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$B-\bar{B}$-Mixing with Domain Wall Fermions in the Static Approximation

RBC and UKQCD Collaborations

C. Albertus[a], Y. Aoki[b], P.A. Boyle[c], N.H. Christ[d], L. Del Debbio[c], T.T. Dumitrescu[d], J.M. Flynn[a], T. Izubuchi[e], O. Loktik[d], C.T. Sachrajda[a], A. Soni[b], J. Wennekers[*c]

[a] *School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK*
[b] *Brookhaven National Laboratory, Upton, NY 11973, USA*
[c] *SUPA, School of Physics, The University of Edinburgh, Edinburgh EH9 3JZ, UK*
[d] *Physics Department, Columbia University, New York, NY 10027, USA*
[e] *Institute for Theoretical Physics, Kanazawa University, Kanazawa, Ishikawa 920-1192, Japan*

We present preliminary numerical results for the pseudoscalar decay constants f_B and f_{B_s} and the $\Delta B = 2$ mixing matrix elements in the B^0 and B_s^0 systems. We use the static approximation for the b-quark with two variants of gauge field smearing (APE and HYP2) in the static lattice action. The light quarks are 2+1 dynamical flavours of Domain Wall Fermions at fixed lattice spacing of 0.12 fm with the lightest pion mass at 400 MeV. At this lattice spacing we have found large differences in our observables computed with the different smearings. The matching to the renormalised continuum theory is done at one loop in perturbation theory.

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1. Introduction

The mixing of particles and antiparticles in the B_q^0 systems ($q = d,s$) has been studied in recent experiments. These measurements of oscillation frequencies given in terms of mass differences ΔM_q have achieved a remarkable precision,

$$\Delta M_d = (0.507 \pm 0.005)\,\mathrm{ps}^{-1} \quad [1], \tag{1.1}$$

$$\Delta M_s = (17.77 \pm 0.10\,(\mathrm{stat}) \pm 0.07\,(\mathrm{sys}))\,\mathrm{ps}^{-1} \quad [2]. \tag{1.2}$$

In the Standard Model the frequencies are related to CKM matrix elements by

$$\Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 S_0 \left(\frac{m_t^2}{m_W^2}\right) \eta_B m_{B_q} f_{B_q}^2 B_{B_q}. \tag{1.3}$$

The relation allows a determination of the ratio $|V_{ts}/V_{td}|$ when provided with a theoretical input for the relevant $\Delta B = 2$ weak matrix elements $f_{B_q}^2 B_{B_q}$. Lattice simulations are a unique non-perturbative way to compute f_{B_q} and B_{B_q}. However it may be impossible to match the experimental precision.

Here we present a study using static b-quarks and Domain Wall Fermion light quarks. The large scale difference prohibits the direct simulation of relativistic b-quarks with current (super-) computers. Our choice of effective action is described in the following section alongside details of our simulation. In Section 4 we summarise results on the decay constants f_B and f_{B_s}. Section 5 contains our results on the mixing matrix elements and there is a short conclusion in the final section.

2. Lattice action

The static approximation is the lowest order of a systematic expansion in the b-quark mass. In the infinite mass limit the b-quark becomes literally static, it propagates in time only,

$$S_{\mathrm{static}} = \sum_x \overline{h}(x)[h(x) - V_0^\dagger(x-\hat{0})h(x-\hat{0})], \tag{2.1}$$

where V is the smeared gauge field. In the case of $V_0 = U_0$ (the original gauge link) this action was proposed by Eichten and Hill [3]. Large statistical fluctuations have made is necessary to used smeared gauge fields in the static action. We consider two different smearings: one-level APE smearing [4] with $\alpha = 1$ and Hypercubic blocking (HYP smearing) [5] with $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5)$ as advertised in [6] and usually referred to as HYP2.

The gauge configurations used in this project have been generated with the Iwasaki gauge action [7] and Domain Wall fermion action [8, 9] with 2+1 flavours. They are part of the current research programme of the RBC and UKQCD collaborations [10]. Details of the used $L = 2\,\mathrm{fm}$ ensembles can be found in [11]. The $16^3 \times 32$ lattices at $\beta = 2.13$ have a measured lattice cut-off of $a^{-1} = 1.62(4)\,\mathrm{GeV}$. Domain Wall fermions have an approximate chiral symmetry whose breaking is determined from the violation of a five-dimensional Ward identity. The resulting residual mass for these lattices is $am_{\mathrm{res}} = 0.00308(4)$ which corresponds to a bare mass of $5\,\mathrm{MeV}$. The input quark masses are $am_l = 0.01, 0.02, 0.03$ for the light quarks (u,d) and $am_s = 0.0359$ for the strange quark. This corresponds to the measured value of m_s on these lattices [11]. The lowest pion mass reached is $400\,\mathrm{MeV}$.

3. Renormalisation

The perturbative renormalisation and matching to the continuum of the static-light axial current A_0^{stat} and the four-fermion operator O_{VV+AA} have been described in Thomas Dumitrescu's contribution [12]. The renormalisation factors for both APE and HYP smearing will be published in [13]. Here we just cite the results for two possible definitions of the gauge coupling, mean-field improved and $\overline{\text{MS}}$ for $\mu = a^{-1}$. We use the average of the two as central value and half the difference as an additional systematic error. The parity even operator $V_\mu V_\mu + A_\mu A_\mu$ mixes in the

		Z_Φ	Z_{VA}	Z_{SP}
no-smear	MF	0.899	0.642	-0.133
no-smear	$\overline{\text{MS}}$	0.794	0.458	-0.123
APE	MF	0.961	0.769	-0.133
APE	$\overline{\text{MS}}$	0.902	0.674	-0.123
HYP	MF	0.985	0.819	-0.133
HYP	$\overline{\text{MS}}$	0.946	0.761	-0.123

Table 1: Overall perturbative matching factors evaluated for unsmeared, APE smeared or HYP smeared static quark gauge links and for two choices of coupling constant α^{MF} and $\alpha^{\overline{\text{MS}}}$.

static-light case with $SS+PP$ under renormalisation. The renormalised matrix element is given by $M = Z_{VA} M_{VV+AA} + Z_{SP} M_{SS+PP}$.

4. Decay constants calculation

The pseudoscalar decay constants for the B^0 and B_s^0 meson are determined from the two-point functions of the static-light axial current $A_\mu^{\text{stat}} = \overline{h}\gamma_\mu\gamma_5 q$. To obtain spatial volume averaging we use (gauge-fixed) wall sources which extend over a whole timeslice (L^3 points). The normalisation of these sources is not known a priori. So we compute a ratio in which it drops out,

$$\Phi_q = \frac{\sqrt{2}C^{\text{WL}}(t)}{\sqrt{C^{\text{WW}}(t)\mathrm{e}^{-m_B^* t} L^3}}, \tag{4.1}$$

where C^{WL} and C^{WW} are the wall-local/wall-wall two-point functions of A_0^{stat}. m_B^* is the effective mass obtained from C^{WL} which is used to cancel the asymptotic time dependence of the ratio Φ_q, Fig.1a. Since $Z_\Phi \Phi_q = \sqrt{m_{B_q}} f_{B_q}$ it is sufficient to compute Φ_q to get the bare decay constants. Fitting $\Phi_{d/s}$ in the interval $12-16$ then gives our results summarised in Table 2.

With the existing data the extrapolation to the physical light quark mass is done linearly. Here we use $m_{l,\text{phys}}^{\text{bare}} + m_{\text{res}} = 0.00162(8)$ [11].

Combining these results with Z_Φ from Table 1 and using $m_B = 5279$ MeV and $m_{B_s} = 5368$ MeV

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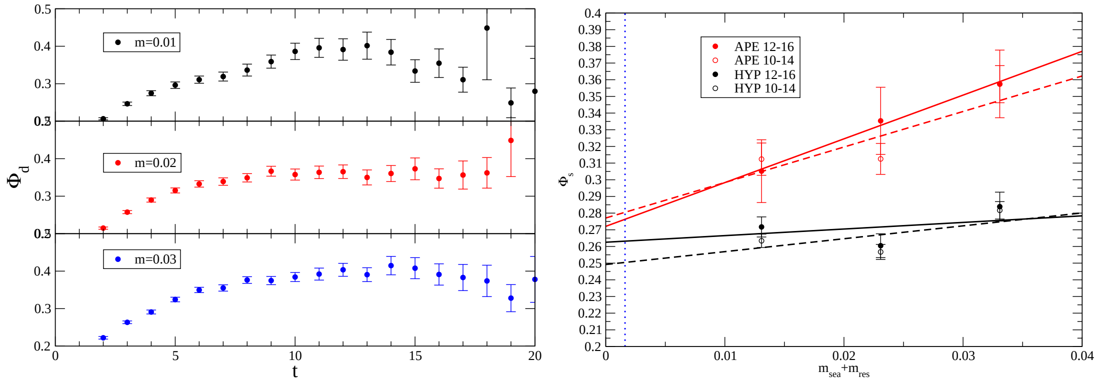

(a) Time dependence of Φ_d for the three ensembles from the data with HYP smearing.

(b) Chiral extrapolation of Φ_s for the two static actions and two different fit ranges. The filled symbols correspond to the data in the bottom half of Table 2. The blue dotted line marks the physical point.

Figure 1: Plateaux for Φ_d^{HYP} and chiral extrapolation for Φ_s for both static actions

$m_{\mathrm{sea}}+m_{\mathrm{res}}$	$m_{\mathrm{val}}+m_{\mathrm{res}}$	Φ_q	
		APE	HYP
0.01308	0.01308	0.274(29)	0.255(14)
0.02308	0.02308	0.319(21)	0.251(11)
0.03308	0.03308	0.354(19)	0.280(10)
0.01308	0.039	0.305(19)	0.272(6)
0.02308	0.039	0.335(20)	0.260(7)
0.03308	0.039	0.357(20)	0.283(9)

Table 2: Results for Φ_d and Φ_s from a fit to timeslices 12-16.

[1] we obtain

$$f_{B_d}^{\mathrm{stat}} = \begin{cases} 193(35)(^{+16}_{-28})\,\mathrm{MeV} & (\mathrm{APE}), \\ 198(18)(^{+15}_{-27})\,\mathrm{MeV} & (\mathrm{HYP}), \end{cases} \tag{4.2}$$

$$f_{B_s}^{\mathrm{stat}} = \begin{cases} 229(26)(18)\,\mathrm{MeV} & (\mathrm{APE}), \\ 216(6)(17)\,\mathrm{MeV} & (\mathrm{HYP}). \end{cases} \tag{4.3}$$

The first error is statistical while the second is systematic and contains uncertainties due to the chiral extrapolation, the renormalisation factor and the setting of the lattice scale. The asymmetric error is chosen to reflect a possible logarithmic contribution [14] in the chiral extrapolation. While we do not observe such a behaviour, assuming it sets in just below our lightest data point leads to a six percent downwards error.

The ratio $\xi_f = f_{B_s}/f_B$ is independent of the renormalisation factor Z_Φ and is expected to give better agreement between the two smearing procedures. We get

$$\frac{f_{B_s}^{\rm stat}}{f_{B_d}^{\rm stat}} = \frac{\Phi_{B_s}}{\Phi_{B_d}}\sqrt{\frac{m_{B_d}}{m_{B_s}}} = \begin{cases} 1.14(4)(^{+10}_{-3}) & (\text{APE}), \\ 1.08(2)(^{+10}_{-3}) & (\text{HYP}). \end{cases} \tag{4.4}$$

In the extrapolation to the physical light quark mass we use that Φ_s/Φ_l is constrained to unity for $m_l = m_s$, Fig. 2.

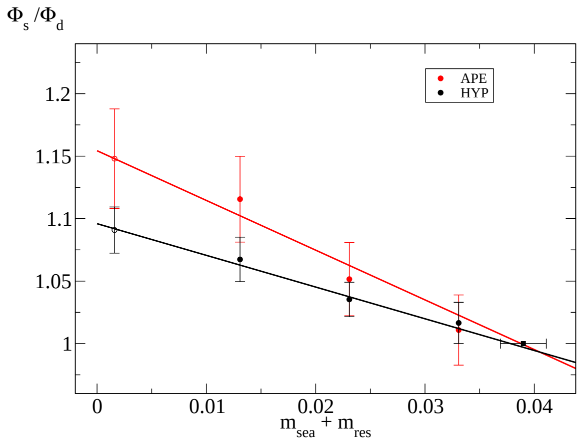

Figure 2: Chiral extrapolation of the ratio Φ_s/Φ_d for the two static actions. The black square shows the constraint at $m_l = m_s$ and the open symbols are the extrapolated results at the physical point. The errors are statistical only.

5. Matrix elements calculation

The relevant $\Delta B = 2$ matrix element is parametrised as $f_{B_q}\sqrt{B_{B_q}}$. We determine this product in two separate ways. One is computing

$$M_{O_i}(t_1,t) \equiv \frac{C^{\rm B}_{O_i}(t_1,t)\mathrm{e}^{m_B^* t_1/2}}{\sqrt{C^{\rm BB}(t,t_1)C^{\rm BB}(t,0)}} \xrightarrow{t_1 \gg t \gg 0} \langle \bar{B}|O_i^{\rm latt}(0)|B\rangle. \tag{5.1}$$

Here we use a box source of size 8^3 and APE-smearing in the static action. The data is summarised in Table 3. The renormalised matrix element is obtained using the perturbative Z-factor in Table 1.

The other method is computing the bag parameter from

$$B_{O_i} \equiv \frac{C^{\rm W}_{O_i}(t_1,t,0)}{C^{\rm WL}(t,0)C^{WL}(t_1,t)} \xrightarrow{t_1 \gg t \gg 0} \frac{\langle \bar{B}(t_1)|O_i^{\rm latt}(t)|B(0)\rangle}{\frac{8}{3}m_{B_q}^2 f_{B_q}^2}. \tag{5.2}$$

The analogue of this ratio is a standard method used in the kaon system. We analyse the HYP data in this way (Table 3). The values for the bag parameters at the physical point are $B_B^{\rm HYP} = 0.74(10)(3)$ and $B_{B_s}^{\rm HYP} = 0.79(3)(3)$. Here we assume the that the chiral logarithm in B_B is small.

$m_{\rm sea}$	$m_{\rm val}$	$M^{\rm APE}_{VV+AA}$	$M^{\rm APE}_{SS+PP}$	$M^{\rm APE}_{q}$	$B^{\rm HYP}_{VV+AA}$	$B^{\rm HYP}_{SS+PP}$	$B^{\rm HYP}_{B_q}$
0.01	0.01	0.235(19)	−0.142(8)	0.199(14)	0.821(32)	−0.544(16)	0.719(27)
0.02	0.02	0.272(15)	−0.179(7)	0.233(11)	0.947(22)	−0.558(10)	0.820(18)
0.03	0.03	0.272(11)	−0.176(5)	0.232(8)	0.900(18)	−0.578(9)	0.786(15)
0.01	0.0359	0.276(13)	−0.169(6)	0.235(9)	0.901(11)	−0.565(6)	0.785(9)
0.02	0.0359	0.309(15)	−0.187(7)	0.262(11)	0.920(11)	−0.573(5)	0.801(9)
0.03	0.0359	0.295(15)	−0.188(7)	0.252(11)	0.902(11)	−0.572(6)	0.786(9)

Table 3: Lattice results for the matrix element with APE smearing and the B parameter with HYP smearing for three ensembles and two valence masses each. The errors are statistical only.

Combining the results for B_{B_q} with the decay constants from the previous section we can make a comparison of the two static actions. Extrapolating both data sets linearly we obtain

$$f^{\rm stat}_{B_d}\sqrt{B^{\rm stat}_{B_d}(m_b)} = \begin{cases} 237(13)(^{+19}_{-26})\,\text{MeV} & (\text{APE}), \\ 171(16)(^{+15}_{-21})\,\text{MeV} & (\text{HYP}), \end{cases} \tag{5.3}$$

$$f^{\rm stat}_{B_s}\sqrt{B^{\rm stat}_{B_s}(m_b)} = \begin{cases} 262(12)(22)\,\text{MeV} & (\text{APE}), \\ 192(6)(17)\,\text{MeV} & (\text{HYP}). \end{cases} \tag{5.4}$$

Again there is a large discrepancy between the two data sets even taking possible $O(a^2)$ lattice artefacts into account. We are investigating other possible sources for this effect at the moment.

We also compute the SU(3) flavour-breaking ratio

$$\xi = \frac{f^{\rm stat}_{B_s}\sqrt{B^{\rm stat}_{B_s}}}{f^{\rm stat}_{B_d}\sqrt{B^{\rm stat}_{B_d}}} = \begin{cases} 1.11(7)(^{+13}_{-4}) & (\text{APE}), \\ 1.14(8)(^{+13}_{-4}) & (\text{HYP}). \end{cases} \tag{5.5}$$

As expected the ratio is consistent between the two calculations. This can be due to the large cancellation in the renormalisation.

6. Conclusions

We have presented a 2+1 flavour DWF calculation of static-light decay constants and bag parameters/mixing matrix elements. The results at $a^{-1} = 1.62(4)\,\text{GeV}$ have been extrapolated to the physical point using three independent ensembles, unitary masses for the light quarks and the physical value for the strange quark mass. We use perturbative estimates for the renormalisation of the static-light axial current and the operator mixing of the parity even part of the left-left four fermion operator.

Our preliminary values for the decay constants are $f_{B_s} = (220 \pm 32)\,\text{MeV}$ and $f_{B_s}/f_B = 1.10(^{+11}_{-5})$. For the bag parameters we get $B_{B_s} = 0.79(4)$ and $B_B = 0.74(10)$. We also determine the full matrix element directly; the results can be found in Section 5. We are investigating the large differences found between the two static actions. The quoted uncertainties contain an estimate of the error induced by chiral extrapolation, perturbative renormalisation, lattice artefacts and the statistical error. Recently there has been a new determination of the lattice scale at this coupling but on a larger volume. The new value of $a^{-1} = 1.73(3)\,\mathrm{GeV}$ [15] leads to an additional 7% error which has to be added in quadrature. Possible corrections to the limit of infinite b-quark mass are beyond the scope of this project. A full account of our work will be published in [13].

References

[1] **Particle Data Group** Collaboration, W. M. Yao *et. al.*, *Review of particle physics*, *J. Phys.* **G33** (2006) 1–1232.

[2] **CDF** Collaboration, A. Abulencia *et. al.*, *Observation of b/s0 anti-b/s0 oscillations*, `hep-ex/0609040`.

[3] E. Eichten and B. Hill, *An effective field theory for the calculation of matrix elements involving heavy quarks*, *Phys. Lett.* **B234** (1990) 511.

[4] **APE** Collaboration, M. Albanese *et. al.*, *Glueball masses and string tension in lattice qcd*, *Phys. Lett.* **B192** (1987) 163.

[5] A. Hasenfratz and F. Knechtli, *Flavor symmetry and the static potential with hypercubic blocking*, *Phys. Rev.* **D64** (2001) 034504 [`hep-lat/0103029`].

[6] M. Della Morte, A. Shindler and R. Sommer, *On lattice actions for static quarks*, *JHEP* **08** (2005) 051 [`hep-lat/0506008`].

[7] Y. Iwasaki, *Renormalization group analysis of lattice theories and improved lattice action. 2. four-dimensional nonabelian su(n) gauge model*, *UTHEP-118* (1983).

[8] Y. Shamir, *Chiral fermions from lattice boundaries*, *Nucl. Phys.* **B406** (1993) 90–106 [`hep-lat/9303005`].

[9] V. Furman and Y. Shamir, *Axial symmetries in lattice qcd with kaplan fermions*, *Nucl. Phys.* **B439** (1995) 54–78 [`hep-lat/9405004`].

[10] P. A. Boyle, *2+1 flavour domain wall fermion simulations by the RBC and UKQCD collaborations*, `PoS(LATTICE 2007)005`.

[11] **RBC and UKQCD** Collaboration, C. Allton *et. al.*, *2+1 flavor domain wall qcd on a (2-fm)**3 lattice: Light meson spectroscopy with l(s) = 16*, `hep-lat/0701013`.

[12] N. H. Christ, T. T. Dumitrescu, T. Izubuchi and O. Loktik, *The static approximation to b meson mixing using light domain-wall fermions: Perturbative renormalization and ground state degeneracies*, `PoS(LATTICE 2007)351`.

[13] C. Albertus, Y. Aoki, P. A. Boyle, N. H. Christ, L. Del Debbio, T. T. Dumitrescu, J. M. Flynn, T. Izubuchi, O. Loktik, C. T. Sachrajda, A. Soni and J. Wennekers, *Unquenched B meson decay constants and $B^0-\overline{B}^0$ mixing parameters from chiral lattice QCD*, *in preparation*.

[14] S. R. Sharpe and Y. Zhang, *Quenched chiral perturbation theory for heavy-light mesons*, *Phys. Rev.* **D53** (1996) 5125–5135 [`hep-lat/9510037`].

[15] **RBC and UKQCD** Collaboration, D. J. Antonio *et. al.*, *Neutral Kaon Mixing from 2+1 Flavor Domain Wall QCD*, `hep-ph/0702042`.