficient Z_2 of $\widetilde{O}_{SS+PP}$ in Eq. (3.3) is $\mathscr{O}(\alpha_s)$, only the domain-wall overlap factors contribute to the lattice-continuum matching for this operator. While formally inconsistent, we use the one-loop mean-field improved values of the overlap factors throughout to ensure tadpole-safety. Combining Eqs. (3.3) and (3.10) we get:

$$O_{VV+AA} = Z_{VA}(\mu_b,a^{-1})a^{-6}O_{VV+AA}^{\rm lat} + Z_{SP}(\mu_b,a^{-1})a^{-6}O_{SS+PP}^{\rm lat}, \tag{3.11}$$
$$Z_{VA}^{\overline{\rm MS}} = 0.902, \quad Z_{VA}^{\rm MF} = 0.769, \quad Z_{SP}^{\overline{\rm MS}} = -0.123, \quad Z_{SP}^{\rm MF} = -0.133. \tag{3.12}$$

4. Ground State Degeneracies of Static-Light Mesons and f_B, B_B on the Lattice

Let H be the Hamiltonian corresponding to the full lattice action in Sec. 2. For any t, the heavy quark action in Eq. (2.1) is invariant under $h(\vec{x}) \to e^{i\theta(\vec{x})}h(\vec{x})$ for a set of V/a^3 parameters $\theta(\vec{x})$, where $V = L^3$ is the spatial lattice volume. If $\Theta(\vec{x})$ is the generator corresponding to $\theta(\vec{x})$ then

$$[\Theta(\vec{x}),h(\vec{y})] = h(\vec{x})\delta_{\vec{x}\vec{y}}, \quad \left[\Theta(\vec{x}),\overline{h}(\vec{y})\right] = -\overline{h}(\vec{x})\delta_{\vec{x}\vec{y}}, \quad [\Theta(\vec{x}),\Theta(\vec{y})] = 0, \quad [\Theta(\vec{x}),H] = 0. \tag{4.1}$$

Simultaneously diagonalize H and all $\Theta(\vec{x})$. Since Eq. (4.1) implies that $h(\vec{x})$ and $\overline{h}(\vec{x})$ raise and lower the eigenvalues of $\Theta(\vec{x})$ by 1, and the charge conjugation invariance of QCD implies $\Theta(\vec{x})|0\rangle = 0$, the spectrum of $\Theta(\vec{x})$ contains $\mathbb{Z}$. Define the unit-norm state $|B(\vec{x})\rangle$ to be the lowest energy state with the quantum numbers of a B meson which also satisfies $\Theta(\vec{y})|B(\vec{x})\rangle = \delta_{\vec{x}\vec{y}}|B(\vec{x})\rangle$. Thus $\langle B(\vec{x})|B(\vec{y})\rangle = \delta_{\vec{x}\vec{y}}$, and we can interpret these states as having the heavy quark localized at a fixed lattice site with the light quark smeared out around it. Since $T(\hat{i})\Theta(\vec{x})T(\hat{i})^{-1} = \Theta(\vec{x}+\hat{i})$, where $T(\hat{i})$ is a lattice translation by a in the spatial direction $\hat{i}$, all B meson ground states $|B(\vec{x})\rangle$ are degenerate. We also define total spatial momentum eigenstates $|\widetilde{B}(\vec{k}_l)\rangle$, where $l_i \in \mathbb{Z}$ $(i = 1,2,3)$:

$$|\widetilde{B}(\vec{k}_l)\rangle = \sqrt{2a^3}\sum_{\vec{x}} e^{-i\vec{k}_l\cdot\vec{x}}|B(\vec{x})\rangle, \;\; \vec{k}_l = \frac{2\pi}{L}(l_1,l_2,l_3), \;\; -\frac{L}{2a} < l_i \leq \frac{L}{2a}, \;\; \langle\widetilde{B}(\vec{k}_{l'})|\widetilde{B}(\vec{k}_l)\rangle = 2V\delta_{l'l}. \tag{4.2}$$

As $a \to 0, V \to \infty$, these states reduce to continuum momentum eigenstates $|\widetilde{B}(\vec{p})\rangle^{\rm c}$ with conventional static effective theory normalization ${}^{\rm c}\langle\widetilde{B}(\vec{p}\,')|\widetilde{B}(\vec{p})\rangle^{\rm c} = 2(2\pi)^3\delta^{(3)}(\vec{p}\,'-\vec{p})$. In the $m_b \to \infty$ limit, these states only differ from the corresponding full QCD states by a factor of $\sqrt{m_B}$. Thus:

$$f_B\sqrt{m_B} \equiv \langle 0|A_0(\vec{0},0)|\widetilde{B}(\vec{p}=\vec{0})\rangle^{\rm c} = Z_\Phi^{\rm MF} a^{-3}\langle 0|A_0^{\rm lat}(\vec{0},0)\left(\sqrt{2a^3}\sum_{\vec{x}}|B(\vec{x})\rangle\right) = \\ = \sqrt{2}Z_\Phi^{\rm MF}a^{-3/2}\langle 0|A_0^{\rm lat}(\vec{0},0)|B(\vec{0})\rangle \equiv \sqrt{2}Z_\Phi^{\rm MF}a^{-3/2}\Phi_B^{\rm lat}. \tag{4.3}$$

In complete analogy to the above, we can construct $\overline{B}$ meson ground states states $|\overline{B}(\vec{x})\rangle$. Using these and Eq. (3.11), the calculation of the $B-\overline{B}$ mixing matrix element $\frac{8}{3}m_B^2 f_B^2 B_B = \langle \overline{B}|O_{VV+AA}|B\rangle$ is reduced to the calculation of the lattice quantities $\langle \overline{B}(\vec{0})|O_i^{\rm lat}(\vec{0},0)|B(\vec{0})\rangle$, $i\in\{VV+AA, SS+PP\}$.

The degeneracy of the states $|B(\vec{x})\rangle$ complicates the extraction of $\Phi_B^{\rm lat}$ and $\langle \overline{B}(\vec{0})|O_i^{\rm lat}(\vec{0})|B(\vec{0})\rangle$, since even a large time separation of source and sink may not project onto a unique B meson ground state: different combinations of the $|B(\vec{x})\rangle$ may enter the correlation functions used for calculating the matrix elements and those used for normalization. To see this, consider the extraction of $\Phi_B^{\rm lat}$; we now work exclusively in the lattice theory. Define local and smeared B meson interpolation operators $A_0^L(\vec{x},t) = \overline{h}(\vec{x},t)\gamma_0\gamma_5 q(\vec{x},t)$, $A_0^S(t) = \sum_{\vec{y}\in\Delta V}\sum_{\vec{z}\in\Delta V}\overline{h}(\vec{y},t)\gamma_0\gamma_5 q(\vec{z},t)$, where ΔV is a fixed subvolume of V and the smeared operators are Coulomb gauge fixed. From experience, local-local correlation functions in the static effective theory are prohibitively noisy; instead calculate the local-smeared and smeared-smeared correlation functions. Inserting a complete set of states $\sum_{\vec{w}}|B(\vec{w})\rangle\langle B(\vec{w})| +$ (higher energy states) with the correct quantum numbers, we have as $t\to\infty$:

$$\mathscr{C}^{LS}(t) \equiv \sum_{\vec{x}\in V}\langle 0|A_0^L(\vec{x},t)A_0^S(0)^\dagger|0\rangle = \Phi_B^{\rm lat}e^{-m_B^* t}\left(\sum_{\vec{w}\in V}\langle B(\vec{w})|\sum_{\vec{y}\in\Delta V}\sum_{\vec{z}\in\Delta V}\overline{q}(\vec{y},0)\gamma_0\gamma_5 h(\vec{z},0)|0\rangle\right), \tag{4.4}$$

$$\mathscr{C}^{SS}(t) \equiv \langle 0|A_0^S(t)A_0^S(0)^\dagger|0\rangle = e^{-m_B^* t}\left(\sum_{\vec{w}\in V}\left|\langle B(\vec{w})|\sum_{\vec{y}\in\Delta V}\sum_{\vec{z}\in\Delta V}\overline{q}(\vec{y},0)\gamma_0\gamma_5 h(\vec{z},0)|0\rangle\right|^2\right). \tag{4.5}$$

where m_B^* is the unphysical mass of the lattice B meson. Since $\mathscr{C}^{SS}(t)$ contains a sum over squares, the use of a naive ratio $\sim \mathscr{C}^{LS}(t)/\sqrt{\mathscr{C}^{SS}(t)}$ requires a translationally invariant wall source $\Delta V = V$ to project onto the unique state of zero-momentum. In this case the sums over $\vec{w}$ only give a factor of V/a^3 and $\Phi_B^{\rm lat} = \mathscr{C}^{LS}(t)/\sqrt{\mathscr{C}^{SS}(t)e^{-m_B^* t}V/a^3}$. To remedy the poor overlap of the wall source with the B meson ground state - especially on large lattices - consider a fixed box source and a series of box sinks summed over an entire timeslice to project onto zero momentum; this approach also allows more general types of smearing, such as the use of an atomic wavefunction. Let $\widetilde{A}_0^S(\vec{w},t) = \sum_{\vec{y}\in\Delta V_{\vec{w}}}\sum_{\vec{z}\in\Delta V_{\vec{w}}}\overline{h}(\vec{y},t)\gamma_0\gamma_5 q(\vec{z},t)$ where $\Delta V_{\vec{w}}$ is a box of fixed size located at $\vec{w}$ and $\Delta V_{\vec{0}} = \Delta V$, $\widetilde{A}_0^S(\vec{0},t) = A_0^S(t)$. Define a corresponding smeared-smeared correlation function and insert a complete set of momentum eigenstates $\frac{1}{2V}\sum_{\vec{k}_l}|\widetilde{B}(\vec{k}_l)\rangle\langle\widetilde{B}(\vec{k}_l)| +$ (higher energy states); then as $t\to\infty$,

$$\mathscr{C}^{\widetilde{S}\widetilde{S}}(t) \equiv \sum_{\vec{w}}\langle 0|\widetilde{A}_0^S(\vec{w},t)A_0^S(0)^\dagger|0\rangle = \frac{e^{-m_B^* t}}{2V}\sum_{\vec{w}}\langle 0|\widetilde{A}_0^S(\vec{w},t)|\widetilde{B}(\vec{0})\rangle\langle\widetilde{B}(\vec{0})|A_0^S(0)^\dagger|0\rangle = \frac{e^{-m_B^* t}}{2a^3}\left|\langle\widetilde{B}(\vec{0})|A_0^S(0)^\dagger|0\rangle\right|^2. \tag{4.6}$$

Since $|\widetilde{B}(\vec{0})\rangle = (2a^3)^{1/2}\sum_{\vec{w}} |B(\vec{w})\rangle$, we can rewrite the right side of Eq. (4.4) and obtain another ratio for Φ_B^{lat} which reaches a plateau more quickly due to the improved ground state overlap:

$$\mathscr{C}^{LS}(t)e^{m_B^* t/2} \Big/ \sqrt{\mathscr{C}^{\widetilde{S}\widetilde{S}}(t)} = \Phi_B^{\text{lat}} \langle \widetilde{B}(\vec{0})|A_0^S(0)^\dagger|0\rangle \Big/ \sqrt{\left|\langle \widetilde{B}(\vec{0})|A_0^S(0)^\dagger|0\rangle\right|^2} = \Phi_B^{\text{lat}}. \quad (4.7)$$

The calculation of $\langle \overline{B}(\vec{0})|O_i^{\text{lat}}(\vec{0})|B(\vec{0})\rangle$, $i \in \{VV+AA, SS+PP\}$ is considerably simpler. Define

$$\mathscr{C}_{O_i}(T,t) \equiv \sum_{\vec{x}\in V} \langle 0|\overline{A}_0^S(T) O_i^{\text{lat}}(\vec{x},t) A_0^S(0)^\dagger|0\rangle, \quad (4.8)$$

where $\overline{A}_0^S(T) = \sum_{\vec{y}\in\Delta V}\sum_{\vec{z}\in\Delta V} \overline{q}(\vec{y},T)\gamma_0\gamma_5 h(\vec{z},T)$. Proceeding as above, we have as t, $T-t \to \infty$:

$$\langle \overline{B}(\vec{0})|O_i^{\text{lat}}(\vec{0},0)|B(\vec{0})\rangle = \mathscr{C}_{O_i}(T,t) \Big/ \mathscr{C}^{SS}(T) = \mathscr{C}_{O_i}(T,t)e^{m_B^* T/2} \Big/ \sqrt{\mathscr{C}^{SS}(T-t)\mathscr{C}^{SS}(t)}. \quad (4.9)$$

Here no zero momentum projection is necessary; the use of $\mathscr{C}^{SS}$ for smaller time separations simply reduces noise. Using Eqs. (4.7) and (4.9) we can thus calculate f_B and B_B using only box sources and sinks. These are preferable to wall sources, whose poor ground state overlap led to late plateaus in the $V = (2\text{ fm})^3$ RBC-UKQCD calculation and presents an even bigger problem for the ongoing extension to $V = (3\text{ fm})^3$. It is worth emphasizing that this simple method relies on the particular properties of the static effective theory, and further such improvements might be possible.

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