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$B \to D^* \ell \nu$ with 2+1 flavors

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We present a calculation of the form factor for $B \to D^* l \nu$ using a 2+1 improved staggered action for the light quarks (on the MILC configurations), and the Fermilab action for the heavy quarks. The form factor is computed at zero recoil using a new double ratio method which yields the form factor more directly than previous approaches.

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1. Introduction

The CKM element V_{cb} is important for the phenomenology of flavor physics in determining the apex of the unitarity triangle in the complex plane. For example, the Standard Model prediction of ε_K depends sensitively on V_{cb} (where it appears to the fourth power), and the present errors on this quantity contribute errors to ε_K of the same size as those due to B_K, the kaon mixing parameter which has been the focus of much recent work [1, 2, 3]. It is possible to determine $|V_{cb}|$ from both inclusive and exclusive semileptonic B decays, and they are both limited by theoretical uncertainties. The inclusive method makes use of the heavy quark expansion [4, 5], but is limited by the breakdown of local quark-hadron duality, the errors of which are difficult to estimate. The exclusive method requires reducing the uncertainty of the form factor $\mathscr{F}_{B\to D^*}$, which has been calculated using lattice QCD in the quenched approximation [6]. Given the phenomenological importance of this quantity we have revisited this calculation of $\mathscr{F}_{B\to D^*}$ using the 2+1 flavor MILC lattices with improved light staggered quarks [7]. The quenching error is thus eliminated, and the systematic error associated with the chiral extrapolation is reduced significantly.

This calculation was done using a blind analysis as follows: the perturbation theory calculation needed to renormalize the lattice current was done separately from the rest of the numerical analysis, and the renormalization constants needed to compare results at different lattice spacings to the continuum were given an overall offset which was not revealed until the systematic errors in the rest of the numerical analysis had been determined.

2. Obtaining $|V_{cb}|$

The differential rate for the semileptonic decay $\overline{B} \to D^* l \overline{\nu}_l$ is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2-1}\mathscr{G}(w)|V_{cb}|^2 |\mathscr{F}_{B\to D^*}(w)|^2 \tag{2.1}$$

where $w = v' \cdot v$ is the velocity transfer from the initial state to the final state, and $\mathscr{G}(w)|\mathscr{F}_{B\to D^*}|^2$ contains a combination of four form factors which must be calculated nonperturbatively. At zero recoil $\mathscr{G}(1) = 1$, and $\mathscr{F}_{B\to D^*}(1)$ reduces to a single form factor, $h_{A_1}(1)$. This is sufficient to determine $|V_{cb}|$ from experiment. Heavy quark symmetry plays an important role in constraining $h_{A_1}(1)$, leading to the heavy quark expansion [8, 9]

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{2m_c 2m_b} - \frac{\ell_P}{(2m_b)^2} \right], \tag{2.2}$$

up to order $1/m_Q^2$ and where η_A is a factor which matches QCD and heavy quark effective theory (HQET). The ℓ's are long-distance matrix elements of the heavy quark effective theory. The earlier work by the Fermilab lattice collaboration [6] used a series of three double ratios in order to obtain separately each of the three $1/m_Q^2$ coefficients in Eq. (2.2). These three double ratios also determine three out of the four coefficients appearing at $1/m_Q^3$ in the heavy quark expansion. It was shown

in [10] that for the Fermilab method matched to tree level in α_s and to leading order in HQET, the leading discretization errors for the double ratios for this quantity are of order $\alpha_s(\overline{\Lambda}/m_Q)^2$ and $\overline{\Lambda}/m_Q^3$.

In the calculation reported here, the form factor $h_{A_1}(1)$ is computed more directly using only one double ratio,

$$\mathscr{R}_{A_1} = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\overline{B}|\bar{b}\gamma_4 b|\overline{B}\rangle} = |h_{A_1}(1)|^2 . \tag{2.3}$$

which is exact to all orders in the heavy quark expansion (modulo discretization errors for the corresponding lattice ratio). The errors in this ratio do not rigorously scale as $\mathscr{R}-1$ because Eq. (2.3) is not one in the limit of equal bottom and charm quark masses (it becomes one only in the static quark limit). Nevertheless, this double ratio still retains the desirable features of the previous double ratios, i.e. large statistical error cancellations, and the cancellation of most of the lattice current renormalization. The quenching error has been eliminated by including the fermion determinant in the weighting of the gauge configurations, and so the rigorous scaling of all the errors as $\mathscr{R}-1$, including the quenching error, is no longer as important. The more direct method introduced here has the significant advantage that extracting coefficients from fits to HQET expressions as a function of heavy quark masses is not necessary, and no error is introduced from truncating the heavy quark expansion to a fixed order in $1/m_Q^n$.

Most of the current renormalization cancels in the lattice double ratio, leaving only a small correction factor, ρ, defined such that $\rho\sqrt{R_{lat}} = \sqrt{\mathscr{R}_{cont}} = h(1)$, as discussed in [11]. This ρ factor has been calculated perturbatively [12], and was found to contribute less than a 0.5% correction.

3. Lattice calculation

The lattice calculation was done on the MILC lattices for three lattice spacings ($a \approx 0.15$, 0.125, and 0.09 fm) where the light quarks were computed with the "AsqTad" staggered action. The heavy quarks were computed using the clover action with the Fermilab interpretation in terms of HQET [13]. We have several light masses at both full QCD and partially quenched points ($m_{valence} \neq m_{sea}$), and our light quark masses range between $m_s/10$ and $m_s/2$.

Extracting correlation functions that contain staggered quarks presents an extra complication because of the contributions of wrong parity excited states which introduce oscillations into the usual plateau fits. The average,

$$C^{X\to Y}_{avg}(0,t,T) \equiv \frac{1}{2}C^{X\to Y}(0,t,T) + \frac{1}{4}C^{X\to Y}(0,t,T+1) + \frac{1}{4}C^{X\to Y}(0,t+1,T+1), \tag{3.1}$$

is equivalent to a smearing which suppresses the oscillating states, and has been applied to all of the data for the double ratios. Figure (3) shows a plateau fit to the double ratio used to obtain $h_{A_1}(1)$. The source is at time slice 0, the sink is at T, and the operator position is varied along t. Two different extended propagators were constructed at even and odd source sink separations ($T = 16, 17$). The average of these two extension points was taken according to Eq. (3.1), and this average was fit to a constant as shown in Figure 1. There is no detectable oscillation even before

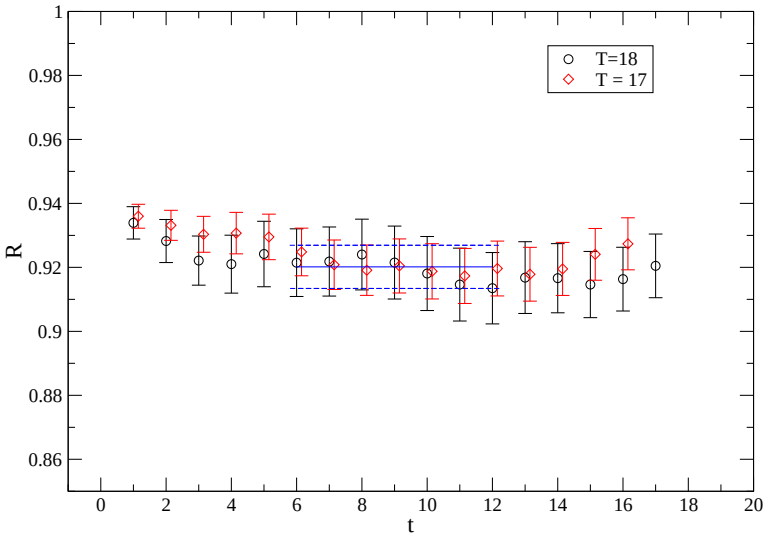

Figure 1: Double ratio on the $m_\ell = 0.0124$ fine ensemble. The source was fixed to time slice 0, and the operator position was varied as a function of time. Two different sink (extension) points were used with even and odd time separations between source and sink [$C(0,t,T)$ and $C(0,t,T+1)$] in order to study the effect of non-oscillating wrong parity states. The fit is to the average of the source sink separations given in Eq. (3.1).

the average is taken; the oscillating contributions are reduced even further in the average so that their systematic errors can be safely neglected.

The chiral extrapolation errors can be controlled by using the appropriate rooted staggered chiral perturbation theory (rSχPT) for heavy light quantities [14]. Eq. (34) of [15] gives the expression needed for fits to $h_{A_1}(1)$ for partially quenched data with degenerate up and down quark masses (the 2+1 case). This partially quenched expression parameterizes the dependence on both valence and sea quark masses, and includes taste breaking violations coming from the light quark sector. The expression contains explicit dependence on the lattice spacing a, and requires as inputs the parameters of the staggered chiral lagrangian δ'_V, δ'_A, in addition to the staggered taste splittings $\Delta_{P,A,T,V,I}$. These parameters can be obtained from chiral fits to the pseudoscalar sector and are held fixed in the chiral extrapolation of $h_{A_1}(1)$. The continuum low energy constant $g_{D^*D\pi}$ appears, and this can be taken from phenomenology; we take a generous range of values for this term to estimate the error it contributes to $h_{A_1}(1)$. The only other parameter which appears at NLO is an overall constant that is determined by a fit to our data for $h_{A_1}(1)$.

For the chiral fits we find it useful to form two ratios that normalize results for $h_{A_1}(1)$ at a "fiducial point,"

$$R_{sea}(m_L,m_S,a) = \frac{h_{A_1}(m_x^{\rm fid},m_L,m_S,a)}{h_{A_1}(m_x^{\rm fid},m_L^{\rm fid},m_S^{\rm fid},a)}, \quad R_{val}(m_L,m_S,a) = \frac{h_{A_1}(m_x,m_L,m_S,a)}{h_{A_1}(m_x^{\rm fid},m_L,m_S,a)}. \tag{3.2}$$

where fid stands for fiducial, m_x is the light valence quark, m_L is the light sea quark, m_S is the strange

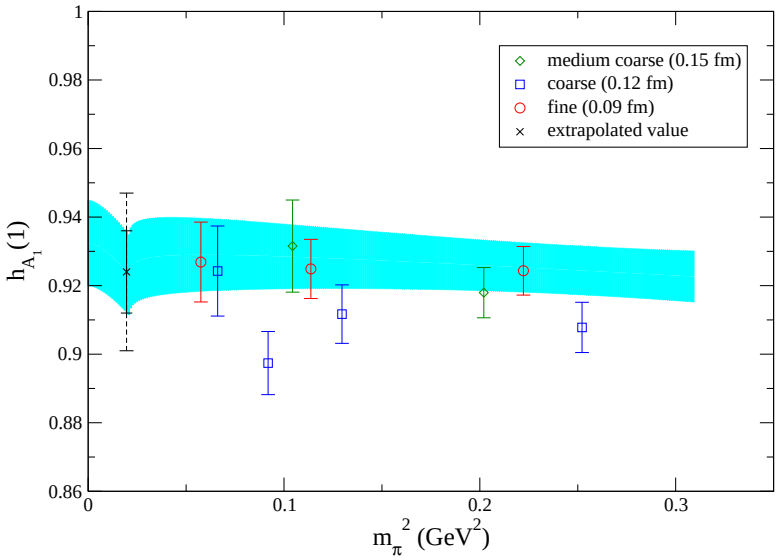

Figure 2: All of the data at the full QCD points ($m_{valence} = m_{sea}$) on the three lattice spacings. The cyan (light grey) band is the continuum extrapolated full QCD curve. The cross is the value at the physical light pion mass, where the solid line is the statistical error, and the dashed line is the total systematic error added to the statistical error in quadrature.

sea quark. Here we take $m_x^{\rm fid} \approx 0.5 m_{\rm strange}^{\rm physical}$, $m_L^{\rm fid} \approx 0.5 m_{\rm strange}^{\rm physical}$, and $m_S^{\rm fid} \approx m_{\rm strange}^{\rm physical}$. The ratios in Eq. (3.2) are now quadruple ratios; thus the statistical errors and excited state contamination are further suppressed over that of the double ratio. The main advantage of these ratios, however, is that heavy quark discretization effects largely cancel, so that we can disentangle the heavy quark discretization effects and those of the staggered chiral logs. This isolates the discretization effects coming from non-analytic taste violations, and these can be removed using rSχPT. We have chosen the fiducial point to be $\approx 0.5 m_{\rm strange}^{\rm physical}$ because it would be feasible to simulate this mass point on very fine lattices and smaller volumes without running into finite size effects, thus normalizing our data at a point where the heavy quark discretization effects are much smaller. For now we use the point with $m \approx 0.5 m_{\rm strange}^{\rm physical}$ on the finest lattice spacing available ($a \approx 0.09$ fm) as our fiducial point. By taking the chiral extrapolation and the continuum limit of the two ratios, multiplying them together and then multiplying that by the value of $h_{A_1}(1)$ at the fiducial mass on the finest available lattice spacing, we can construct the value of the form factor at the physical light quark mass, $h_{A_1}^{phys} = h_{A_1}^{fid} \times [R_{sea}(m_\ell^{phys}, m_s^{phys}, 0) \times R_{val}(m_\ell^{phys}, m_s^{phys}, 0)]$. This quantity is shown in Figure 2.

4. Results and conclusions

The final error budget is presented in Table 1. The error labelled "$g_{D^*D\pi}$ uncertainty" comes from the error in the chiral low energy constant $g_{D^*D\pi}$, which we take to vary between 0.3 and 0.6. The next error is the difference between doing NLO chiral fits for the chiral extrapolation, versus

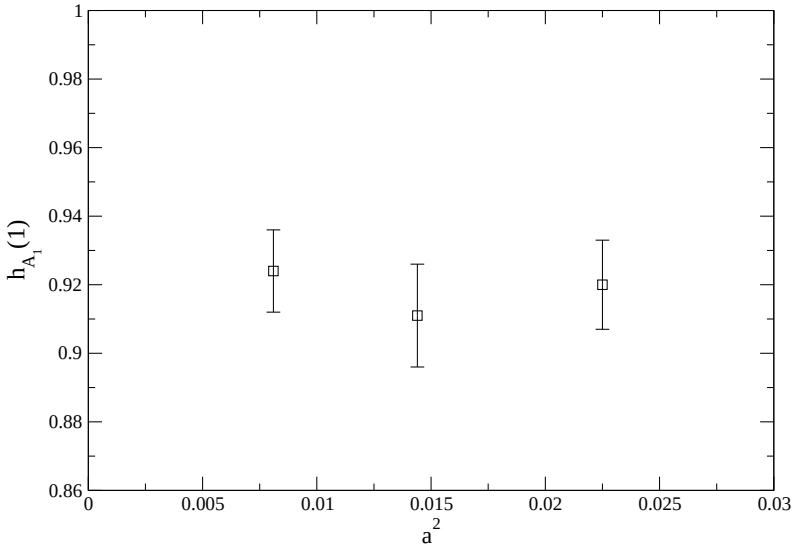

Figure 3: The values for h_{A_1} using the continuum extrapolated ratios determined in the fiducial point procedure to extrapolate the fiducial points on each of the three lattice spacings to the physical light quark masses. The fiducial point procedure allows us to remove the taste violations coming from staggered chiral logs, but it does not remove the analytic terms associated with the light quark sector, nor does it remove the heavy quark discretization errors. Although it is appropriate to extrapolate this curve to the continuum, a first principles extrapolation formula is not known. We therefore compare the value of h_{A_1} using the fiducial point on the fine lattice with the results obtained by using fiducial points on coarser lattice spacings. A comparison of the scatter of these results allows us to estimate the size of the remaining light quark and heavy quark discretization errors.

fits which include the NNLO analytic terms but not the 2-loop logarithmic terms, which have not been calculated. Both fits give acceptable confidence levels.

Our largest systematic uncertainty comes from discretization errors. The fiducial point procedure described above allows us to remove the effect of the splittings in the staggered chiral logs, but it does not determine and remove the analytic a^2 dependence in the light quark sector, nor the heavy quark discretization errors. Comparing the values obtained with different fiducial points on various lattice spacings gives an estimate of the size of the remaining light quark and heavy quark discretization errors. The scatter of the points in Figure 2 gives an estimate of the size of these effects, which cannot be resolved within statistics. The difference between the fine ($a = 0.09$ fm) and coarse ($a = 0.12$ fm) lattice spacings is a 1.3% difference, which is about the size one would expect for heavy quark discretization errors in this quantity from power counting arguments and a reasonable choice for the HQET parameter $\overline{\Lambda}$.

The error labelled "kappa tuning" comes from the parametric uncertainty associated with tuning the charm and bottom quark masses. The next error is from the perturbative matching of the lattice currents in the double ratio. As mentioned above, this renormalization factor is small because most of the renormalization cancels nonperturbatively in the ratio. We take the entire 1-loop correction of 0.4% as a conservative estimate of the error due to the omission of higher orders.

Table 1: Error budget

uncertainty	$h_{A_1}(1)$
statistics	1.2%
$g_{D^*D\pi}$ uncertainty	0.6%
NLO vs partial NNLO ChPT fits	0.9%
discretization errors	1.3%
kappa tuning	1.0%
perturbation theory	0.4%
Total	2.3%

We quote a preliminary result for the form factor $h_{A_1}(1) = 0.924(12)(19)$, where the first error is statistical, and the second is the sum of all systematic errors in quadrature. Taking the latest world average of $\mathscr{F}(1)|V_{cb}| = (36.0 \pm 0.6) \times 10^{-3}$ from experiment [16], we find $|V_{cb}| = (38.7 \pm 0.7_{exp} \pm 0.9_{theo}) \times 10^{-3}$. We estimate that the theoretical error on this determination of $|V_{cb}|$ from exclusive $B \to D^* \ell \nu$ can be reduced significantly by making use of the existing extra-fine MILC lattices ($a = 0.06$ fm) and higher statistics on the coarser ensembles.

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