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A determination of the B_s^0 and B_d^0 mixing parameters in 2+1 lattice QCD

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We report on the advances in our unquenched calculation of the matrix elements relevant for the analysis of $B^0-\bar{B}^0$ mixing using the Asqtad (light quark) and Fermilab (heavy quark) actions. We have calculated the hadronic parameters for the mass and width differences in the neutral B meson system. Preliminary results are presented for $f_{B_q}^2 B_q$ as well as for the ratio $\xi^2 = f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d}$.

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1. Introduction

The very accurate experimental measurements of the mass differences between the heavy and light B_s^0 and B_d^0 mass eigenstates, ΔM_s [1] and ΔM_d [2], that describe the $B_s^0-\bar{B}_s^0$ and $B_d^0-\bar{B}_d^0$ mixings respectively, make improving the theoretical study of these quantities crucial. In the standard model (SM), the mass difference is given by [3]

$$\Delta M_{s(d)}|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2}|V_{ts(d)}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_{s(d)}} f_{B_{s(d)}}^2 \hat{B}_{B_{s(d)}}\,, \tag{1.1}$$

where $x_t = m_t^2/M_W^2$, η_2^B is a perturbative QCD correction factor, $S_0(x_t)$ is the Inami-Lim function and the products $f_{B_{s(d)}}^2 \hat{B}_{B_{s(d)}}$ parameterize the hadronic matrix elements in the effective theory with $f_{B_{s(d)}}$ the $B_{s(d)}^0$ decay constants and $\hat{B}_{B_{s(d)}}$ the (renormalization group invariant) bag parameters. The hadronic matrix elements can be calculated in lattice QCD. Our current knowledge of them limits the accuracy with which the CKM matrix elements appearing in Eqn. (1.1) can be determined from the experimental measurements of $\Delta M_{s(d)}$. The goal of our project is to calculate all the hadronic matrix elements which are relevant for the mass and width differences in the $B_{s(d)}^0$ systems in unquenched lattice QCD at the few percent level.

Many of the uncertainties that affect the theoretical calculation of the decay constants and bag parameters cancel totally or partially if one takes the ratio $\xi^2 = f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d}$. Hence, this ratio and therefore the combination of CKM matrix elements related to it by Eqn. (1.1) can be determined with a significantly smaller error than the individual matrix elements. This is a crucial ingredient in the unitarity triangle analysis. In these proceedings we report our preliminary results for the determination of ξ, as well as for the quantities $f_{B_q}^2 B_{B_q}$.

Other work on this subject using $2+1$ lattice QCD methods can be found in [4].

2. Operators, actions and matching calculation

The whole set of operators whose matrix elements are needed to determine the $B_{s(d)}^0$ mixing parameters are

$$\begin{aligned} Q^{s(d)} &= \left[\bar{b}^i \gamma_\mu (1-\gamma_5) s^i(d^i)\right]\left[\bar{b}^j \gamma^\mu (1-\gamma_5) s^j(d^j)\right]\,, \\ Q_S^{s(d)} &= \left[\bar{b}^i (1-\gamma_5) s^i(d^i)\right]\left[\bar{b}^j (1-\gamma_5) s^j(d^j)\right]\,, \\ Q_3^{s(d)} &= \left[\bar{b}^i (1-\gamma_5) s^j(d^j)\right]\left[\bar{b}^j (1-\gamma_5) s^i(d^i)\right]\,, \end{aligned} \tag{2.1}$$

where i,j are color indices. In these proceedings we focus on the results for the first two pairs of operators, enough to determine $\Delta M_{s(d)}$, and leave the study of the third pair, needed for an improved determination of $\Delta\Gamma_{s(d)}$, for a forthcoming publication [5].

We use the Fermilab action [6] for the b valence quarks and the Asqtad action [7], for the light sea and valence quarks, u, d and s. The Fermilab action has errors starting at $O(\alpha_s \Lambda_{QCD}/M)$ and $O((\Lambda_{QCD}/M)^2)$, while the errors of the Asqtad action are $O(\alpha_s a^2, a^4)$.

The products $f_{B_{s(d)}}^2 B_{B_{s(d)}}^{\overline{MS}}$ in Eqn. (1.1) parametrize the matrix elements by

$$\langle \bar{B}_s^0 | Q^{s(d)} | B_s^0 \rangle^{\overline{MS}}(\mu) = \frac{8}{3} M_{B_{s(d)}}^2 f_{B_{s(d)}}^2 B_{B_{s(d)}}^{\overline{MS}}(\mu)\,. \tag{2.2}$$

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The lattice matrix elements $\langle \bar{B}^0_{s(d)}|Q^{s(d)}|B^0_{s(d)}\rangle^{\rm lat}$ determine $f^2_{B_{s(d)}}B_{B_{s(d)}}$ at tree level. Beyond tree-level, the operators $Q^{s(d)}$, mix with $Q_S^{s(d)}$ both on the lattice and in the continuum. Including one-loop corrections, the renormalized matrix element is given by

$$\frac{a^3}{2M_{B_{s(d)}}}\,\langle Q^{s(d)}\rangle^{\overline{MS}}(\mu) = [1+\alpha_s\cdot\rho_{LL}(\mu,m_b)]\langle Q^{s(d)}\rangle^{\rm lat}(a)+\alpha_s\cdot\rho_{LS}(\mu,m_b)\langle Q_S^{s(d)}\rangle^{\rm lat}(a)\,. \quad (2.3)$$

The $O\left(\frac{\Lambda_{QCD}}{M}\right)$ improvement is implemented by a rotation of the b quark as explained in [6], so the perturbative matching errors start at $O(\alpha_s^2,\alpha_s\Lambda/aM)$. The matching coefficients ρ_{LL} and ρ_{LS} are the differences between the continuum $\overline{MS}$ and lattice renormalization coefficients calculated at one-loop order. We have calculated these coefficients for the same choice of lattice actions as used in the numerical simulations. We have checked that our results have the correct infrared behavior, that they are correct in the massless limit, and that they are gauge invariant. However, our results for the matching coefficients are still preliminary, because not all diagrams have been independently checked.

The optimal value of the strong coupling constant to be used in Eqn. (2.3) is $\alpha_V(q^*)$. Missing a calculation of q^* for the specific processes we are studying, we choose $q^*=2/a$, very close to the q^* calculated for heavy-light currents. The specific values for α_s we use are given in Table 1.

3. Simulation details

The matrix elements needed to determine both $f^2_{B_q}B_{B_q}$ and B_{B_q}, are extracted from the following three-point and two-point functions

$$C_O(t_1,t_2) = \sum_{\vec{x},\vec{y}}\langle \bar{b}(\vec{x},t_1)\gamma_5 q(\vec{x},t_1)|O(0)|\bar{b}(\vec{y},t_2)\gamma_5 q(\vec{y},t_2)\rangle,$$
$$C_Z(t) = \sum_{\vec{x}}\langle \bar{b}(\vec{x},t)\gamma_5 q(\vec{x},t)\bar{q}(0)\gamma_5 b(0)\rangle,\quad C_{A_4}(t) = \sum_{\vec{x}}\langle \bar{b}(\vec{x},t)\gamma_0\gamma_5 q(\vec{x},t)\bar{q}(0)\gamma_5 b(0)\rangle\,, \quad (3.1)$$

where the operator O is any $Q^{s(d)}$ or $Q_S^{s(d)}$ defined in Eqn. (2.1). The B meson operators are smeared at the sink with a 1S onium wavefunction. All the correlation functions in Eqn. (3.1) are calculated using the open meson propagator method described in [8].

We have performed these calculations on the MILC coarse lattices ($a=0.12\,fm$) with 2+1 sea quarks and for three different sea light quark masses. The strange sea quark mass is always set to 0.050. The light sea masses, $m_l \equiv m^{sea}_{light}$, number of configurations and other simulation details are collected in Table 1. The mass of the bottom quark is fixed to its physical value, while for each sea quark mass we determine the different matrix elements for six different values of the light valence quark mass in a generic meson B_q^0, $m_q = 0.0415, 0.03, 0.020, 0.010, 0.007, 0.005$. We use $m_q=0.0415$ for the valence strange mass in our simulations. It is close to the physical strange quark mass, $m_s^{phys.}=0.036$ [9]. The matrix elements of the operators are extracted from simultaneous fits of three-point and two-point functions using Bayesian statistics.

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m_l/m_s^{sea}	Volume	N_{confs}	a^{-1}(GeV)	$\alpha_s = \alpha_V(2/a)$	$N_{sources}$
0.020/0.050	$20^3\times 64$	460	1.605(29)	0.31	4
0.010/0.050	$20^3\times 64$	590	1.596(30)	0.31	4
0.007/0.050	$20^3\times 64$	890	1.622(32)	0.31	4

Table 1: Simulation parameters and α_s used in the matching with the continuum. m_l is the light sea quark mass.

4. Results

The results in Fig. 1 show $f_{B_q}\sqrt{B_{B_q}^{\overline{MS}}(m_b)}$ in lattice units as a function of the light valence mass am_q. The errors shown are statistical errors only; the analysis of the systematic errors is not yet complete.

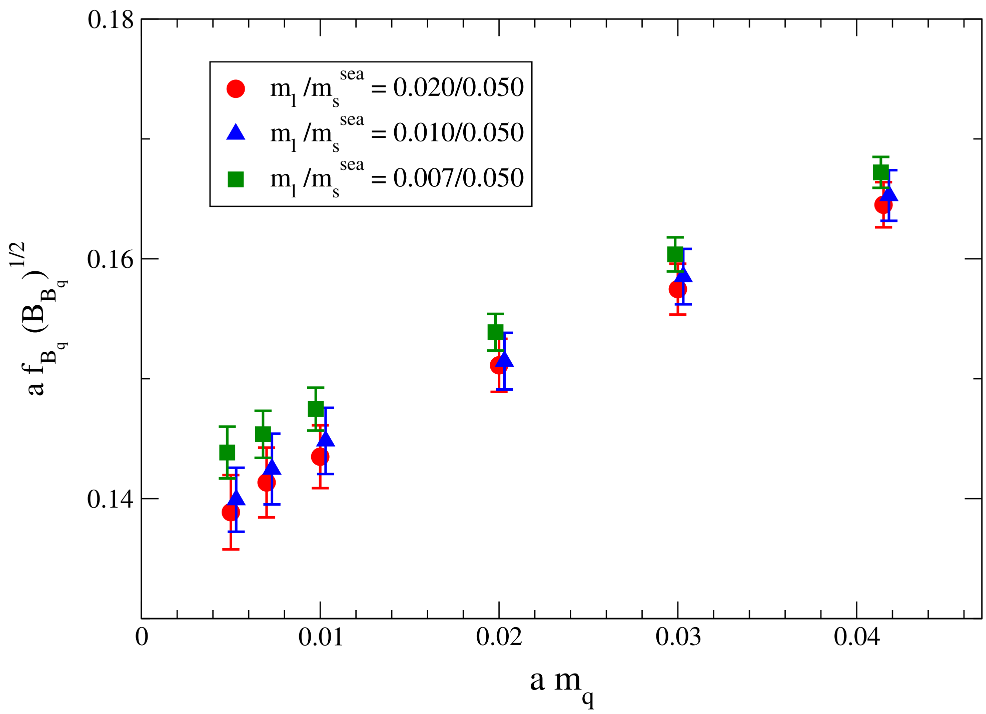

Figure 1: $f_{B_q}\sqrt{B_{B_q}^{\overline{MS}}(m_b)}$ in lattice units. The different symbols and colors correspond to different values of the sea light quark masses, m_l.

The statistical errors range between $1-3\%$. Some conclusions can be already extracted from this plot. The light sea quark mass dependence of $f_{B_q}\sqrt{B_{B_q}}$ is small compared to the statistical errors. The dependence on the light valence quark mass, however, is noticeable within statistics. In order to get a value for $f_{B_s}\sqrt{B_{B_s}^{\overline{MS}}(m_b)}$, since the s valence quark mass we are using is slightly larger than the physical one, we need an interpolation in the s valence quark mass together with a chiral

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extrapolation to the physical sea quark masses. To determine $f_{B_d}\sqrt{B_{B_d}^{\overline{MS}}(m_b)}$ we need extrapolations in both the d valence quark mass and the sea quark masses. This is in progress.

In Fig. 2 we plot the ratio $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$ as a function of the d valence quark mass in the denominator. Again, our results are preliminary for the same reasons as mentioned before and the errors are only statistical. Most of the systematic errors cancel in the ratio, but not those associated with the chiral extrapolation in the light valence quark mass.

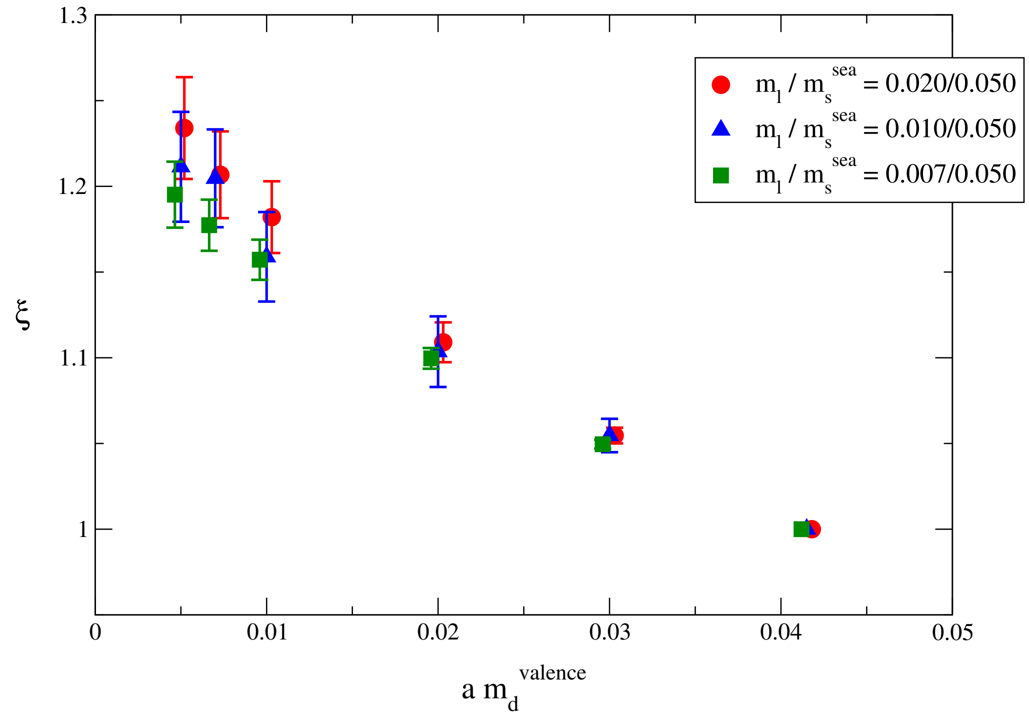

Figure 2: ξ as a function of the valence d mass for three different values of the light sea quark masses.

4.1 Chiral extrapolation

The continuum chiral expansion of the hadronic matrix element $\langle \bar{B}_q|Q|B_q\rangle$ at NLO in (partially quenched) heavy meson chiral perturbation theory (HMChPT) is given by [10]

$$\langle \bar{B}_q|Q|B_q\rangle = \beta(1+w(T_q+W_q+S_q))+c_0 m_q + c_1(m_U+m_D+m_S) \tag{4.1}$$

where m_U, m_D, m_S are the sea quark masses and m_q the light valence quark mass. β, w, c_0 and c_1 are low energy constants (LECs) to be determined from the fits. The functions T_q, W_q, and S_q contain the chiral logs and correspond to tadpole-, wave-function, and sunset-type contributions, respectively.

The effects of $O(a^2)$ taste changing interactions can be included in Eqn. (4.1) using staggered chiral perturbation theory (SChPT). In that case, the chiral log functions are modified to depend

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	$f_{B_q}\sqrt{B_{B_q}}$	ξ
statistics	$1-3$	$1-2$
scale(a^{-1})	0.9	0
Higher order matching	~ 4.5	cancel to a large extent
Heavy quark discret.	$2-3$	< 0.5
Light quark discret. + χPT fits	Work in progress	

Table 2: Error budget for $f_{B_q}\sqrt{B_{B_q}}$ and ξ in percent.

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on the masses of the different taste multiplets. Explicit expressions from SChPT for heavy-light bilinear quantities can be found in [11, 12]. Similar terms are expected to contribute to our four-quark operators. The modified chiral logs contain other fixed LEC's, most of which are already determined to a high degree of certainty [12]. The logs also contain constants from heavy quark effective theory, in particular the mass splitting between the vector and pseudoscalar heavy mesons, $\Delta* = M_B* - M_B$, and the mass splittings between the pseudoscalar heavy mesons containing different valence and sea light quarks, $\delta_{qr} = M_{B_r} - M_{B_q}$. These HQET constants can be determined directly from the two-point function fits and used as input into the chiral fits, with the experimental values then used in the extrapolation.

We are still in the process of determining the exact SChPT form of Eqn. (4.1). Once the functional SChPT form of Eqn. (4.1) is completely determined, we plan to use it to simultaneously fit it to our lattice data points for all sea and valence quark masses, and to determine the unknown LEC's in the process. For the systematic error analysis, we plan to study the effects of changing the SChPT form, for example by adding NNLO analytic terms, and the effects of allowing the more poorly known fixed parameters to vary. Our physical results will then be obtained by turning the taste-violations off, and extrapolating (interpolating) to the physical light (strange) sea and valence quark masses.

We will quote results for ξ, $f_{B_s}\sqrt{B_{B_s}}$, $f_{B_d}\sqrt{B_{B_d}}$, B_{B_s}, and B_{B_d} when this step is completed. We expect light quark discretization effects to be an important source of uncertainty until we calculate the three-point correlators at several lattice spacings and use these in the chiral fits.

5. Summary and future work

We have presented preliminary results for $f_{B_q}\sqrt{B_{B_q}}$ for six different values of m_q as well as for the ratio ξ with five different values of $m_d^{valence}$. Our nalysis on three ensembles with different sea light quark masses gives statistical errors between $1-3\%$ for $f_{B_q}\sqrt{B_{B_q}}$ and $1-2\%$ for the ratio ξ. The systematic error analysis is in progress, see Table 2.

Two important sources of error in the calculation of $f_{B_q}\sqrt{B_{B_q}}$, the matching uncertainties and heavy quark discretization errors, are expected to cancel to a large extent when taking the ratio. We have already checked that the difference between the tree level and the one-loop results for ξ with our preliminary results for the renormalization constants is less than 0.5%. The higher order matching errors in Table 2 have been naively estimated as being $O\left(1\times\alpha_s^2\right)$ for the coarse lattice. Heavy quark discretization effects in the table are estimated by power counting [13].

We are in the process of generating lattice data on the coarse lattice with a smaller light sea quark mass, $m_l = 0.005$, which will further constrain the chiral extrapolations. Better fitting approaches and different smearings that could reduce statistical errors further are also being investigated. Results from these improvements will be presented in a future publication [5]. We also plan to present results for the decay width differences $\Delta\Gamma_s$ and $\Delta\Gamma_d$, for which we have already calculated all the hadronic matrix elements needed. Finally, we plan to improve this analysis by repeating this calculation at other lattice spacings to study the discretization errors in detail. Simulations on finer lattices will reduce both discretization and perturbative matching errors.

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