

Chiral behaviour of matrix elements of $\Delta B = 2$ and $\Delta C = 2$ operators*

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We investigate the light-quark mass and spatial volume dependence of matrix elements of $\Delta B = 2$ and $\Delta C = 2$ four-fermion operators. These operators are relevant for $B_{(s)}^0 - \bar{B}_{(s)}^0$ and $D^0 - \bar{D}^0$ mixing in and beyond the Standard Model. An important conclusion of this work is that the chiral extrapolations for matrix elements of heavy-light meson mixing beyond the Standard Model are more complicated than that for the Standard Model mixing matrix elements.

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1. Introduction

A precise calculation for neutral B mixing matrix elements has been an urgent task since the recent measurement of Δm_s [1]. Such a calculation is crucial in obtaining stringent constraints on the unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and searching for new physics. On the other hand, the $D^0 - \bar{D}^0$ mixing system is a good channel to search for new physics [2], because the Standard Model contribution is strongly suppressed. In the Standard Model, the short distance contribution to the mass differences of these heavy neutral meson mixing systems is predominantly determined by the matrix elements of a single set of four quark operators:

$$\mathcal{O}_{1,aa} = \bar{h}^\alpha \gamma_\mu (1 - \gamma_5) q_a^\alpha \bar{h}^\beta \gamma_\mu (1 - \gamma_5) q_a^\beta, \quad (1.1)$$

where h is a heavy quark field (either a b or a c quark), q_a is a light-quark field with flavour a (a is not summed over), and α and β are colour indices. Models containing flavour-changing currents other than the $V - A$ form (arising in supersymmetric extensions of the Standard Model and other scenarios) usually result in mass differences that additionally depend on matrix elements of the four-quark operators [3]

$$\begin{aligned} \mathcal{O}_{2,aa} &= \bar{h}^\alpha (1 - \gamma_5) q_a^\alpha \bar{h}^\beta (1 - \gamma_5) q_a^\beta, \\ \mathcal{O}_{3,aa} &= \bar{h}^\alpha (1 - \gamma_5) q_a^\beta \bar{h}^\beta (1 - \gamma_5) q_a^\alpha, \\ \mathcal{O}_{4,aa} &= \bar{h}^\alpha (1 - \gamma_5) q_a^\alpha \bar{h}^\beta (1 + \gamma_5) q_a^\beta, \\ \mathcal{O}_{5,aa} &= \bar{h}^\alpha (1 - \gamma_5) q_a^\beta \bar{h}^\beta (1 + \gamma_5) q_a^\alpha, \end{aligned} \quad (1.2)$$

Generically we can represent these operators as

$$\mathcal{O}_{i,aa} = \bar{h} \Gamma_1 q \bar{h} \Gamma_2 q, \quad (1.3)$$

for the appropriate choice of spin and colour matrices, $\Gamma_{1,2}$. In lattice calculations, it is convenient to perform a Fierz transformation which renders linear combinations of the operators in Eq. (1.2) into products of colour-singlet currents. Nevertheless, we choose to work in the basis of Eq. (1.2).

In this article, we present a study for the light-quark mass and spatial volume dependence in matrix elements relevant to the mass differences in the neutral B and D meson mixing systems, using (partially quenched) heavy meson chiral perturbation theory¹. So far, only one exploratory numerical calculation in quenched lattice QCD has been carried out for the full set of these matrix elements [5]. However, with experimental progress in CKM physics, an accurate determination of these matrix elements, which involves reliable chiral extrapolations, will become necessary in the foreseeable future. In this work, we show that the chiral extrapolation for matrix elements of $\mathcal{O}_{1,aa}$ is considerably less complicated than that for matrix elements of the operators in Eq. (1.2). This is due to the fact $\mathcal{O}_{1,aa}$ preserves heavy quark spin symmetry, as explained in Sections 3 and 4. Details of the formulae for the chiral extrapolations of these matrix elements have been presented in Ref. [6].

¹Some of the matrix elements studied in this work are also relevant to the width difference in the B mixing system [4].

2. Heavy meson chiral perturbation in finite volume

Heavy meson chiral perturbation theory (HM χ PT) was formulated in Refs. [7, 8, 9], and generalised to the quenched and partially-quenched versions in Refs. [10, 11]. Finite spatial volume effects in this effective theory have also been investigated in Ref. [12]. Here we only sketch the ingredients necessary to study neutral heavy-light meson mixing systems. Details of this effective theory can be found in the above references.

In HM χ PT, the heavy-light meson field appears in the covariant form

$$H_a^{(Q)} = \frac{1 + \not{v}}{2} \left(P_{a,\mu}^{*(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5 \right), \quad (2.1)$$

where $P_a^{(Q)}$ and $P_{a,\mu}^{*(Q)}$ annihilate pseudoscalar and vector mesons containing a heavy quark Q and a light anti-quark of flavour a . In the heavy particle formalism, such mesons have momentum $p^\mu = M_P v^\mu + k^\mu$ with $|k^\mu| \ll M_P$ and v^μ is the velocity of the particle. Under a heavy quark spin $SU(2)$ transformation S and a generic light-flavour transformation U [*i.e.*, $U \in SU(3)$ for full QCD and $U \in SU(6|3)$ for partially-quenched QCD (PQQCD)],

$$H_a^{(Q)} \longrightarrow S H_b^{(Q)} U_{ba}^\dagger. \quad (2.2)$$

The conjugate field, which creates heavy-light mesons containing a heavy quark Q and a light anti-quark of flavour a , is defined as

$$\bar{H}_a^{(Q)} = \gamma^0 H^{(Q)\dagger} \gamma_0 = \left(P_{a,\mu}^{*(Q)\dagger} \gamma^\mu + P_a^{(Q)\dagger} \gamma_5 \right) \frac{1 + \not{v}}{2}, \quad (2.3)$$

which transforms under S and U as

$$\bar{H}_a^{(Q)} \longrightarrow U_{ab} \bar{H}_b^{(Q)} S^\dagger. \quad (2.4)$$

These heavy-light meson fields couple to the Goldstone meson fields

$$\xi \equiv e^{i\Phi/f} = \sqrt{\Sigma}, \quad (2.5)$$

where Σ is the ordinary nonlinear Goldstone field. The field ξ transforms as

$$\xi \longrightarrow U_L \xi U_R^\dagger = U \xi U_R^\dagger, \quad (2.6)$$

where $U_{L(R)}$ is an element of the left-handed (right-handed) $SU(3)$ and $SU(6|3)$ groups for QCD and PQQCD respectively.

The mesons containing a heavy anti-quark \bar{Q} and a light quark of flavour a can be included in the theory by applying the charge conjugation operation to the above heavy-light meson field $H_a^{(Q)}$. The field that annihilates such mesons is

$$H_a^{(\bar{Q})} = \left(P_{a,\mu}^{*(\bar{Q})} \gamma^\mu - P_a^{(\bar{Q})} \gamma_5 \right) \frac{1 - \not{v}}{2}, \quad (2.7)$$

which transforms under S and U as

$$H_a^{(\bar{Q})} \longrightarrow U_{ab} H_b^{(\bar{Q})} S^\dagger. \quad (2.8)$$

The main effects of heavy quark symmetry breaking in HM χ PT Lagrangian is the splitting between vector and pseudoscalar heavy-light meson masses [13]

$$\frac{\lambda_2}{M_P} \text{tr}_D \left(\bar{H}_a^{(Q)} \sigma_{\mu\nu} H_a^{(Q)} \sigma^{\mu\nu} \right), \quad (2.9)$$

where λ_2 is a low energy constant (LEC), and tr_D means trace in the spinor indices.

3. Four-fermion operators

Under a chiral transformation, the four-quark operators in Eq. (1.2) fall into two categories:

$$\begin{aligned} \mathcal{O}_{LL} &= \bar{h} \Gamma_{LL} q_L \bar{h} \Gamma_{LL} q_L, \\ \mathcal{O}_{LR} &= \bar{h} \Gamma_{LR}^{(1)} q_L \bar{h} \Gamma_{LR}^{(2)} q_R, \end{aligned} \quad (3.1)$$

where

$$q_{L,R} = \frac{1 \pm \gamma_5}{2} q. \quad (3.2)$$

Operators $\mathcal{O}_{1,aa}$, $\mathcal{O}_{2,aa}$ and $\mathcal{O}_{3,aa}$ are of the first type and transform in the symmetric $(\mathbf{6}_L, \mathbf{1}_R)$ representation built from the direct product $(\mathbf{3}_L, \mathbf{1}_R) \otimes (\mathbf{3}_L, \mathbf{1}_R) = (\mathbf{6}_L, \mathbf{1}_R) \oplus (\bar{\mathbf{3}}_L, \mathbf{1}_R)$ under chiral rotations while $\mathcal{O}_{4,aa}$ and $\mathcal{O}_{5,aa}$ are of the second type and transform in the $(\mathbf{3}_L, \mathbf{3}_R)$ representation. Here we refer to the SU(3) flavour transformation properties, leaving the partially quenched extension to the following subsection. Note that the colour indices in Eq. (1.2) are relevant to short-distance physics, and hence play no role in the chiral properties of these operators. Treating Γ_{LL} , $\Gamma_{LR}^{(1)}$ and $\Gamma_{LR}^{(2)}$ as spurions transforming as

$$\begin{aligned} \Gamma_{LL} &\longrightarrow S \Gamma_{LL} U_L^\dagger, \\ \Gamma_{LR}^{(1)} &\longrightarrow S \Gamma_{LR}^{(1)} U_L^\dagger, \\ \Gamma_{LR}^{(2)} &\longrightarrow S \Gamma_{LR}^{(2)} U_R^\dagger, \end{aligned} \quad (3.3)$$

the operators in Eq. (3.1) remain invariant under heavy-quark spin and chiral rotations. The bosonisation of the operators in Eqs. (1.1) and (1.2) can be performed using these spurion transformation properties. The procedure is explained in details in Ref. [6]. It leads to the following set of operators involving the individual heavy meson fields:

$$\begin{aligned} \mathcal{O}_{1,aa}^{\text{HM}\chi\text{PT}} &= \beta_1 \left[\left(\xi P^{(h)\dagger} \right)_a \left(\xi P^{(\bar{h})} \right)_a + \left(\xi P_\mu^{*(h)\dagger} \right)_a \left(\xi P^{*(\bar{h}),\mu} \right)_a \right], \\ \mathcal{O}_{2(3),aa}^{\text{HM}\chi\text{PT}} &= \beta_{2(3)} \left(\xi P^{(h)\dagger} \right)_a \left(\xi P^{(\bar{h})} \right)_a + \beta'_{2(3)} \left(\xi P_\mu^{*(h)\dagger} \right)_a \left(\xi P^{*(\bar{h}),\mu} \right)_a, \\ \mathcal{O}_{4(5),aa}^{\text{HM}\chi\text{PT}} &= \beta_{4(5)} \left(\xi P^{(h)\dagger} \right)_a \left(\xi^\dagger P^{(\bar{h})} \right)_a + \hat{\beta}_{4(5)} \left(\xi^\dagger P^{(h)\dagger} \right)_a \left(\xi P^{(\bar{h})} \right)_a \\ &\quad + \beta'_{4(5)} \left(\xi P_\mu^{*(h)\dagger} \right)_a \left(\xi^\dagger P^{*(\bar{h}),\mu} \right)_a + \hat{\beta}'_{4(5)} \left(\xi^\dagger P_\mu^{*(h)\dagger} \right)_a \left(\xi P^{*(\bar{h}),\mu} \right)_a, \end{aligned} \quad (3.4)$$

where the β_i , β'_i , $\hat{\beta}_i$, and $\hat{\beta}'_i$ are LECs.

It is important to note that in the above equation, the operator $\mathcal{O}_{1,aa}^{\text{HM}\chi\text{PT}}$ behaves somewhat differently from the other operators as only a single LEC, β_1 , occurs. This greatly simplifies any chiral extrapolation of corresponding lattice data for neutral heavy-light meson mixing in the Standard

Model, as discussed in the next section. We stress that this simplification is not obvious from the operator structure and is particular to the $V - A$ form of the Standard Model currents. In general, one would expect that operators for pseudoscalar and vector meson mixing processes are accompanied by different LECs. This is the case for all the non-Standard-Model operators, as shown in Eq. (3.4).

To understand the origin of the above simplification in the Standard Model operator $\mathcal{O}_{1,aa}^{\text{HM}\chi\text{PT}}$, we turn to heavy quark effective theory (HQET). In this effective theory, the operators that produce the same matrix elements as those in Eq.(1.3) are [14]

$$\mathcal{O}_{i,aa}^{\text{HQET}} = \tilde{Q}\Gamma_1 q_a Q^\dagger \Gamma_2 q_a + Q^\dagger \Gamma_1 q_a \tilde{Q}\Gamma_2 q_a, \quad (3.5)$$

where $\Gamma_{1,2}$ are the appropriate Dirac and colour structures from Eq. (1.2). Here, Q and \tilde{Q} denote fields annihilating a heavy quark and heavy anti-quark, respectively (these fields do not create the corresponding anti-particles). Additional terms in HQET which create two heavy quarks or annihilate two heavy anti-quarks will not contribute to neutral heavy-meson mixing and are ignored.

The standard model operator in HQET, $\mathcal{O}_{1,aa}^{\text{HQET}}$, satisfies the relation

$$\left\{ S_Q^3, \mathcal{O}_{1,aa}^{\text{HQET}} \right\} |P\rangle = \left\{ S_Q^3, \mathcal{O}_{1,aa}^{\text{HQET}} \right\} |\bar{P}\rangle = 0, \quad (3.6)$$

where $|P\rangle$ is pseudoscalar heavy-light meson state, and $|\bar{P}\rangle$ is the state of its anti-particle. The operator

$$S_Q^3 = \varepsilon^{ij3} [Q^\dagger \sigma_{ij} Q - \tilde{Q} \sigma_{ij} \tilde{Q}^\dagger], \quad (3.7)$$

is the heavy quark spin operator [15] that changes the spin of the heavy-light meson state by one. Therefore, Eq. (3.6) implies that the mixing matrix elements for vector and pseudoscalar heavy-light mesons are equal and opposite in the heavy-quark limit. This symmetry is reflected in HM χ PT, leading to the result for $\mathcal{O}_{1,aa}^{\text{HM}\chi\text{PT}}$ in Eq. (3.4).

For the non-Standard-Model operators, it is straightforward to show that

$$\left\{ S_Q^3, \mathcal{O}_{i,aa}^{\text{HQET}} \right\} |P\rangle \neq 0, \quad \left\{ S_Q^3, \mathcal{O}_{i,aa}^{\text{HQET}} \right\} |\bar{P}\rangle \neq 0, \quad (3.8)$$

and

$$\left[S_Q^3, \mathcal{O}_{i,aa}^{\text{HQET}} \right] |P\rangle \neq 0, \quad \left[S_Q^3, \mathcal{O}_{i,aa}^{\text{HQET}} \right] |\bar{P}\rangle \neq 0, \quad (3.9)$$

where $i = 2, 3, 4, 5$. This means that the pseudoscalar and vector meson mixing processes via these operators are not proportional to each other, hence the appearance of the terms accompanied by $\beta'_{2,3,4,5}$ and $\hat{\beta}'_{4,5}$ in Eq. (3.4).

We end this section by noting that equations of motion for the heavy quark [5] result in $\mathcal{O}_{3,aa} = -\mathcal{O}_{1,aa}/2 - \mathcal{O}_{2,aa}$, and can be used to relate some of the LECs in Eq. (3.4).

4. Neutral meson mixing matrix elements

Calculations at NLO in the chiral expansion require the evaluation of the one-loop diagrams shown in Figure 1. We perform these calculations both at infinite volume and in a cubic spatial box of dimensions L^3 (the time extent is assumed to be infinite). In this section we summarise the

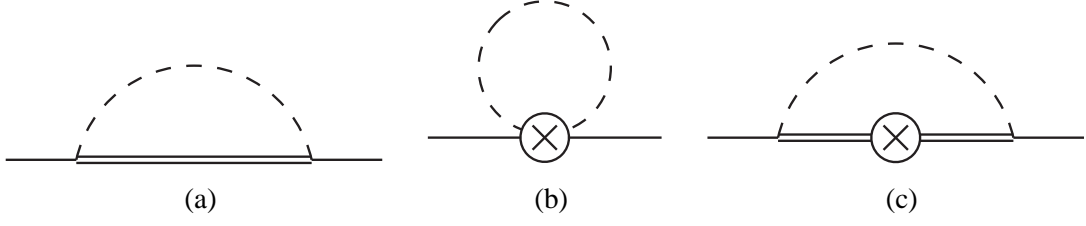


Figure 1: Diagrams contributing to the matrix elements of four-quark operators at NLO in the chiral expansion. Solid, double and dashed lines correspond to propagators of pseudoscalar and vector heavy-light mesons, and Goldstone mesons, respectively. The crossed circle denotes the four-quark operator and diagram (a) is the wave-function renormalisation.

results, relegating details of the calculations to Ref. [6]. Although we present results specifically in the B -meson systems, note that they are also applicable to D -meson systems, under the assumption that the charm-quark mass is large enough compared to Λ_{QCD} .

For the standard model operator we find the following matrix elements

$$\begin{aligned} \langle \bar{B}^0 | \mathcal{O}_{1,dd} | B^0 \rangle &= \beta_1 \left(1 + \mathcal{F}_d^{(1)} + \frac{\mathcal{W}_{\bar{B}^0} + \mathcal{W}_{B^0}}{2} + \mathcal{Q}_d^{(1)} \right) + \text{analytic terms}, \\ \langle \bar{B}_s^0 | \mathcal{O}_{1,ss} | B_s^0 \rangle &= \beta_1 \left(1 + \mathcal{F}_s^{(1)} + \frac{\mathcal{W}_{\bar{B}_s^0} + \mathcal{W}_{B_s^0}}{2} + \mathcal{Q}_s^{(1)} \right) + \text{analytic terms}. \end{aligned} \quad (4.1)$$

The “analytic terms” here include Goldstone meson mass squared terms, a term $\sim \alpha_s(M_b)/4\pi$ (arising from mixing) and a term $\sim \Lambda_{\text{QCD}}/M_b$. The wave-function contributions, \mathcal{W}_M (diagram (a) in Figure 1), and tadpole- and sunset-type operator renormalisations, $\mathcal{F}_a^{(i)}$ and $\mathcal{Q}_a^{(i)}$ (diagrams (b) and (c) in Figure 1, respectively) are non-analytic functions of the light quark mass and lattice volume and are defined in Ref. [6].

For the operators that contribute to the B -meson mixing processes beyond the Standard Model, we obtain:

$$\begin{aligned} \langle \bar{B}^0 | \mathcal{O}_{2(3),dd} | B^0 \rangle &= \beta_{2(3)} \left(1 + \mathcal{F}_d^{(2(3))} + \frac{\mathcal{W}_{\bar{B}^0} + \mathcal{W}_{B^0}}{2} \right) + \beta'_{2(3)} \mathcal{Q}_d^{(2(3))} + \dots, \\ \langle \bar{B}_s^0 | \mathcal{O}_{2(3),ss} | B_s^0 \rangle &= \beta_{2(3)} \left(1 + \mathcal{F}_s^{(2(3))} + \frac{\mathcal{W}_{\bar{B}_s^0} + \mathcal{W}_{B_s^0}}{2} \right) + \beta'_{2(3)} \mathcal{Q}_s^{(2(3))} + \dots, \\ \langle \bar{B}^0 | \mathcal{O}_{4(5),dd} | B^0 \rangle &= [\beta_{4(5)} + \hat{\beta}_{4(5)}] \left(1 + \mathcal{F}_d^{(4(5))} + \frac{\mathcal{W}_{\bar{B}^0} + \mathcal{W}_{B^0}}{2} \right) + [\beta'_{4(5)} + \hat{\beta}'_{4(5)}] \mathcal{Q}_d^{(4(5))} + \dots, \\ \langle \bar{B}_s^0 | \mathcal{O}_{4(5),ss} | B_s^0 \rangle &= [\beta_{4(5)} + \hat{\beta}_{4(5)}] \left(1 + \mathcal{F}_s^{(4(5))} + \frac{\mathcal{W}_{\bar{B}_s^0} + \mathcal{W}_{B_s^0}}{2} \right) + [\beta'_{4(5)} + \hat{\beta}'_{4(5)}] \mathcal{Q}_s^{(4(5))} + \dots, \end{aligned} \quad (4.2)$$

where “...” represents the “analytic terms” as in Eq. (4.1). The terms $\sim \mathcal{Q}_q^{(i)}$ arising from the sunset diagrams [Fig. 1(c)] involve the neutral heavy-light vector meson mixing amplitudes. As discussed in the preceding section, it is only in the case of $\mathcal{O}_{1,qq}$ that these amplitudes are related to those of the pseudoscalar heavy-light mesons. For $i = 2, 3, 4, 5$ these terms are consequently accompanied by different LECs.

The above results lead to the fact that the chiral extrapolations for B mixing matrix elements in and beyond the Standard Model have different features. Generically, the chiral expansion for $\langle \bar{B}_{(s)}^0 | \mathcal{O}_{1,aa} | B_{(s)}^0 \rangle$ takes the form

$$\langle \bar{B}_{(s)}^0 | \mathcal{O}_{1,aa} | B_{(s)}^0 \rangle \xrightarrow{\text{chiral}} \gamma_1 (1 + L) + \text{analytic terms}, \quad (4.3)$$

where γ_1 is the leading-order LEC, L denotes the non-analytic one-loop contributions (chiral logarithms), and the analytic terms are from the next-to-leading-order counter-terms in the chiral expansion. However, for the operators in Eq. (1.2), the chiral expansion has the generic feature:

$$\langle \bar{B}_{(s)}^0 | \mathcal{O}_{i,aa} | B_{(s)}^0 \rangle \xrightarrow{\text{chiral}} \gamma_i (1 + L) + \gamma'_i L' + \text{analytic terms}, \quad (4.4)$$

where $i = 2, 3, 4, 5$, γ_i and γ'_i are unknown leading-order LECs, and L and L' are different one-loop chiral logarithms. The appearance of the second non-analytic term complicates the chiral extrapolation in Eq. (4.4) because an additional unknown parameter must be determined.

We end this article by noting that the effects of scalar resonances in these matrix elements have been studied in Ref. [16] and found to be negligible.

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