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Loop Quantum Gravity and Effective Theory

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To descend from a fundamental theory to low energy phenomena one usually makes use of effective equations which capture the main quantum effects in a specified regime. A Hamiltonian procedure to derive effective equations is first illustrated by the example of an anharmonic oscillator and then applied to quantum cosmology. Extending this to full gravity demonstrates in which sense loop quantum gravity provides an emergence scenario for space-time from a microscopic theory.

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1. Effective theory

Quantum gravity is a "fundamental" theory formulated at extremely high energies which, for most practical applications, needs contact with low energies and thus phenomenology. A widely used tool to achieve this is that of effective equations. Such equations provide useful approximations, but not to the whole quantum theory. They rather describe certain regimes, such as low energy ones, and thus require additional information or assumptions to select such a regime. For instance, one needs to know some properties of a suitable low energy state, such as the vacuum of the theory. Once this is given, effective equations provide quantum effects through systematic derivations of perturbations around this state.

The tool of effective equations is thus not completely general, its applicability depending on whether sufficient information on interesting states can be obtained without actually completely solving the quantum theory to be evaluated. Partial solutions must be known, or assumptions are necessary which must be justified differently. The tool is most powerful if a solvable theory close enough to the one of interest is available. Such a solvable theory is then usually called "free theory" as it is the absence of complicated interactions which makes it explicitly solvable. In such a situation, a free vacuum state or other state of interest is known fully, and modifications to the free vacuum state due to interactions can be included perturbatively. More generally, without reference to a specific low energy regime, the availability of a free theory allows perturbative constructions of dynamical coherent states even for complicated theories.

Often, it is not the interacting vacuum or the dynamical coherent state itself which is of interest but rather a semiclassical description of the selected sector. As shown below, the kind of solvability required to make effective equations feasible implies that the free theory has coherent states whose expectation values follow the classical trajectories exactly. This is no longer true once interactions are switched on and classical equations have to be amended by quantum corrections. Effective equations manage to translate state properties from the behavior of dynamical coherent states into quantum corrections to the classical dynamics. Intuitively, this captures the back-reaction of the spreading and deformations of an evolving state on its expectation values.

2. Illustration: anharmonic oscillator

The best known example is that of quantum field theory where interactions are included perturbatively starting from a free theory with a quadratic action in the canonical fields. Perturbation terms are organized in Feynman expansions for the coupling of n-point functions, but often one can also compute an effective action through path integration. The same techniques can be applied to an anharmonic oscillator, where the harmonic oscillator represents the free theory. Here, it is well-known that coherent states exist whose expectation values follow the classical trajectory precisely, and they can in fact easily be written down. This is no longer true when an anharmonicity $U(q)$ is added to the potential. Then, expectation values of states do not follow the classical trajectory anymore but rather satisfy equations of motion which derive from the effective action

$$\Gamma_{\rm eff}[q] = \int {\rm d}t \left(\left(m + \frac{\hbar (U''')^2}{32 m^2 \left(\omega^2 + \frac{U''}{m}\right)^{\frac{5}{2}}} \right) \frac{\dot{q}^2}{2} - \frac{1}{2} m\omega^2 q^2 - U - \frac{\hbar\omega}{2}\left(1 + \frac{U''}{m\omega^2}\right)^{\frac{1}{2}} \right) \qquad (2.1)$$

to first order in $\hbar$ [1]. In this section, we briefly illustrate how this result can be derived, but using the previously described picture of back-reaction of spread and deformations on expectation values rather than generating functions of 1-particle irreducible n-point functions.

We use a Hamiltonian $\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{q}) = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{q}^2 + \frac{1}{3}\lambda\hat{q}^3$ with cubic anharmonicity for illustrative purposes. Expectation values of a state are subject to equations of motion

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{q}\rangle &= \frac{1}{i\hbar}\langle[\hat{q},\hat{H}]\rangle = \frac{1}{m}\langle\hat{p}\rangle \\
\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{p}\rangle &= \frac{1}{i\hbar}\langle[\hat{p},\hat{H}]\rangle = -m\omega^2\langle\hat{q}\rangle - \lambda\langle\hat{q}^2\rangle = -m\omega^2\langle\hat{q}\rangle - \lambda\langle\hat{q}\rangle^2 - \lambda(\Delta q)^2 = -V'(\langle\hat{q}\rangle) - \lambda(\Delta q)^2
\end{aligned}$$

with a straightforward derivation as, e.g., in the Ehrenfest theorem. The second equation makes it obvious that the anharmonicity couples expectation values to fluctuations, here to $(\Delta q)^2 = \langle(\hat{q}-\langle\hat{q}\rangle)^2\rangle$. This requires quantum corrections to the classical equations when the evolution of expectation values is to be described.

However, fluctuations are themselves dynamical. Unlike in the classical case, the above equations do not provide a closed system since Δq is in general a function of time which must be known to solve for $\langle\hat{p}\rangle$. We can extend the system of equations by one for the fluctuation which is derived as above, also referring to the Leibniz rule:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Delta q)^2 = \frac{\mathrm{d}}{\mathrm{d}t}(\langle\hat{q}^2\rangle - \langle\hat{q}\rangle^2) = \frac{1}{i\hbar}\langle[\hat{q}^2,\hat{H}]\rangle - 2\langle\hat{q}\rangle\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{q}\rangle = \frac{1}{m}\langle\hat{q}\hat{p}+\hat{p}\hat{q}\rangle - \frac{2}{m}\langle\hat{q}\rangle\langle\hat{p}\rangle = \frac{2}{m}C_{qp}\,.$$

This tells us how Δq evolves, but only if the covariance $C_{qp} = \frac{1}{2}\langle\hat{q}\hat{p}+\hat{p}\hat{q}\rangle - \langle\hat{q}\rangle\langle\hat{p}\rangle$ is known, which enters as a new dynamical quantum degree of freedom. We have to go ahead and compute its equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}C_{qp} = \frac{1}{m}C_{qp} + m\omega^2(\Delta q)^2 + 6\lambda\langle\hat{q}\rangle(\Delta q)^2 + 3\lambda G^{0,3}\,.$$

This equation does not only provide a coupling term $\langle\hat{q}\rangle(\Delta q)^2$ between expectation values and fluctuations, but again introduces a new independent variable: the third order moment $G^{0,3} = \langle(\hat{q}-\langle\hat{q}\rangle)^3\rangle = \langle\hat{q}^3\rangle - 3\langle\hat{q}\rangle(\Delta q)^2 - \langle\hat{q}\rangle^3$ which is related to skewness and thus deformations of a wave function away from a Gaussian.

Proceeding in this way shows that all infinitely many quantum variables

$$G^{a,n} := \left\langle\psi\left|\left((\hat{q}-\langle\psi|\hat{q}|\psi\rangle)^{n-a}(\hat{p}-\langle\psi|\hat{p}|\psi\rangle)^a\right)_{\mathrm{symm}}\right|\psi\right\rangle\,, \tag{2.2}$$

which one can use to describe a state $|\psi\rangle$, are coupled to each other and to the expectation values. This whole system of infinitely many ordinary differential equations is equivalent to the partial Schrödinger equation.

So far, we have only reformulated the usual equations of quantum mechanics in a way which in general is much harder to solve than the Schrödinger equation. We have not yet included any approximations or effective techniques. To arrive at effective equations, which are to amend the classical equations by quantum corrections and can thus involve only finitely many local degrees of freedom, we have to truncate the dynamical equations coupling the whole set of infinitely many local quantum degrees of freedom collected in the $G^{a,n}$. In our example, this can be achieved if

$(\Delta q)(q,p)$ is known as a function of q and p. Inserting it into $\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{p}\rangle = -V'(\langle\hat{q}\rangle) - \lambda(\Delta q)^2$ then results in effective equations for $q = \langle\hat{q}\rangle$ and $p = \langle\hat{p}\rangle$ which are closed but do include quantum effects.

The crucial step in the derivation of effective equations and for a justification of the approximation they provide is such a truncation to finitely many independent variables (or finitely many local ones in field theories). For perturbative potentials around the harmonic oscillator, an adiabatic approximation combined with the semiclassical one achieves this decoupling and allows one to compute Δq order by order. To first order in $\hbar$ and second in the adiabatic approximation, the result formulated as a second order equation for q, rather than two coupled first order equations for q and p, is [2, 3, 4]

$$\left(m + \frac{\hbar U'''(q)^2}{32m^2\omega^5\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{5}{2}}}\right)\ddot{q} + \frac{\hbar\left(4m\omega^2 U'''(q)U''''(q)\left(1+\frac{U''(q)}{m\omega^2}\right) - 5U'''(q)^3\right)}{128m^3\omega^7\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{7}{2}}}\dot{q}^2$$
$$+m\omega^2 q + U'(q) + \frac{\hbar U'''(q)}{4m\omega\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}} = 0 \qquad (2.3)$$

with a general anharmonic potential $U(q)$. As one can see, this indeed agrees with the equation of motion implied by the 1-particle irreducible low energy effective action (2.1).

3. General procedure

While the above example demonstrates that the basic principle of quantum back-reaction of a spreading state reproduces results from the 1-particle irreducible effective action, the procedure is more widely applicable. In particular, the entirely canonical formulation makes it suitable for canonical quantizations such as loop quantum gravity. As we saw, the central requirement for its feasibility is the availability of a relation to a free system where quantum variables decouple. In the example, this was the harmonic oscillator for which, with $\lambda = 0$, no coupling terms between expectation values and fluctuations arise. In this case, the Hamiltonian is quadratic in canonical variables, and thus $[\cdot,\hat{H}]$ is linear in basic operators for any basic operator in the left slot of the commutator. The same is available in quantum field theory, where free theories provide quadratic Hamiltonians. However, gravity in general is very far from a harmonic oscillator or a free field theory; at least measure terms $\sqrt{\det q_{ab}}$ with the spatial metric q_{ab} appear in any Hamiltonian and are certainly not quadratic once the metric becomes a dynamical variable rather than being treated as a background.

But free systems are more general than quadratic ones. What is necessary for a free theory in the above sense is the linear nature of commutators between basic variables and the Hamiltonian. This implies a closed set of equations of motion for expectation values of basic operators alone. For canonical basic variables, this requires a quadratic Hamiltonian, but for non-canonical variables one has the more general situation of a linear system. They have basic variables J_i which together with the Hamiltonian $\hat{H}$ form a linear commutator algebra. For such systems, equations of motion for expectation values and moments of a state decouple and can be solved explicitly [2].

Perturbations around free systems can then be analyzed for interacting theories where the decoupling happens only approximately. The coupling terms, solved for by approximations, give rise to quantum corrections in effective equations.

For perturbations around general linear systems, the adiabaticity assumption used before to derive (2.3) may not always be justified. Then, $G^{a,n}$ may not be solvable as functions of the basic J_i to be inserted in equations for expectation values. But the semiclassical approximation by itself allows a decoupling of almost all quantum variables, which still gives us effective equations of finitely many (local) variables. One may, however, be forced to keep more than the expectation values independent, leading to higher dimensional effective system.

4. Isotropic cosmology

Is a linear system of this type available for gravity? It is reasonable to look first at models of the same dimensionality as the harmonic oscillator in classical mechanics. In gravity, this brings us to isotropic cosmology with a single gravitational degree of freedom, the scale factor a. Its dynamics is given by the Friedmann equation

$$c^2\sqrt{p} = \frac{4\pi G}{3} p^{-3/2} p_\phi^2 \,, \tag{4.1}$$

written here in variables $c = \dot{a}$ for extrinsic curvature and $p = a^2$ where p is the component of an isotropic densitized triad, assumed positive. (The densitized triad component can take both signs due to the orientation of the triad. This is important for the singularity issue [5, 6], but can usually be ignored for effective equations which typically break down too close to a classical singularity.) These variables are more closely related to those used in loop quantum gravity.

We have specified this equation to a particular matter content, given by a free massless scalar. (As a perfect fluid, this corresponds to stiff matter whose pressure is identical to its energy density.) This matter ingredient has the advantage of providing a quadratic expression for p_ϕ in terms of the canonical gravitational variables (c, p): Solving the Friedmann equation yields $|p_\phi| \propto |cp| =: H$. One can interpret this as the Hamiltonian generating the flow in the variable ϕ which plays the role of an internal time. Such a formulation in terms of ϕ-evolution is equivalent to working with the Friedmann equation and solving for functions in terms of a coordinate such as proper or conformal time. The reason is that the Friedmann equation is, from the canonical perspective, a constraint. The corresponding gauge freedom is the choice of the time coordinate which, as in full general relativity, is free. Using the evolution of gravitational variables with respect to ϕ as an internal time has two advantages compared to a time coordinate. We eliminate the choice of a coordinate time and a discussion of possible interpretations such as how to quantize quantities referring to time coordinates in the absence of an absolute time. More importantly, the ϕ-Hamiltonian is quadratic and provides the desired free system for gravity.

As with any quadratic Hamiltonian expectation values and moments decouple. In this case, equations of motion corresponding to a Hamiltonian operator $\hat{H} = \frac{1}{2}(\hat{c}\hat{p} + \hat{p}\hat{c})$ are

$$\dot{\langle\hat{c}\rangle} = \langle\hat{c}\rangle \quad , \quad \dot{\langle\hat{p}\rangle} = \langle\hat{p}\rangle \quad , \quad \dot{G}^{0,2} = 2G^{0,2} \quad , \quad \dot{G}^{1,2} = 0 \quad , \quad \dot{G}^{2,2} = -2G^{2,2}$$

derived by the same method as before. They can be solved easily by $\langle\hat{c}\rangle(\phi) = c_1 e^{\phi}$, $\langle\hat{p}\rangle(\phi) = c_2 e^{-\phi}$, $G^{0,2}(\phi) = c_3 e^{-2\phi}$, $G^{1,2}(\phi) = c_4$ and $G^{2,2}(\phi) = c_5 e^{2\phi}$. This system can be analyzed and perturbed around [7], but here it is more interesting to discuss an alternative quantization of the same classical model, provided by loop quantum cosmology. In loop quantum gravity, as described in more detail in the following section, holonomies are used as basic operators rather than components of extrinsic curvature. For loop quantum cosmology [8], this essentially implies (skipping several steps in the derivation) that $\sin c$ appears in the Friedmann equation instead of c which one can interpret as the inclusion of higher curvature terms due to quantum geometry. Proceeding as before, the loop Hamiltonian $\hat{H} = \widehat{p\sin c}$ (in a yet to be specified factor ordering) for ϕ-evolution is then non-quadratic.

Remarkably, the system does remain linear under this change [9]. To see this, we introduce a new basic operator $\hat{J} = \widehat{pe^{ic}}$, for which we have a linear Hamiltonian $\hat{H} = -\frac{1}{2}i(\hat{J} - \hat{J}^\dagger)$ as a specific factor ordering of $\widehat{p\sin c}$. Since we now use non-canonical variables $(\hat{p}, \hat{J})$, linearity of the Hamiltonian does not immediately imply solvability. However, here the algebra of basic operators, which includes the Hamiltonian as a linear combination, is linear, providing a centrally extended $\mathrm{sl}(2,\mathbb{R})$ algebra

$$[\hat{p},\hat{J}] = \hbar\hat{J} \quad , \quad [\hat{p},\hat{J}^\dagger] = -\hbar\hat{J}^\dagger \quad , \quad [\hat{J},\hat{J}^\dagger] = -2\hbar\hat{p} - \hbar^2 .$$

The system is thus linear and provides a free system even in the context of a loop quantization.

This can also be seen in the equations of motion

$$\langle\dot{\hat{p}}\rangle = -\tfrac{1}{2}(\langle\hat{J}\rangle + \langle\hat{J}^\dagger\rangle) \quad , \quad \langle\dot{\hat{J}}\rangle = -\tfrac{1}{2}(\langle\hat{p}\rangle + \hbar) = \langle\dot{\hat{J}}^\dagger\rangle$$

for expectation values, forming a closed set. They have general solutions

$$\langle\hat{p}\rangle(\phi) = \tfrac{1}{2}(c_1 e^{-\phi} + c_2 e^{\phi}) - \tfrac{1}{2}\hbar \tag{4.2}$$

$$\langle\hat{J}\rangle(\phi) = \tfrac{1}{2}(c_1 e^{-\phi} - c_2 e^{\phi}) + i\langle\hat{H}\rangle \tag{4.3}$$

where $\langle\hat{H}\rangle$ is a constant of motion since no explicit time dependence is present in the absence of a matter potential. However, not all these solutions correspond to expectation values computed in a physically normalized state. Although we have made use of the fact that $\langle\psi|\psi\rangle = 1$ in deriving the equations of motion and thus referred to some part of the normalization conditions, there is no guarantee yet that the system is defined with respect to an inner product respecting the correct adjointness conditions of basic operators. Implementing the correct inner product is far from trivial in constrained systems, and it may appear that it is even more difficult in our treatment since no states are being used explicitly. The implementation is indeed indirect, but bears several advantages in its concrete realization.

For $\hat{p}$ we must guarantee that it is self-adjoint, which we can easily do in our equations for expectation values (and fluctuations below) by requiring $\langle\hat{p}\rangle$ to be real for solutions we accept as physical. The condition for $\hat{J}$ is more involved: Classically we have $J\bar{J} = p^2$ for $J = p\exp(ic)$. This provides a reality condition for c which, in addition to $\langle\hat{p}\rangle$ being real, must be imposed in the quantization by determining a physical inner product for which $\exp(ic)$ is quantized to a unitary operator. Although an explicit form for a physical inner product in this system may be difficult to find, for the equations used here this reality condition can also be implemented rather easily through reality conditions for $\langle\hat{J}\rangle$. Taking expectation values of the identity $\hat{J}\hat{J}^\dagger = \hat{p}^2$, we have [10]

$$|\langle\hat{J}\rangle|^2 - (\langle\hat{p}\rangle + \tfrac{1}{2}\hbar)^2 = G^{pp} - G^{J\bar{J}} + \frac{1}{4}\hbar^2 \tag{4.4}$$

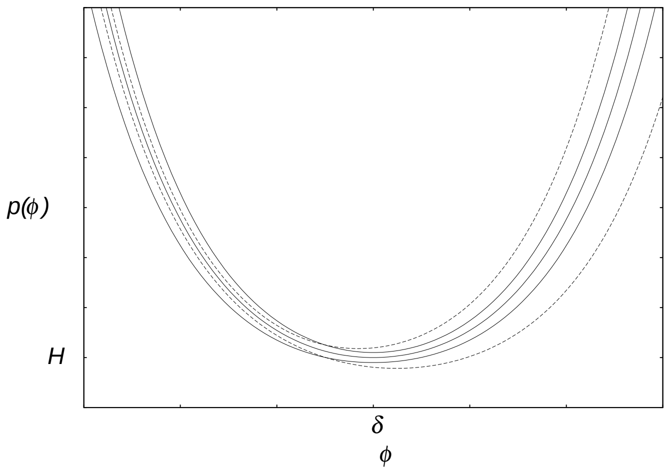

Figure 1: State spreading through a bounce: Mean trajectories for expectation values and spreads from fluctuations are plotted. The fluctuations may or may not be symmetric around the bounce

where we now denote quantum variables explicitly by their operator products since more than two real variables are involved in the basic operators. Here, this implies $c_1c_2 = H^2 + O(\hbar)$ and thus leaves only the family of bouncing solutions

$$\langle\hat{p}\rangle(\phi) = \langle\hat{H}\rangle\cosh(\phi-\delta) - \hbar \quad , \quad \langle\hat{J}\rangle(\phi) = -\langle\hat{H}\rangle(\sinh(\phi-\delta) - i) \tag{4.5}$$

out of (4.2) and (4.3), where p is bounded away from zero. (The parameter δ, defined by $e^{2\delta} = c_2/c_1$, is the only freedom left in solutions for expectation values after imposing the reality condition.)

Similarly, we can derive equations of motion for fluctuations to get a more precise picture of the spreading state [10]. In this parameterized model, the only quantum degree of freedom is the volume of the universe since the matter field ϕ plays the role of time. Fluctuations thus primarily refer to those of the total isotropic geometry, which is a rather non-intuitive notion. On solutions, however, the geometrical and matter variables are correlated, and so geometry fluctuations are directly related to fluctuations of the homogeneous matter field. One can see this clearly in Fig. 1, where one can interpret the spread vertically as fluctuations of p, or horizontally as fluctuations of ϕ. (Physically more interesting fluctuations as they are used in inflationary structure formation would require perturbations of the model by inhomogeneities.)

For $\langle\hat{H}\rangle \gg \hbar$, solutions are given by $(\Delta p)^2 = G^{0,2} \approx \hbar\langle\hat{H}\rangle\cosh(2(\phi-\delta_2))$ with a new integration constant δ_2. For $\delta_2 \neq \delta$ (which can be seen to be related to squeezing), fluctuations of the state are not symmetric around the bounce. Examples are illustrated in Fig. 1, and physical implications are further discussed in [11]. For instance, it turns out that from within one branch of the universe there is only limited access to some properties of the other branch which makes it impossible to determine whether the state before the bounce resembles that after the bounce in its semiclassical appearance.

5. Inhomogeneities

What we have seen so far provides example for the use of effective equations in quantum cosmology, even addressing thorny issues such as implications of the physical inner product. For phenomenological applications, this has to be extended to inhomogeneous situations which requires more background material than provided so far. Loop quantum gravity [12, 13, 14] is based on canonical gravity formulated as an SU(2) gauge theory in Ashtekar variables $\{A_a^i(x), E_j^b(y)\} = 8\pi\gamma G\delta_a^b\delta_j^i\delta(x,y)$ [15, 16]. The gauge group is that of rotations of the triad e_i^a which is used instead of a metric $q_{ab} = e_a^i e_b^i$ (where e_i^a is the matrix inverse of e_a^i). All points $x, y \in \Sigma$ are spatial, making use of a space-time splitting as in any canonical formulation. The variables are then defined as the densitized triad $E_i^a = |\det e_b^j| e_i^a$ and the connection $A_a^i = \Gamma_a^i + \gamma K_a^i$ with the spin connection $\Gamma_a^i = -\varepsilon^{ijk} e_j^b(\partial_{[a} e_{b]}^k + \frac{1}{2} e_k^c e_a^l \partial_{[c} e_{b]}^l)$ and $K_a^i := e_i^b K_{ab}$ related to extrinsic curvature. (The Barbero–Immirzi parameter $\gamma > 0$ [16, 17] can be freely specified classically without affecting the theory. After quantization it determines the discreteness scale of spatial geometry.)

Gravitational dynamics of these variables is implemented by constraints. But before addressing those one can notice an important advantage of connection variables compared to others such as the older ADM variables directly referring to q_{ab} [18]. They allow the introduction of scalar-type variables as in lattice gauge theories: For any curve e and surface S in space, we define holonomies and fluxes [19]

$$h_e(A) = \mathscr{P}\exp\int_e A_a^i \tau_i \dot{e}^a \mathrm{d}t \quad , \quad F_S(E) = \int_S \mathrm{d}^2y\, n_a E_i^a \tau^i \tag{5.1}$$

with the tangent vector $\dot{e}^a$ to a curve, the co-normal n_a to a surface and Pauli matrices τ_i. These variables satisfy a closed algebra determined by their Poisson brackets which is well-defined and, unlike that of the fields, free of delta-functions. It can thus be represented on a Hilbert space, which provides a rigorous quantization.

In this way, loop quantum gravity does not directly and straightforwardly quantize the classical variables as they are presented in general relativity. It rather uses mathematical consistency conditions and principles such as background independence at the very first step of the quantization, which leads to the selection of the quantized variables and thus the eventual microscopic degrees of freedom of the theory. The classical theory is certainly used, but only as a guideline so as to have a chance of obtaining the correct classical limit. Still, the correct classical limit is not guaranteed and has to be verified explicitly. In particular, a classical space-time picture has to emerge from the underlying microscopic dynamics in low-curvature regimes.

After quantization, holonomies play the role of "creation" operators of geometry. In a connection representation, they act by multiplication and add new dependence of a state on the connection along the curve used in the holonomy. Degrees of freedom are thus graph-based, a general state taking the form $\sum_{g,j,C} c_{g,j,C}(\phi) T_{g,j,C}$ in a spin network basis $T_{g,j,C}(A) = \prod_{v\in g} C_v \cdot \prod_{e\in g} \rho_{j_e}(h_e(A))$ [20]. Labels are graphs g in space, half-integers j on edges of the graph for irreducible SU(2) representations and vertex labels C of intertwining operators. In the coefficients $c_{g,j,C}(\phi)$ of a general state we have included the dependence on possible matter fields such as a scalar ϕ. These functions can themselves be expanded in a basis of the matter Hilbert space, but this will not be necessary for what follows.

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Such a state represents quantum geometry, which can be extracted by acting with geometrical operators. Elementary geometry expectation values are given by fluxes (or areas) $\langle\rho_{j_e}(h_e)|\hat{F}_S|\rho_{j_e}(h_e)\rangle \sim j_e$ using a flux operator quantizing $F_S = \int_S \mathrm{d}^2 y E^a_i \tau^i n_a$ for a 2-surface S intersecting an edge e. In a basis state, the edge labels j_e thus determine local elementary areas building up an inhomogeneous geometry described by a labeled graph (g, j, C). (The remaining labels C_v are relevant for the volume.)

These basis states can, however, not be physical when one considers the imposition of constraints to capture the correct dynamics of quantized general relativity. Physical states are annihilated by the Hamiltonian constraint (which in the isotropic case gave us the Friedmann equation (4.1)). As an operator, this constraint typically creates new edges and vertices [21, 22] because it involves holonomy operators quantizing the curvature of the Ashtekar connection. As a consequence, physical states cannot be based on a single graph but must be superpositions of different graph states. Such superpositions can be understood as encoding the relational dynamics in the absence of an absolute time. As in the cosmological example, one chooses an internal time variable from the dynamical fields. In general situations, no global time such as ϕ for a free massless scalar exists, but locally evolution can still be described in other suitable variables. It is often convenient to use the spatial volume as internal time and, at least formally, expand a state in volume eigenstates. (The volume operator [23, 24] is complicated and its spectrum is not known explicitly in the full theory, but it is convenient for the conceptual argument.) Since each action of the Hamiltonian constraint changes spins and the graph, and thus the volume, its basic action can be seen to provide elementary moves of a dynamically changing lattice. Typically, larger volumes require finer graphs and thus the underlying lattice is being refined as the universe expands. New degrees of freedom emerge while the universe grows; see also [25] for more details. This is complicated in general, but treatable in models. At this place, further assumptions must be used in effective models describing the refinement, which are to be substantiated by consistency relations arising from the action of the underlying Hamiltonian constraint operator.

6. Near isotropy and emergence

Suitable approximations refer to perturbations of inhomogeneities around an isotropic background, which is of importance for observational cosmology. This can be implemented at the quantum level: If all labels of a single graph state in the volume decomposition are nearly equal, the total volume of an isotropic "background" geometry is determined by the total spin $\langle j\rangle = \sum_e j_e$. (We assume that not many different graphs contribute to the superposition at a fixed volume eigenvalue, such that, at a given volume, one essentially has a single lattice.) Small inhomogeneities are given by $\delta j(e) = j_e - \langle j\rangle$, seen as a discrete spatial function on edges. A continuum limit finally provides the spatial metric modes of cosmological perturbation theory through the interpolated $\delta j(e)$. After a mode decomposition, based on the availability of a background geometry in the perturbation theory, classical modes such as the scalar metric mode or gravitons emerge.

While the Hamiltonian constraint equation at the quantum level would be difficult to solve, especially for interesting cosmological solutions, one can more easily extract its crucial properties in effective equations. Those equations, as described in the examples before, are derived from expectation values of Hamiltonians or constraints. These expectation values can then be treated by approximation schemes to arrive at simpler expressions which still capture the essential physics. Only then would one start to solve equations, which is more feasible than before deriving effective equations. Even properties of suitable semiclassical states can be derived at the effective level, which is based on equations of motion for fluctuations and higher moments. Thus, neither solutions to the quantum constraints nor precise functional expressions of semiclassical states are required in this procedure. Nevertheless, there are certainly several hurdles in an application to inhomogeneous situations, such as the actual computation of expectation values in general states.

Detailed calculations are still in progress [26, 27, 28, 29] for applications in cosmological phenomenology, but this picture already illustrates the emergence of classical excitations in this framework [30]: Microscopic degrees of freedom are given by graph states with labels j_e determining elementary geometrical excitations. The fundamental Hamiltonian constraint changes edges and spins, and thus provides local moves of an evolving irregular lattice in the relational interpretation. Such local changes of j_e now evolve $\langle j\rangle$ (related to the total volume) as well as $\delta j(e)$ (the discrete inhomogeneities). Continuum excitations emerge from the latter with a dynamics coming from lattice moves. This is one example for emergence in the sense of a microscopic theory providing degrees of freedom subject to dynamics considerably different from the classical evolution on macroscopic scales.

Dynamical semiclassical states capturing these detailed local changes are difficult to write, especially for a graph-changing Hamiltonian. Here we see again the advantages of effective techniques: they do not require a full state with a precise connection dependence on all edges. Properties of approximately coherent states can rather be constructed order by order in a semiclassical expansion. The relevant information is again extracted from expectation values $\langle\hat{H}\rangle$ in general states parameterized by quantum variables with suitable semiclassicality conditions. Now in the field theoretical context, independent quantum variables arise for each fundamental degree of freedom, i.e. for each edge. Due to the discrete nature of loop quantum gravity there are still finitely many variables in a bounded volume, even though one is dealing with the quantization of a field theory. This suggests that an extension of effective equations techniques to such a full situation can be done without having to face too many field theoretical subtleties.

7. Conclusions

Effective theories provide means to extract phenomenological information from fundamental quantum theories. With recent developments, this is available for canonical quantizations and can take into account requirements for loop quantum gravity. Explicit examples are provided in cosmology, based on an isotropic universe sourced by free massless scalar as solvable zeroth order. This model is thus analogous to the harmonic oscillator in classical mechanics and provides the basis for a perturbation theory to include other degrees of freedom and interactions. These principles, in an inhomogeneous context, illustrate the emergence of classical excitations from quantum states.

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