PROCEEDINGS OF SCIENCE

Mode creation
and phenomenology of inflationary spectra.

David Campo*
Lehrstuhl für Astronomie, Universität Würzburg
E-mail: dcampo@astro.uni-wuerzburg.de

Quantum field theories are thaught to be valid descriptions of physical phenomena below a certain cutoff. On an expanding background, modes are continuously created to keep the density of degrees of freedom constant. It is argued that the question of the state of these newly born modes cannot be examined independently from the dynamics. The primordial spectra predicted from a consistent treatment are unlikely to be useful probes in the phenomenological approach to Quantum Gravity.

From Quantum to Emergent Gravity: Theory and Phenomenology
June 11-15 2007
Trieste, Italy

*Speaker.

PoS(QG-Ph)014

1. Introduction

Observations have established the nearly Gaussian and scale invariant character of the spectrum of curvature perturbations on super-Hubble scales [1]. This unique type of spectrum is a fairly generic prediction of inflation. However this universality prevents us from learning about the microphysics at the scale of inflation from the sole measurement of the primordial power spectrum. Fortunately, future experiments will soon enlarge the number of accessible observables. For the primordial spectra alone they include measurements of the bi- and tri-spectrum (i.e. non-Gaussianities), of the B-polarization spectrum and of gravitational waves, as well as an increased precision of the temperature power spectrum at the level of less than a percent (at the scales not limited by the cosmic variance). The prospect of this increased precision requires a keen knowledge of the subleading effects predicted by the host of scenarios currently on the market.

The object of this paper is to discuss the class of corrections known as the trans-Planckian effects. This appellation is somewhat misleading since true trans-Planckian effects cannot be calculated. It refers to any unkown high energy effect characterized by a mass scale $M \leq M_{\rm Pl}$ and which can be modeled by a Quantum Field Theory (QFT). Previously considered phenomena include a breaking of Lorentz invariance through modified dispersion relations [2, 3, 4] or irrelevant operators [5, 6], non commutative structure of space time [7, 8], effects of heavy particles [9, 10] and modified initial states [11, 12] (see also [13, 14]).

Since proper momenta are redshifted by the expansion of the background, the assumption of a cutoff breaking Lorentz Invariance (LI) raises the question of the ”initial” state of the modes. Several studies [5, 6, 10, 11, 12] assumed a pure state on an initial boundary and predict that the leading corrections to the primordial power spectra have a distinctive oscillatory pattern that passes on to the CMB power spectrum. The frequency and amplitude of these oscillations is of course model dependent, but their existence is generic given these two assumptions [*see note added in proof*]. We will show that these oscillations are an interference effect which reflects both the existence of the boundary and the coherence of the initial state.

In [15] we argued that making both assumptions is physically questionable, and we showed how the superimposed oscillations are suppressed if one of these hypothesis is abandonned. For that purpose we considered a model where the modes are prepared in an initial pure state at a ”fuzzy” boundary, i.e. with a cutoff M that belongs to a statistical ensemble. The leading corrections to the primordial power spectra in each realization of the ensemble are oscillatory with a frequency linear in M. The oscillatory corrections are therefore damped in the ensemble averaged power spectrum.

We take the opportunity of these proceedings to develop the physical motivation for this picture. We argue that in a consistent Effective Field Theory (EFT) with a cutoff on an expanding background, one must encode the past dynamics into an interacting vacuum. The Bunch-Davis vacuum is then recovered in the limiting case of weak interactions and exact adiabaticity. We also complete the analysis of [15] in two ways. The physical interpretation of the phase of the afor-mentionned corrections is explained in an Appendix. The presentation is generalized to include a discussion of the boudary EFT formulation of [5]. Sec. 2 is a reminder of the standard settings. In Sec. 3, the conditions for a consistent EFT on an expanding background with a background are presented. These conditions are put together in a coherent picture in Sec. 4 and the phenomenology of primordial spectra is derived.

2. The standard inflationary spectra

2.1 Settings

In inflationary models with one inflaton, the power spectra of both linear curvature perturbations ζ and gravitational waves h_{ij} during inflation can be related to that of a scalar test field φ by [16]

$$\zeta = \varphi \, \frac{\sqrt{4\pi G}}{a\sqrt{\varepsilon_1}} \, , \qquad h_{ij} = \varphi \, \frac{\pi^s_{ij}}{a} \, , \tag{2.1}$$

where π^s_{ij} is the polarization tensor of the gravitational waves and ε_1 is the first slow-roll parameter

$$\varepsilon_1 = -\frac{d\ln H}{d\ln a} = -\frac{\partial_t H}{H^2} \, . \tag{2.2}$$

Given this correspondence, it is sufficient to understand the statistical properties of φ.

The scalar field is massless, minimally coupled to gravity, and freely propagating. We introduce the Fourier decomposition

$$\hat{\varphi}(\tau,\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} \, e^{i\mathbf{qx}} \left(a_{\mathbf{q}} \varphi_q(\tau) + a^\dagger_{-\mathbf{q}} \varphi^*_q(\tau) \right) \tag{2.3}$$

The wave functions φ_q are solutions of

$$\left(\partial^2_\tau + \omega^2_q(\tau) \right) \varphi_q = 0 \, . \tag{2.4}$$

with a conformal frequency of the form

$$\omega^2_q(\tau) = q^2 - \frac{f}{\tau^2} \tag{2.5}$$

where τ is the conformal time defined by $d\tau = dt/a(t)$ and f is a function of the slow-roll parameters of order unity. Its explicit expression is not needed here.

Knowledge of the 'intial' state of $\hat{\varphi}$

$$\rho = \otimes_{\frac{1}{2}\mathbf{q}} \, \rho_{\mathbf{q},-\mathbf{q}} \tag{2.6}$$

and of the solution of (2.4) allows to calculate the power spectrum $\mathscr{P}_\rho$ which is related to the Fourier transform of the equal time two-point correlation function of $\hat{\varphi}$ by

$$\mathrm{Tr}\left[\rho \, \hat{\varphi}(t,\mathbf{x}) \hat{\varphi}(t,\mathbf{y})\right] = \int_0^{+\infty} \frac{dq}{q} \, \frac{\sin(qr)}{qr} \, \mathscr{P}_\rho(q,t) \, , \tag{2.7}$$

where $r = |\mathbf{x} - \mathbf{y}|$.

Because of the time dependence of ω_q in (2.4), the decomposition (2.3) of the field into positive and negative solutions is ambiguous. Different prescriptions for φ_q and ρ give different expressions of $\mathscr{P}_\rho$. If ρ is chosen to be the ground state associated with a particular choice of positive frequency modes, the power spectrum depends only on the boundary conditions in the definition of the modes, to wit

$$\mathscr{P}(q,t) = \frac{q^3}{2\pi^2} |\varphi_q(t)|^2 \, . \tag{2.8}$$

PoS(QG-Ph)014

2.2 The Bunch-Davis approximation

The standard choice for ρ is the so-called Bunch-Davis (BD) vacuum [17]. This state can be defined from the solutions of Eq. (2.4) with positive frequency in the asymptotic past. Using the fact that $\omega_q \to q$ for $\tau \to -\infty$ (see Eq. 2.5), the asymptotic positive frequency modes obey

$$(i\partial_\tau - q)\,\varphi_q^{-\infty}|_{\tau\to-\infty} = 0\,. \tag{2.9}$$

The corresponding power spectrum is thus

$$\mathscr{P}_\infty(q,t) \;=\; \frac{q^3}{2\pi^2}|\varphi_q^{-\infty}(t)|^2 \to \left(\frac{H_q}{2\pi}\right)^2 . \tag{2.10}$$

The second expression holds in the long wavelength limit $q/aH \ll 1$ where terms of order $(q/aH)^2$ are neglected. The stricking feature of this expression is that it depends only on the scale H_q, the scale of inflation at the time of horizon crossing

$$q = H(t_q)\,a(t_q) = H_q\,a_q \tag{2.11}$$

The second equality fixes the notation. The reason for this is the scale independence of the definition (2.9) and the stationarity of de Sitter space. The soft breaking of the de Sitter symmetry which constitutes the slow-roll regime produces a logarthmic correction to the otherwise scale-invariant power spectrum.

3. Consistency requirements for a field theoretic treatment

3.1 Adiabaticity

As natural as it may be from the point of view of the geometry, the BD vacuum does not make any sense from the point of view of EFT because in (2.9) the modes have a physical momentum larger than the Planck mass, where both the notions of field theory and smooth manifold are expected to break down. A consistent field theoretic treatment requires to consider physical momenta below a certain cutoff scale M,

$$\frac{q}{a} \le M \le M_{\rm Pl}\,. \tag{3.1}$$

The modified power spectra now depend on H_q and M. To be consistent with observations, they must assume a perturbative form

$$\mathscr{P}_M(q) = \mathscr{P}_\infty(q)\,\times\,[1 + \text{corrections}] \tag{3.2}$$

where corrections are suppressed by powers of H/M.

To understand under which conditions both requirements can be fulfilled, consider a scattering experiment between ”light” particles of a field φ at a typical energy E in Minkowski space. Massive degrees of freedom (d.o.f.) ψ of mass $M \gg E$ are exponentially rarely created and therefore appear in transition amplitudes only as intermediate states. This is the decoupling theorem of Appelquist and Carazzone [18, 19], see also the review of C.P. Burgess in these proceedings. The heavy d.o.f. can therefore be integrated in

$$\langle T \exp\left(i\int \mathscr{L}(\varphi,\psi)\right)\rangle \equiv T \exp\left(i\int \mathscr{L}_{\text{eff}}(\varphi)\right) \tag{3.3}$$

where $\langle ... \rangle$ is the expectation value in the ground state of ψ. The effective Lagrangian $\mathscr{L}_{\text{eff}}(\varphi)$ summarizes the dynamical influence of ψ. It is in general non local and can be expanded a series of irrelevant operators suppressed by power of $1/M^2$.

Applied to the cosmological perturbations, the corrections in (3.2) are generically predicted to be $O(H^2/M^2)$ [9], and are therefore unobservable unless M is close to H. This conclusion rests on the assumption that the heavy field ψ is not excited, a condition which might be violated in inflationary cosmology. For instance, time dependent background fields may bring the effective mass of ψ close to H, therefore modifying the propagation of the light modes φ. When this altered dynamics can be modeled by a modifed dispersion relation for φ, it means that the condition of adiabaticity

$$\frac{|\partial_\tau \omega_q|}{\omega_q^2} \ll 1\,, \tag{3.4}$$

is violated in this regime. We refer to the neat analysis of [4] and [10] for more details.

We can now return to the question of the "initial" state of the modes used in replacement to the BD vacuum. This state is thaught to result from the past evolution of the modes of φ above the cutoff. If this evolution is adiabatic, this state is well approximated by the adiabatic vacuum at the time $q = aM$, and corrections are found to be $O(H^3/M^3)$ [14]. If non adiabatic transitions occur, corrections could be much larger [1]. It has been proposed to encode this possible non-trivial dynamics of ψ into less adiabatic vacua for φ which diagonalize quadratic Hamiltonians at a time $q/aH \gg 1$ but finite [2] (while the dispersion relation is kept linear) [11, 12]. The rest of this section is concerned with the atypical phenomenology of this class of models. We then argue that this choice is unphysical.

---

[1] Notice that the amplitude of the corrections cannot be infered from the dimension of the operators in the effective Lagrangian because the latter only refects the structure of the interacting vacuum. For instance, corrections of order $10^{-5} \sim H/M$ do not necessarily come from Lorentz-breaking operators such as $M^{-1}\,\varphi(\Delta/a^2)^{3/2}\varphi$. When ψ participate actively to the dynamics, inducing for instance resonances, the amplitude of the corrections of dimension 6 bulk operators can be enhanced by large factors, see [10] for instructive examples.

[2] We remind that the addition of total derivatives to the action of the linear perturbations corresponds to different choices of canonical variables and yields to different functional forms of the Hamiltonian. They have a common limiting form $H = \frac{1}{2}\int d^3q\,\left(|\partial_\tau \varphi_q|^2 + q^2|\varphi_q|^2\right)$ when $q/aH \to \infty$. However diagonalization at a finite time or a finite proper momentum yields different ground states. The latters are related by unitary transformations (two-mode squeezing operators). The positive frequency modes associated with the states minimizing these quadaratic Hamiltonians are equivalently defined by [15]

$$(i\partial_\tau - \Omega_q)\left(\frac{\varphi_q^M}{a^n}\right)(\tau_*) = 0\,,$$

where τ_* is either τ_M or τ_0. n is a real number, and Ω_q is a function of the form

$$\Omega_q = q\left(1 + A\sigma_q^m + O\left(\sigma_q^{m+1}\right)\right)\,.$$

3.2 Two types of boundary surfaces

There are two obvious but inequivalent ways to modify the boundary condition (2.9). The first assumes that each mode φ_q is in a given vacuum state $|\Psi_M\rangle$ at the time $t_M(q)$ when the physical momentum q/a crosses the proper scale M [11]

$$q = Ma(t_M)\,. \tag{3.5}$$

We assume that H and M are well separated and introduce the parameter

$$\sigma_q \equiv \frac{H_q}{M} \ll 1\,. \tag{3.6}$$

In the second type of boundary conditions the state is defined at a fixed space-like hypersurface [5]

$$\forall q\,, \qquad \tau = \tau_0 \tag{3.7}$$

It is worth pointing that these boundary conditions explicitely break LI by the introduction of a prefered frame. LI violation is however not mandatory, and a minimal length can be defined in a covariant way. For instance, causal set theory is a discrete approach to Quantum Gravity, therefore with a build in minimal length (the scale of discreteness), which does not violate LI [20]. Instrumental to this is the Poissonian nature of the causet elements which are, roughly speaking, a random sprinckling of a Lorentzian manifold with a well defined causal structure. A covariant cutoff can also be defined as the cutoff on the spectrum of the d'Alambertian operator [21], since this spectrum is an invariant of the manifold. The subject of this note is not whether the local invariance under boosts is broken, but how to consistently choose the "initial" state of the low energy d.o.f. (i.e. the modes of an EFT) given the assumption of a such UV cutoff.

The first stricking difference between these boundaries is that (3.5) preserves the stationarity of both (2.9) and de Sitter space, while (3.7) violates them strongly (since the state of the modes is defined at a different physical wavelength for each value of q). Although it is argued in [5] that (3.7) is the natural choice to encode initial states in an EFT, it is far from obvious (at least to the author) that this is physically sensible since it violates the condition of adiabaticity. This is perhaps better seen from the expression of the modified power spectrum (2.8) (we remind that we consider only ground states) where φ_q is decomposed into the superposition of a positive frequency BD mode and a backscattered wave

$$\varphi_q(\tau) = \alpha_q\, \varphi_q^{-\infty}(\tau) + \beta_q\, \varphi_q^{-\infty *}(\tau) \tag{3.8}$$

The time-independent Bogoliubov coefficients are definied by the overlaps of the two sets of modes

$$\alpha_q = \left(\varphi_q^{-\infty}\right)^* \, i\overleftrightarrow{\partial}_\tau\, \varphi_q\,, \qquad \beta_q = -\varphi_q^{-\infty}\, i\overleftrightarrow{\partial}_\tau\, \varphi_q\,, \tag{3.9}$$

calculated at the boundary. The coefficient β_q measures the degree of non-adiabaticity of $|\Psi_M\rangle$ with respect to the BD vacuum. Hence the EFT treatment is valid only if

$$|\beta_q| \ll 1 \tag{3.10}$$

For both boundary conditions (3.5) and (3.7), the power spectrum (2.8) in terms of the Bogoliubov coefficients (3.8) is given by

$$\mathscr{P}_M(q) \,=\, \mathscr{P}_\infty(q) \times |\alpha_q|^2 \left\{ 1 + 2\mathrm{Re}\left(\frac{\beta_q^*}{\alpha_q^*} \frac{\left(\varphi_q^{-\infty}\right)^2}{|\varphi_q^{-\infty}|^2} \right) + \frac{|\beta_q|^2}{|\alpha_q|^2} \right\} . \tag{3.11}$$

The leading correction to (2.10) is the interference of the positive and negative frequency waves and is therefore an oscillatory term. These oscillations pass on to the CMB power spectra. Alternately, the superposition of positive and negative frequency BD-modes in (3.8) means that the state $|\Psi_M\rangle$ contains pairs of BD quanta, created at the time τ_M or τ_0, and which propagate freely thereafter. These space-time correlations survive in the expectation values (2.7) because of the coherence of the state $|\Psi_M\rangle$. This picture, developped in the Appendix, gives the physical interpretation of the phase of this oscillatory term.

With a boundary condition in momentum space (3.5), the amplitude of leading correction is equal to a certain power of the parameter σ_q (3.6)

$$|\alpha_q^M|^2 = 1 + |\beta_q^M|^2 , \qquad |\beta_q^M| = O\left(\sigma_q^p\right) , \qquad p \geq 1 . \tag{3.12}$$

where p is a model dependent parameter, see footnote 2. The corrections $\delta\mathscr{P}/\mathscr{P}_\infty$ have a nearly constant amplitude with logarithmic deviations induced by the slow rolling of the inflaton, i.e.

$$H_q \simeq H_{q_0} \left[1 + \varepsilon_1 \ln\left(\frac{q}{q_0}\right)\right] \tag{3.13}$$

The condition of adiabaticity (3.10) therefore holds over an exponentially large range of comoving momenta $q/q_0 \sim \exp(\varepsilon_1 M/H_{q_0})$.

When the boundary condition is imposed on a spacelike hypersurface $\tau = \tau_0$, the leading correction in (3.11) is proportional to the ratio of the physical momentum at τ_0 with the scale of new physics M

$$|\alpha_q^{\tau_0}\beta_q^{\tau_0}| = O\left(\frac{q}{Ma(t_0)}\right) . \tag{3.14}$$

The corrections scale linearly in the comoving momentum q, in sharp contrast with (3.12), and the condition of adiabaticity is therefore violated for $q \geq q_0 = Ma(t_0)$. Notice that it implies that the condition (3.7) is equivalent to imposing a comoving cutoff q_0, i.e. a cutoff on the spectrum of the Laplacian. This implies that the proper cutoff scales like $1/a(t)$, which apparently induces a flow of the Renormalization Group (RG). Adiabaticity can therefore be restored at the price of renormalizing the bare parameters of the irrelevant operator encoding the initial state on the boundary term of the action so as to compensate this RG-flow [5].

3.3 Mode creation

It is apparent from (3.14) that the boundary condition (3.7) amounts to impose a cutoff on the comoving momentum q, while (3.5) is a cutoff on proper momentum. The consequences of favouring one or the other run very deep and are relevant for the choice of the initial state of the modes. We therefore explain them with some details.

In Minkowski space, a UV cutoff has little impact on the description of low energy phenomena. For instance, condensed materials at a critical point are well described by a QFT. On an expanding background, and more generally in the presence of a horizon, the behaviour of quantum fields differs in two respects. First, horizons act as magnifying glasses since the late time/long distance low energy spectra emerge from configurations with frequencies larger than the Planck mass, thus challenging the validity of the predictions of QFT. This is the trans-Planckian question and as we saw, the robustness of the spectra is guaranteed by the adiabatic evolution of the state before crossing of the geometric scale (the mass of the black hole or H in cosmology). The second difference is that on an expanding background a minimal length can be chosen *a priori* to be on comoving (i.e. coordinate) distances, or alternatively on proper (i.e. physical) distances. These two alternatives lead to two radically different situations.

A comoving cutoff means that the number of degrees of freedom in a fixed comoving volume is fixed, and therefore the density of degrees of freedom

$$\rho = \frac{N}{V_{\rm proper}} = \frac{N}{a^3(t)V_{\rm com}} \tag{3.15}$$

is diluted with the expansion like $1/a^3(t)$. [Think of a regular and expanding comoving lattice characterized by the distance l between two sites. The shortest proper wavelength propagating on this lattice is $\lambda_{\rm min} = a(t)l$, and the density is $n(t) = \lambda_{\rm min}^{-3}$.] Assuming that the cutoff is the Planck mass 60 efolds before the end of inflation, we arrive at the absurde conclusion that today it is $10^{-52}M_{\rm Pl} \sim 10^{-33}$GeV. As a consequence, no QFT could be formulated inside our Hubble horizon. Moreover, it is hard to conceive a scenario where the initial state of the modes is not fixed by hand. We no longer consider comoving cutoffs.

A proper minimal length L implies that the density of degrees of freedom is constant, $\rho \sim L^{-3}$, and as a result their number in a fixed comoving volume scales like the proper volume. In other words, new degrees of freedom are created as the universe expands. Let us not dismiss this conclusion on the basis that field theories do not describe the creation of degrees of freedom (i.e. new modes, not to be confused with the creation of quanta which is the *raison d'être* of QFT). Our task is to describe the process of mode creation, and in particular to identify mechanisms which fix the state of the newly born modes [3].

Two strategies are possible. The first is to assume that the cutoff is a "true" cutoff, with nothing above it [22, 23]. In that case, we must face the challenge to formulate Quantum Mechanics with a changing structure, i.e. where a there is no unique Hilbert space and where the operators acting on it are not fixed once and for all (for instance, at two different times, the Hamiltonians differ by the number of modes on which they act non trivialy), see [24] are references therein. It is moreover possible that the theory has nothing to say about the initial state of the dynamical degrees of freedom, unless it selects a unique solution by some consistency requirement.

[3]Note that the approach to Quantum Gravity which models a grainy structure by an expanding lattice is likely to be hopeless. It misses a fundamental ingredient of Quantum Mechanics, namely randomness. At a "mesoscopic" scale, manifolds are more likely to be replaced by an equivalence class of sets of finite structures (which could be causal sets, holonomies...), presumably with a Poissonian distribution (since we know from [20] that it preserves local LI). It is remarkable that mathematical tools exist to deal with this kind of structures, namely sampling theory (although not developped to describe arbitrary Lorentzian manifolds). Fields with a finite density have a sampling property, i.e. they admit a description on both a manifold and on any random sampling of this manifold [21].

The second strategy is to assume the existence of an infinite ”reservoir” of trans-Planckian degrees of freedom in equilibrium with the cis-Planckian modes. The infinite density means that cis-Planckian modes can be solicited as much as need be, while the condition of stationarity guarantees that their (mean) density is constant, see for instance [25]. The degrees of freedom constituting this reservoir could be for instance virtual black-holes as elements of the space-time foam of Wheeler and Hawking, or the massive exited states of the string spectrum. We do not however need to know their precise nature to describe the phenomenology of low energy observables, and in particular for the choice of the initial state of the QFT modes. In that case Green’s functions generically display a dissipative behaviour (due for instance to the absorption of quanta by virtual black holes). Let us mention here one phenomenological approach described in the contribution of R. Parentani to these proceedings [26].

Mode creation can be realized in a unitary QFT as the conversion of confined degrees of freedom φ_q (overdamped modes) into propagating degrees of freedom (underdamped regime). In Minkowski space, dissipation occurs on-mass shell above the scale M if Lorentz invariance is explicitely broken at that scale through derivative interaction with a field ψ (unitarity is encoded in the form of fluctuation-dissipation relations). On an expanding background, the gravitational redshift of the proper momentum is responsible for the effective decrease of the coupling. Notice that the Equivalence Principal plays a central role in this mechanism. The state of the newly born modes is then a straightforward consequence of the dynamics: it is the interacting vacuum (we assume that the trans-Planckian d.o.f. decouple).

4. Dynamical selection of the initial state and consistent EFT treatment

If need was, the previous discussion made it clear that the oscillatory corrections found in (3.11) are not generic but contrived. They are an artefact of choosing $|\Psi_M\rangle$ to be the ground state of a quadratic Hamiltonian on a sharp boundary. This is inconsistent with the assumption that $|\Psi_M\rangle$ encodes the past dynamics of the cis-Planckian modes with the trans-Planckian degrees of freedom (or heavy cis-Planckian d.o.f. [10]). The state of the modes at the time of creation should be the interacting vacuum. In that case, there are no oscillations in the power spectrum. We insist that this conclusion is reached on general grounds.

The abscence of oscillations in the power spectrum can be seen in the following way. If the two-point function is the only measured observable of φ, the true state of the modes is operationaly indistinguishable from a Gaussian state $\rho_{\rm eff}$ with the same anticommutator function. This state has necessarily a non vanishing entropy because, by definition, it is a state for which we have no knowledge of the higher correlation functions (the Gaussian state is the state of maximal entropy among the set of density matrices which have the same two-point correlation function and differ only by the higher vertices). One easely shows [27] that $\rho_{\rm eff}$ has the form (2.6) and is charaterized by the two expectation values

$$n_q = \mathrm{Tr}\left(\rho_{\rm eff} a_{\mathbf{q}}^{-\infty\dagger} a_{\mathbf{q}}^{-\infty}\right) , \qquad c_q = \mathrm{Tr}\left(\rho_{\rm eff} a_{\mathbf{q}}^{-\infty} a_{\mathbf{q}}^{-\infty}\right) \tag{4.1}$$

where all the other expectation values of one and two creation and annihilation operators vanish by statistical homogeneity. The state is pure if, and only if $|c_q|^2 = n_q(n_q+1)$, which is the requirement that it minimizes the Heisenberg uncertainty relations. In that case, we have $n_q = |\beta_q|^2$ and $c_q = \alpha_q\beta_q$. The power spectrum (2.7) now reads

$$\mathscr{P}_\rho(q,t) \,=\, \frac{q^3}{2\pi^2}\left\{(2n_q+1)\,|\varphi_q^{-\infty}(t)|^2 + 2\mathrm{Re}\left[c_q\left(\varphi_q^{-\infty}(t)\right)^2\right]\right\} \tag{4.2}$$

which generalizes (3.11). For mixed states, it cannot be factorized in the form (2.8), and compared to the pure state $|\Psi_M\rangle$ with the same mean occupation number, the interference term is damped by the factor

$$\frac{|c_q|}{n_q(n_q+1)} < 1\,. \tag{4.3}$$

In particular, for $c_q = 0$, the oscillations are completely washed out. In the limiting case $n_q \to 0$, the BD vacuum is recovered. From that point of view, it is the state at zero temperature with the least correlations, as in any equilibrium state. Indeed, in the dynamical settings outlined at the end of the previous section, the two-point function in the interacting vacuum is indistinguishable in the weak interacting limit from the ones in the BD vacuum.

The absence of an oscillatory term can also be understood as the result of the averaging caused by the influence of several incoherent effects. It is likely that the high energy physics has a very rich phenomenology that we may not be able (or even need) to describe exactely, but only on a statistical basis. The statistical average is performed after the quantum expectation value. Averaging over the phase of the parameter c_q damps the oscillations in the power spectrum. That is, if we call f the distribution of the physical phase θ_q centered around a mean value $\bar{\theta}_q$,

$$\bar{c}_q \equiv \int_0^{2\pi}\frac{d\theta_q}{2\pi}\, f(\theta_q - \bar{\theta}_q)\, c_q = e^{i\bar{\theta}_q}\,|\alpha_q\beta_q|\,\langle\langle\, e^{i\theta}\,\rangle\rangle \tag{4.4}$$

where

$$\langle\langle\, e^{i\theta}\,\rangle\rangle < 1 \tag{4.5}$$

A third possible source also emerges from the picture of a reservoir, the elements of which exchange energy with the cis-Planckian modes. This exchange being governed by Quantum Mechanics, it is a random process. In other words the density of cis-Planckian degress of freedom is a field $\rho(t,\mathbf{x})$. Hence the cutoff, defined as $\rho = M^3$, fluctuates locally around a mean value $\bar{M}$ [25]. Since the phase $2q\tau_M$ of the oscillatory corrections depends linearly M, a fluctuating cutoff leads to damped oscillatory corrections. For instance, in the simplest model of a stochastic cutoff with Gaussian statistics

$$\langle\langle\, M\,\rangle\rangle = \bar{M}\,, \qquad \langle\langle\,(M-\bar{M})^2\,\rangle\rangle^{1/2} = \Sigma\,. \tag{4.6}$$

one finds (see [15] for details)

$$\langle\langle\, \sigma_q^p\, e^{i2/\sigma_q}\,\rangle\rangle \,=\, \bar{\sigma}_q^p\, e^{i2/\bar{\sigma}_q} \times \exp\left(-4\frac{\Sigma^2}{H_q^2}\right)\,. \tag{4.7}$$

This leaves the question of the amplitude of Σ open.

In all these cases, the damping may be strong enough so as to render the oscillatory term subleading. In that case, the leading corrections to the power spectra are the last term in (3.11), or more generally the term in (4.2) proportional to n_q, which depend only logarithmically on the momentum. They are therefore undistinguishable from a running of the slow roll parameters or loop corrections, thus closing the door to the hoped opportunity to use primodial power spectra as a probe in Quantum Gravity's phenomenology.

5. Summary

The notion of decoupling is challenged in the presence of a horizon. Predictions made by effective field theories are valid provided the ground state evolves adiabaticaly below a UV cutoff M. Rare nonadiabatic transitions appear as corrections in the power spectra of the cosmological perturbations and of the CMB. If large enough, these corrections can thus be used as a probe in the phenomenological approach to Quantum Gravity. Moreover, a proper cutoff means that modes are continuously created as the universe expands, leading to the question of the initial state of the modes at the creation time. We argued that assuming the existence of a trans-Planckian reservoir in order to maintain constant the density of degrees of freedom (and therefore the cutoff), the dynamics selects the initial state of the cis-Planckian modes to be the interacting vacuum. [The mechanisms listed in Sec. 4 can be considered as decoherence effects.] As a result, the leading corrections to the power spectra are likely to be featureless (unless so fined tuned dynamics occurs). This conclusion extends also to any phenomenological model of high-energy physics (not necessarily trans-Planckian).

Note added in proof: We briefly recall why the corrections to the power spectra in Eq. (3.2) in the models of Sec. 3.2 are in general polynomial in σ rather than exponentionally suppressed. The reader may find more details in [29] and [30]. Let φ_{WKB} be the WKB modes of the equation (2.4). The general solution of (2.4) can be decomposed as $\varphi = c(\tau)\varphi_{WKB}(\tau) + d(\tau)\varphi^*_{WKB}(\tau)$. The degree of non adiabaticity is controlled by the ratio (3.4) $Q = |\partial_\tau \omega|/\omega^2$ which is assumed to be slowly varying in the time interval $[\tau_i, \tau_f]$. We take as initial conditions $c(\tau_i) = 1$ and $d(\tau_i) = 0$. τ_f is the time at which the non-adiabatic corrections, i.e. $d(\tau_f)$, are calculated. If $Q \to 0$ at both $\tau \to \tau_i$ and τ_f, then $d(\tau_f)$ is exponentially small. On the other hand, when $Q \to 0$ at the initial time τ_i but $Q \neq 0$ at a later time τ_f, the corrections are polynomial in Q. This can be seen from the integral representation of $d(\tau) = \int_{\tau_i}^{\tau} d\tau' \frac{\partial_{\tau'}\omega(\tau')}{2\omega} \exp\left(i2\int^{\tau'}\omega\right) c(\tau')$. The leading contribution is then given by the boundary term $\propto Q(\tau_f)$.

Acknowledgments

This is to thank R. Brout, J. Niemeyer, and R. Parentani for many enjoyable discussions over the years about these questions.

This work is supported by the Alfried Krupp Prize for Young University Teachers of the Alfried Krupp von Bohlen und Halbach Foundation and by the European Science Foundation network programme "Quantum Geometry and Quantum Gravity".

A. Physical interpretation of the oscillatory corrections to the power spectrum

The equal time correlation function in the pure state $|\Psi\rangle$ is given by

$$\begin{aligned}\mathscr{P}_\Psi(q,t) &= \frac{q^3}{2\pi^2}\langle\Psi|\hat{\varphi}_{\mathbf{q}}(t)\hat{\varphi}_{\mathbf{q}}(t)|\Psi\rangle \\ &= \frac{q^3}{2\pi^2}\left\{\langle\Psi|\{\hat{a}_{\mathbf{q}}^{-\infty\dagger},\hat{a}_{-\mathbf{q}}^{-\infty}\}|\Psi\rangle|\varphi_q^{-\infty}(t)|^2+2\mathrm{Re}\left[\langle\Psi|\hat{a}_{\mathbf{q}}^{-\infty}\hat{a}_{-\mathbf{q}}^{-\infty}|\Psi\rangle\left(\varphi_q^{-\infty}(t)\right)^2\right]\right\} \end{aligned}\tag{A.1}$$

To calculate this expectation value, we need the expansion of $|\Psi\rangle$ in the basis of Fock states build from the Bunch-Davis vacuum. If $|\Psi\rangle$ is the ground state annihilated by $a_{\mathbf{q}}$ defined by

$$a_{\mathbf{q}} = \alpha_q^* a_{\mathbf{q}}^{-\infty} - \beta_q^* a_{\mathbf{q}}^{-\infty\dagger} \tag{A.2}$$

(obtained by inversion of (3.9)), one finds that $|\Psi\rangle$ is related to the Bunch-Davis vacuum through a unitary transformation (a squeezing operator)

$$|\Psi\rangle = N_q\sum_{n=0}^{+\infty}\left(\frac{\beta_q^*}{\alpha_q^*}\right)^n|n,\mathbf{q},BD\rangle\otimes|n,-\mathbf{q},BD\rangle \tag{A.3}$$

where $\{|n,\mathbf{q},BD\rangle\}$ is the Fock basis of the mode $\mathbf{q}$ constructed from the Bunch-Davis vacuum. N_q is the normalization factor. Written in the Fock basis build upon the BD vacuum, the state $|\Psi_M\rangle$ is entangled. It describes pairs of BD-quanta created at the time $\tau_M(q)$ in the case (3.5), or at τ_0 in the case (3.7). The expectation values in (A.1) are respectively the mean occupation number of the BD quanta and their correlations. They are related to the Bogolibov coefficients by

$$\langle\Psi|\hat{a}_{\mathbf{q}}^{-\infty\dagger}\hat{a}_{-\mathbf{q}}^{-\infty}|\Psi\rangle = |\beta_q|^2 \tag{A.4}$$

$$\langle\Psi|\hat{a}_{\mathbf{q}}^{-\infty}\hat{a}_{-\mathbf{q}}^{-\infty}|\Psi\rangle = \alpha_q\beta_q \tag{A.5}$$

This rewriting of the power spectrum simplifies the interpretation of the phase of inteference term. The latter was shown in [15] to be *generically* of the form

$$\theta_q \equiv \arg\left\{\alpha_q\beta_q^*\left[\varphi_q^{-\infty}(q\tau\ll 1)\right]^2\right\} = 2q\tau_M + O(1)\,. \tag{A.6}$$

This result was derived in the case (3.5) but is trivially generalized to boundary conditions (3.7), in which case one has $2q\tau_0$. It is important to note that this phase is independent of the choices of the phases of the mode functions and has therefore a physical meaning. To wit, θ_q is twice the phase accumulated from the creation time τ_M (or τ_0) until some time long after horizon exit when the power spectrum is evaluated and the phase of the BD modes freeze. This conclusion is reached by forming wave packets and following their semi-classical trajectories [28].

Consider a right moving wave-packet build from the Bunch-Davis modes

$$\bar{\varphi}_{\mathrm{R}}(\tau,\mathbf{x}) = \int\frac{\widetilde{d^3q}}{(2\pi^3)}\left(e^{i\mathbf{qx}}f(\mathbf{q})\varphi_{\mathbf{q}}^{-\infty}(\tau)+\mathbf{c.c.}\right) \tag{A.7}$$

where for definiteness we choose

$$f(\mathbf{q}) = \mathscr{N}\exp\left(-\frac{(\mathbf{q}-\bar{\mathbf{q}})^2}{4\sigma^2}\right)e^{-i\mathbf{qx}_d}e^{iq\tau_d} \tag{A.8}$$

Assuming that (A.7) with (A.8) can be evaluated by the stationary phase condition, one obtains the semi-classical trajectory

$$\mathbf{x}_R(\tau,\bar{\mathbf{q}}) \equiv \mathbf{x}_d + (\tau-\tau_d)\frac{\bar{\mathbf{q}}}{|\bar{\mathbf{q}}|} \tag{A.9}$$

The superposition (A.7) therefore describes a localized field configuration propagating with mean momentum $\bar{\mathbf{q}}$ and detected at $(\tau_d,\mathbf{x}_d)$.

We now consider the two-point function of φ in the state $|\Psi\rangle$ filtered by (A.7),

$$\Theta(\tau,\mathbf{x};\bar{\varphi}_{\mathrm{R}}) \equiv \int d^3y\,\bar{\varphi}(\tau,\mathbf{y})\,\langle\Psi|\hat{\varphi}(\tau,\mathbf{x})\hat{\varphi}(\tau_0,\mathbf{y})|\Psi\rangle \tag{A.10}$$

The function $\Theta(\tau,\mathbf{x})$ is a sum of four wave packets. Two of them are the miror symmetric of the others. This doubling of the information is only due to the filtering (A.10) which does not distinguish between right and left moving wave-packets. This technical problem can be resolved using the Klein-Gordon product in place of (A.10). To keep the presentation straightforward, we pass over this technicality and other details, and give only the relevant information for the interpretation of the phase, refering the interested reader to [28] for details. One obtains two semiclassical field configurations

$$\Theta(\tau,\mathbf{x};\bar{\varphi}_{\mathrm{R}}) = \Phi(\mathbf{x}-\mathbf{x}_R(\tau,\bar{q})) + \Phi(\mathbf{x}-\mathbf{x}_L(\tau,\bar{q})) \tag{A.11}$$

The first term is similar to (A.7) while the second is centered on the trajectory

$$\mathbf{x}_L(\tau,\bar{\mathbf{q}}) \equiv \mathbf{x}_d - (\tau+\tau_d-4\tau_M)\frac{\bar{\mathbf{q}}}{|\bar{\mathbf{q}}|} \tag{A.12}$$

It is the trajectory of a left moving field configuration, i.e. with a mean momentum $-\bar{\mathbf{q}}$. The separation between the centers of the two wave-packets is

$$\mathbf{x}_R(\tau,\bar{\mathbf{q}}) - \mathbf{x}_L(\tau,\bar{\mathbf{q}}) = 2(\tau-\tau_M)\frac{\bar{\mathbf{q}}}{|\bar{\mathbf{q}}|}\,, \tag{A.13}$$

which completes the proof.

References

[1] WMAP Collaboration (D.N. Spergel & al.), *Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology*, *Astrophys. J. Suppl.* **170** (2007) 288, [astro-ph/0603449].

[2] J. Martin and R. Brandenberger, *The TransPlanckian problem of inflationary cosmology*, *Phys. Rev.* **D63** (2001) 123501, [hep-th/0005209]

[3] Jens C. Niemeyer *Inflation with a Planck scale frequency cutoff*, *Phys. Rev.* **D63** (2001) 123502, [astro-ph/0005533].

[4] J.C. Niemeyer and R. Parentani, *Trans-Planckian dispersion and scale invariance of inflationary perturbations*, *Phys. Rev.* **D64** (2001) 101301(R), [astro-ph/0101451].

[5] K. Schalm, G. Shiu, J.P. van der Schaar, *The Cosmological vacuum ambiguity, effective actions, and transplanckian effects in inflation*, *AIP Conf. Proc.* **743** (2005) 362, [hep-th/0412288].

[6] H. Collins and R. Holman, *Trans-Planckian signals from the breaking of local Lorentz invariance*, [0705.4666 [hep-ph]].

[7] A. Kempf, *Mode generating mechanism in inflation with cutoff Phys. Rev.* **D63** (2001) 083514, [astro-ph/0009209]; A. Kempf and J. Niemeyer, *Phys. Rev.* **D64** (2001) 103501, [astro-ph/0103225].

[8] R. Easther, B. R. Greene, W. H. Kinney, G. Shiu, *Inflation as a probe of short distance physics*, *Phys. Rev.* **D64** (2001) 103502, [hep-th/0104102].

[9] N. Kaloper, M. Kleban, A.E. Lawrence, S. Shenker, *Signatures of short distance physics in the cosmic microwave background*, *Phys. Rev.* **D66** (2002) 123510, [hep-th/0201158]; N. Kaloper, M. Kleban, A.E. Lawrence, S. Shenker, L. Susskind, *Initial conditions for inflation*, *JHEP* **0211** (2002) 037, [hep-th/0209231].

[10] C.P. Burgess, J. Cline, F. Lemieux, R. Holman, *Are inflationary predictions sensitive to very high-energy physics? JHEP* **0302**, (2003) 048 [hep-th/0210233]; C.P. Burgess, J. Cline, R. Holman, *Effective field theories and inflation JCAP* **0310** (2003) 004 [hep-th/0306079]; C.P. Burgess, J. Cline, F. Lemieux, R. Holman, *Decoupling, trans-planckia and inflation*, [astro-ph/0306236].

[11] U. Danielsson, *A Note on inflation and transPlanckian physics*, *Phys. Rev.* **D66** (2002) 023511, [hep-th/0203198].

[12] R. Easther, B. R. Greene, W. H. Kinney, G. Shiu, *A Generic estimate of transPlanckian modifications to the primordial power spectrum in inflation*, *Phys. Rev.* **D66** (2002) 023518, [hep-th/0204129].

[13] J. Martin and R. Brandenberger, *On the dependence of the spectra of fluctuations in inflationary cosmology on transPlanckian physics Phys. Rev.* **D68** (2003) 063513, [hep-th/0305161].

[14] D. Campo, J.C. Niemeyer, R. Parentani, *Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff*, *Phys. Rev.* **D66** (2002) 083510, [hep-th/0206149].

[15] D. Campo, J.C. Niemeyer, R. Parentani, *Damped corrections to inflationary spectra from a fluctuating cutoff*, *Phys. Rev.* **D76** (2007) 023513, [arXiv:0705.0747 [hep-th]].

[16] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, *Theory of cosmological perturbations*, *Phys. Rep.* **215**, (1992) 203.

[17] T. S. Bunch and P. C. W. Davies, *Quantum field theory in de Sitter Space: renormalization by point splitting*, *Proc. R. Soc. London* **A360**, (1978) 117.

[18] T. Appelquist and J. Carazzone *Infrared singularities and massive fields*, *Phys. Rev.* **D11** (1975) 2856.

[19] A. Pich, *Effective field theory: Course*, [hep-ph/9806303].

[20] F. Dowker, J. Henson, R.D. Sorkin, *Quantum gravity phenomenology, Lorentz invariance and discreteness*, *Mod. Phys. Lett.* **A19** (2004) 1824, [gr-qc/0311055]; L. Bombelli, J. Henson, R.D. Sorkin, *Discreteness without symmetry breaking: A Theorem*, [gr-qc/0605006].

[21] A. Kempf, *A covariant information-density cutoff in curved space-time*, *Phys. Rev. Lett.* **92** (2004) 221301 [gr-qc/0310035]; A. Kempf and R. Martin, *On information theory, spectral geometry and quantum gravity*, [arXiv:0708.0062 [gr-qc]].

[22] B. Z. Foster and T. Jacobson, *Quantum field theory on a growing lattice*, *JHEP* **0408** (2004) 024 [hep-th/0407019].

PoS(QG-Ph)014

[23] A. Kempf, *Mode generating mechanism in inflation with a cutoff*, *Phys. Rev.* **D. 63** (2001) 083514 [astro-ph/0009209]; A. Kempf and L. Lorenz, *Exact solution of inflationary model with minimum cutoff*, *Phys. Rev.* **D. 74** (2006) 103517 [gr-qc/0609123].

[24] T. Jacobson, *Trans-Planckian redshits and the substance of the space-time river*, *Prog. Theor. Phys. Suppl.* **136** (1999) 1, [hep-th 0001085].

[25] R. Brout, *Condensation of Planckian modes and the inflaton*, [gr-qc/0305054].

[26] R. Parentani, *Constructing QFT's wherein Lorentz invariance is broken by dissipative effects in the UV*, [hep-th/0709.3943].

[27] D. Campo and R. Parentani, *Inflationary spectra and partially decohered distributions Phys. Rev.* **D72** (2005) 045015, [astro-ph/0505379].

[28] D. Campo and R. Parentani, *Space-time correlations within pairs produced during inflation, a wave packet analysis*, *Phys. Rev.* **D67** (2003) 103522, [gr-qc/0301044]; D. Campo and R. Parentani, *Space-time correlations in inflationary spectra: A Wave-packet analysis*, *Phys. Rev.* **D70** (2004) 105020, [gr-qc/0312055].

[29] S. Massar and R. Parentani, *Particle creation and nonadiabatic transitions in quantum cosmology*, *Nucl. Phys.* **B513** (1998) 375, [gr-qc/9706008].

[30] J. P. Davis and P. Pechukas, *Nonadiabatic transitions induced by a time-dependent Hamiltonian in the semiclassical/adiabatic limit: the two-state case*, *J. Chem. Phys.* **64** (1976) 3129; J. Hwang and P. Pechukas, *The adiabatic theorem in the complex plane and the semiclassical calculation of nonadiabatic transition amplitudes*, *J. Chem. Phys.* **67** (1977) 4640.