

Quantum Coherence and Entanglement in Black Hole Information Issue from a Detector (Atom) - Field Analog

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This is a synopsis of our recent work [1] on quantum entanglement, quantum coherence and information flow between a uniformly accelerated detector and a massless quantum scalar field. The availability of exact solutions to this model enables us to explore some important aspects of the black hole information issue with some quantifiable results and new insights. To the extent this model can capture the essence of quantum coherence, entanglement and information flow between a black hole and its Hawking radiation, our result seems to suggest that in the prevalent non-Markovian regime, assuming unitarity holds for the combined system which is a dictum of physical laws, that black hole information is not lost or stored in the correlations between the black hole and the quantum field but totally transferred to the quantum field degrees of freedom. This combined system would evolve into a highly entangled state between a remnant of large area (in Bekenstein's black hole atom analog) losing all information of its initial state, while the quantum field is imbued with complex information content not-so-easily retrievable by a local observer.

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A note on BLH's talk in this meeting

This was meant to be a conference paper based on the invited talk of BLH at the Workshop "From Quantum to Emergent gravity: Theory and phenomenology" 11-15 June, 2007 at SISSA, Trieste, Italy. The overall theme of the talk is on a new view towards quantum gravity as a theory of the microscopic structure of spacetime. The statement is that such a theory may be inequivalent to that obtained by quantizing general relativity. This highly successful theory for the macroscopic structure of spacetime may just be an effective theory valid at the low energy, long wavelength limit of the underlying theory describing the microscopic structures of spacetime. With the metric and connection forms acting as the collective or hydrodynamic variables of the microscopic theory, classical gravity in this view is emergent and these variables will lose their meaning at shorter wavelengths and higher energies. Examples are drawn from hydrodynamics, critical dynamics, quantum fluids, and atomic - condensed matter physics to illustrate how very different the emphasis and approaches, the goals and methodology are between the traditional view of quantizing general relativity and the new view of gravity as an emergent theory. Instead of placing emphasis on quantization and posing the challenge of finding the quantum version of a classical theory, the new challenge is to infer the microscopic structure from the known macroscopic phenomena. This has been the task for physicists for centuries. For this, concepts and methods from nonequilibrium statistical mechanics and examples from strongly correlated many-body systems will probably play an essential role hitherto largely ignored for quantum gravity.

From this general backdrop, special emphasis was placed in this talk on the properties of emergent theories, distinguishing those which can be logically and methodically deduced from a microscopic theory and those which cannot, at least not without knowing in some degree certain attributes of the macroscopic theory¹. The former type poses the difficult task of inferring the unknown micro structures from the known macro phenomena, but the latter type adds to it a more difficult challenge of finding the unknown characteristics of an emergent theory. It is not enough to say something is emergent² – If we don't know what gives rise to the emergent theory, at least we should try to describe the underlying processes or mechanisms that could lead to such phenomena. These are the new challenges of quantum gravity in the emergent vein.

The general theme of BLH's invited talk can be found ($\sim 2/3$ of the slides) in the website of the Loops '07 meeting, 25 - 30 June 2007 in Morelia, Mexico³. The issues of nonlocality and stochasticity in the quantum-classical and micro-macro interfaces and how they bear on emergent gravity will be discussed in Ref. [2].

Instead of repeating what can be found in published papers and essays the speaker finds it more useful to present some of the newest research results, as in this case, the work summarized in this paper. It is hoped that this is deemed excusable or even appropriate, because analog gravity is an important theme in this Workshop.

¹Hydrodynamics or nuclear physics are examples of the former (from molecular dynamics and QCD respectively) and quantum Hall effect is often quoted as an example of the latter: One can construct useful theories only AFTER such an effect was observed, but not (or highly unlikely) BEFORE.

²Oftentimes referring to something as emergent is an euphemism for saying we don't really know what the underlying theories are. It borders on a 'copt-out', one with a philosophical tinge notwithstanding.

³<http://www.matmor.unam.mx/eventos/loops07/>

1. Detector-field System and Black-hole Atom Analog

In this work we wish to understand some aspects of the black hole information problem [3, 4, 5] (see, e.g., [6, 7] for an overview) by probing into one key issue of it, namely, how information is distributed between the black hole and the quantum field throughout its history. We are not in a position to unequivocally decide on the end state of a black hole after emitting Hawking radiation [8] – remnants, naked singularity, baby universe formation or complete evaporation. (See e.g., [9, 10, 11, 12, 13].) Nor are we equipped to engage in debates on whether there is net information lost in a black hole, much less the grander issue of whether unitarity in the laws of physics is violated.

On the specific aspect we are interested in, i.e., where information is registered, stored and transferred, a prevailing thought of the school which upholds the validity of unitarity (and thus advocates no information loss) is that information resides in the correlation between the black hole and its Hawking radiation which persists down to the very end of evaporation. Here we want to examine with the help of an analog model another contrasting view expressed earlier by one of us [14] (see also [12]), assuming no violation of unitarity, namely, that information in the black hole is not lost but transferred and dispersed into the quantum field through Hawking radiation, nor does it reside predominantly in the correlations between the black hole and the quantum field. Ref. [14] proposes to use the correlation functions of an interacting field as registers of information and the dynamics of correlations as a measure of information flow accompanied by a suggested scenario where information in the black hole is transferred to the quantum field. According to the viewpoint put forth there, the appearance of information loss is primarily owing to the fact that actual physical measurements by a local observer are limited in accuracy, i.e., one can only access the lowest order correlation functions, beginning with the mean field and the two-point functions. It also highlights the huge capacity of a quantum field in storing and dispensing information.

We have recently studied an exactly solvable model where an uniformly accelerated detector is linearly coupled to a massless scalar field initially in the Minkowski vacuum. Based on the results of this study we drew some suggestive implications on the black hole information flow issue by invoking Bekenstein's black hole atom analog. We give a brief description of the black hole atom analog below, followed by the detector-field model we studied and our findings.

Bekenstein observed that the black hole behaves like an ensemble of quantum mechanical atoms [15, 16, 17], whose spontaneous emissions correspond to Hawking radiation, and the energy level of the atom is analogous to the area level of a black hole. When black holes are fed with field quanta, they tend to absorb more than emit energy. Indeed, Bekenstein and Meisels showed that for black holes the Einstein B coefficient for stimulated absorption is greater than the coefficient for stimulated emission [16]. Following this idea the atom (or the particle detector considered below) itself can be treated as an analog of the black hole, from which we hope to address some aspects of the black hole information problem from this ordinary atom-field interacting system. (Note that the theory for black hole atoms does not a priori assume any violation of unitarity and hence any information loss.)

Dynamics of quantum coherence of an inertial detector interacting with a quantum field in the ultraweak coupling regime has been studied before by Anglin *et al.* [18] They claimed that, as soon as the coupling is switched on, the oscillator loses the quantum coherence on a very short

decoherence time corresponding to the cut-off time scale. But almost all the quantum coherence will recover in the end, after a much longer relaxation time. They then used these findings to draw some implications on the black hole information problem. A comparison of our findings in this more general case with theirs is contained in an appendix of our recent paper [1].

Consider a moving harmonic oscillator with internal degree of freedom Q (known as the Unruh-DeWitt(UD) detector [19, 20, 21]) interacting with a massless quantum scalar field Φ in four-dimensional Minkowski space. The action of the combined particle detector - quantum field system is given by [22]

$$S = \int d\tau \frac{m_0}{2} \left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2 \right] - \int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \lambda_0 \int d\tau \int d^4x Q(\tau) \Phi(x) \delta^4(x^\mu - z^\mu(\tau)), \quad (1.1)$$

where λ_0 is the coupling constant, m_0 and Ω_0 are the bare mass and natural frequency of the detector, respectively. We will consider the cases when it is uniformly accelerated along the trajectory $z^\mu(\tau) = (a^{-1} \sinh a\tau, a^{-1} \cosh a\tau, 0, 0)$ with proper acceleration a . For the cases of detectors at rest ($a = 0$), we have learnt in [22, 23] that the two-point functions of our UD detector theory in (3+1)D with finite a have no singular behavior as $a \rightarrow 0$. Hence all our results expressed in terms of these two-point functions apply equally well to the case of detectors at rest.

2. Information flow between the detector and the quantum field

We study the case when the initial state of the combined system is a direct product of a quantum state $|q\rangle$ for the detector Q and the Minkowski vacuum $|0_M\rangle$ for the field Φ ,

$$|\psi(\tau_0)\rangle = |q\rangle \otimes |0_M\rangle. \quad (2.1)$$

Since the combined system is linear, the operators evolve in the Heisenberg picture as linear transformations. When sandwiched by the factorized initial state (2.1), the two-point functions of the detector and those of the field split into two parts [22], e.g.,

$$\langle Q(\tau)Q(\tau') \rangle = \langle q|q \rangle \langle Q(\tau)Q(\tau') \rangle_v + \langle Q(\tau)Q(\tau') \rangle_a \langle 0_M|0_M \rangle. \quad (2.2)$$

Here $\langle \dots \rangle_a$ depends on the initial state of the detector only, while $\langle \dots \rangle_v$ depends on the initial state of the field, namely the Minkowski vacuum. Therefore by studying the correlation functions $\langle \dots \rangle_a$ of the detector and those of the field, one can monitor how the information initially in the detector flows into the field.

Indeed, from Ref. [22], we learned that $\langle Q^2 \rangle_a$, $\langle P^2 \rangle_a$ and $\langle P, Q \rangle_a$ all decay after the coupling is switched on, and the information about the initial state of the detector is subsumed into the quantum field (in $\langle \Phi(x)\Phi(x') \rangle_a$, etc.). This view is further supported by the energy conservation law found in [22] between the internal energy of the detector and the radiated energy of a monopole. The energy in $\langle Q^2 \rangle_a$ and $\langle P^2 \rangle_a$ will be converted to monopole radiation while the state of the detector at late times is sustained only by the vacuum fluctuations of the field.

3. Entanglement between the detector and the field

For a bipartite system with the combined system in a pure state, such as the Unruh-DeWitt detector theory with initial state (2.1), the purity of each sub-system

$$\mathcal{P} \equiv \text{Tr}_Q [\rho^R(Q, Q')]^2 = \mathcal{P}_\Phi \equiv \text{Tr}_\Phi (\rho^R[\Phi, \Phi'])^2 \quad (3.1)$$

is equal to the other and gives a measure of the entanglement between them. Here $\rho^R(Q, Q')$ and $\rho^R[\Phi, \Phi']$ are reduced density matrices (RDM) of the detector and of the field, respectively. On the other hand, the purity of a two-level atom is proportional to its polarization, thus providing a measure of quantum coherence in that atom. Indeed, a two-level atom is not fully decohered until it becomes an infinite-temperature thermal state where the purity is zero. A thermal state at finite temperature, though looks classical, is not fully decohered and the purity is not vanishing. (Note that a diagonal RDM can nonetheless possess nonzero off-diagonal elements under a basis transformation.) Here we extend this view to our system and use the value of the purity function as a measure of quantum coherence in the detector and in the field.

The behavior of quantum coherence “flow” is quite different from energy flow. When the coupling is switched on, both the quantum coherence in the detector and the quantum coherence in the field decrease, while the entanglement between them increases. So quantum coherence does not flow from one subsystem to the other; It goes into sustaining the entanglement between the two subsystems.

A lower purity means lesser quantum coherence in the detector or the field and stronger entanglement between the detector and the field. To show this one may define the linear entropy in terms of the purity as

$$S_L \equiv 1 - \mathcal{P}. \quad (3.2)$$

Now the value of S_L is zero for a detector in a pure state and is positive for a detector in a mixed state. By definition the linear entropy seen by the detector will be equal to the linear entropy seen by the field. Also, the greater the von Neumann entropy of the detector, the greater S_L . Thus S_L could serve as a measure of entanglement between the detector and the field, just as good as the von Neumann entropy, for the bipartite system in a pure state.

As an example, suppose the detector is in a cat state at the initial moment τ_0 ,

$$|q(\tau_0)\rangle = \cos \varphi |E_0\rangle + e^{i\delta} \sin \varphi |E_1\rangle, \quad (3.3)$$

where $|E_0\rangle$ and $|E_1\rangle$ are the ground state and the first excited state of the free detector, φ is the mixing angle and δ is a constant phase. The RDM of the detector for this initial state reads

$$\rho^R(Q, Q'; \tau) = \sqrt{\frac{G^{11} + G^{22} + 2G^{12}}{\pi}} e^{-G^{ij} Q_i Q_j} \left\{ \cos^2 \varphi + \sin^2 \varphi (C + A^{ij} Q_i Q_j) + \sin \varphi \cos \varphi \left[(e^{i\delta} B^1 + e^{-i\delta} B^{2*}) Q + (e^{-i\delta} B^{1*} + e^{i\delta} B^2) Q' \right] \right\}, \quad (3.4)$$

where $i, j = 1, 2$, $Q_i = (Q, Q')$. The coefficients C , A^{ij} , B^j , and G^{ij} could be expressed in terms of the two-point correlation functions of the detector. Explicit expressions for the two-point functions needed in this paper are listed in Appendix A of Ref. [23]. Actually all A^j and B^j will vanish at

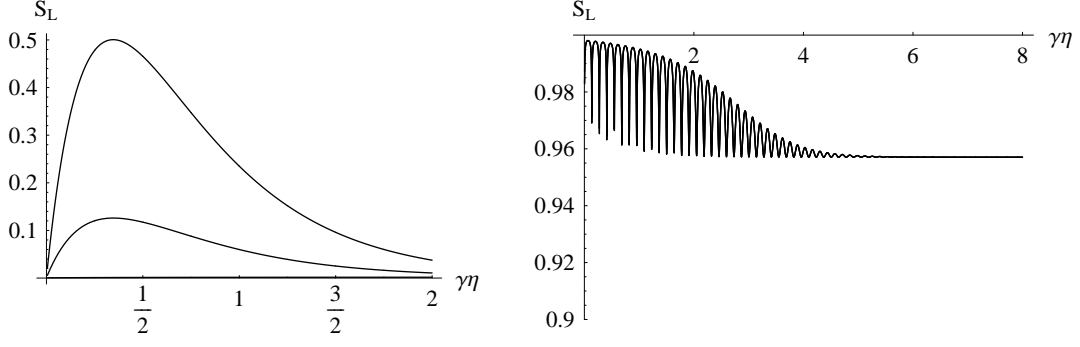


Figure 1: Evolution of linear entropies S_L of (3.4) in detector's proper time. Here $\Omega = 2.3$, $a = 2$, $m_0 = \hbar = 1$, $\Lambda_1 = \Lambda_0 = 10000$, $\delta = 0$, $\gamma = 10^{-7}$ (left) and 0.1 (right). The three curves from top to bottom in each plot (the bottom one in the left plot is very close to the $\gamma\eta$ axis, while the three curves are indistinguishable in the right plot) have $\varphi = \pi/2, \pi/4, 0$, respectively.

late times ($\gamma\eta \gg 1$ with $\gamma \equiv \lambda_0^2/8\pi m_0$ and $\eta \equiv \tau - \tau_0$) when the RDM for all choices of δ and φ in initial states become a universal one,

$$\rho^R|_{\gamma\eta \gg 1} = \rho^R|_{\varphi=0, \gamma\eta \gg 1}. \quad (3.5)$$

Hence the purity (or the linear entropy) goes to a universal value at late times.

3.1 Spontaneous emission in the ultraweak coupling regime

The linear entropies of the RDM (3.4) with different parameters are illustrated in Figure 1. In the ultraweak coupling regime ($\gamma\Lambda_1 \ll a, \Omega$, where Λ_1 is a large constant corresponding to the time-resolution or the frequency cut-off of this theory, $\Omega \equiv \sqrt{\Omega_r^2 - \gamma^2}$ with Ω_r the renormalized natural frequency of the detector) [23], for a/Ω sufficiently small, one has

$$\mathcal{P} \approx 1 + 2e^{-2\gamma\eta} (e^{-2\gamma\eta} - 1) \sin^4 \varphi. \quad (3.6)$$

One can see that in this regime \mathcal{P} is very close to unity at late times, when each subsystem re-gains almost all quantum coherence and turns into a nearly pure state.

Indeed, observing the left plot of Figure 1, the linear entropy S_L in the detector increases from zero right after the coupling is switched on, reaches a maximum $(1/2) \sin^4 \varphi$ at $\eta \approx \ln 2/2\gamma$, then decays to a small common value that detectors with all other initial states will asymptote to. Anglin, Laflamme, Zurek and Paz (ALZP) used the decay of the von Neumann entropy as an indication of the restoration of quantum coherence and called it ‘‘recoherence’’ [18]. Our results show that late-time ‘‘recoherence’’ manifests only in the ultraweak coupling regime at sufficiently low acceleration (temperature), where the late-time RDM of the detector closely resembles the density matrix of the ground state of a free detector. Thus ALZP’s ‘‘recoherence’’ characterizes the process of spontaneous emission by which the detector initially in an excited state will finally fall into a steady state which is very close to the ground state of the free detector. Nevertheless, *full* ‘‘recoherence’’ is impossible once the coupling is on, since the late-time linear entropy

$$S_L|_{\gamma\eta \rightarrow \infty} \approx 1 - \tanh \frac{\pi\Omega}{a} + \gamma \frac{2 \tanh^2 \frac{\pi\Omega}{a}}{\pi\Omega} \left[\Lambda_1 + \ln \frac{\Omega}{a} - \text{Re} \left[\psi \left(\frac{i\Omega}{a} \right) + \frac{i\Omega}{a} \psi^{(1)} \left(\frac{i\Omega}{a} \right) \right] \right] + O(\gamma^2) \quad (3.7)$$

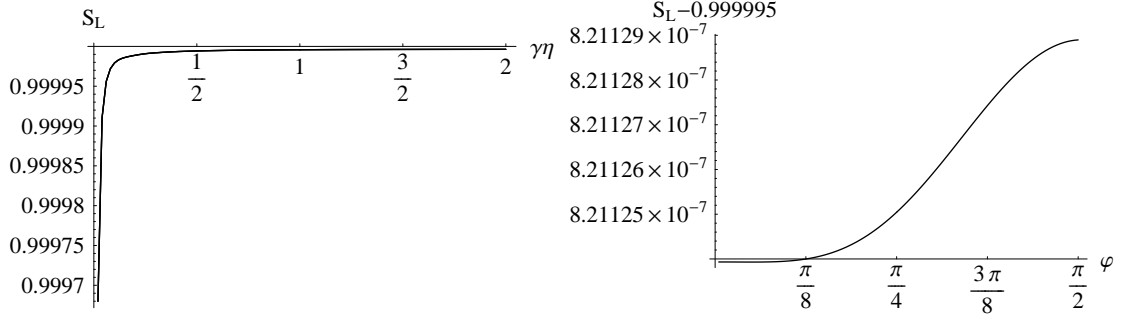


Figure 2: S_L in ultrahigh acceleration limit: $a = 2 \times 10^6$, $\gamma = 0.1$, other parameters have the same values as those in Figure 1. As indicated in the right plot, the curves with different φ are not distinguishable in the left plot.

$(\psi^{(1)}(x) \equiv d\psi(x)/dx)$ remains nonzero for any positive γ , even when $a \rightarrow 0$.

If the detector is initially in its first excited state ($\varphi = \pi/2$), the two-point function $\langle \mathcal{Q}^2 \rangle$ at $\eta = \ln 2/2\gamma$ will have the same value as the average of those $\langle \mathcal{Q}^2(\tau_0) \rangle$ from the ground state and from the first excited state at the initial moment. It may seem that the intermediate state of the detector during the transition is a cat state which is a superposition of the ground state and the first excited state of the detector, but this is not true. Large S_L in transient indicates that the intermediate state during the spontaneous emission is a mixed state, in contrast to the zero S_L for a pure cat state (at the initial moment of the middle curve in the left plot of Figure 1). The value of $\langle \mathcal{Q}^2 \rangle$ in transient is mainly an ensemble (probabilistic) average of the population in the ground state and the population in the first excited state.

3.2 Beyond the ultraweak coupling regime

When $\gamma\Lambda_1$ is comparable with or greater than a and Ω , the system is in the non-Markovian regime and the purity is always small (right plot in Figure 1), which implies that the detector experiences strong decoherence associated with strong entanglement between the detector and the field. The behavior of the detector is dominated by the physical cut-offs and the differences between various initial states of the detector can be negligible. For example, when $\varphi = \pi/2$, just like the case with the detector initially in its ground state, the initial distribution of the RDM in the energy-eigenstate representation $\rho_{m,n}^R$ peaked at the element $\rho_{1,1}^R$ would, upon the switch-on of the coupling, collapse rapidly into a distribution widely spread over the whole density matrix, for which the energy eigenstates of the free detector cannot form a good basis because the off-diagonal terms of the RDM do not vanish even in a steady state [23]. The late-time linear entropy

$$S_L|_{\gamma\eta \gg 1} \approx 1 - \frac{\pi/2}{\sqrt{\frac{2}{\Omega}\gamma\Lambda_1 \operatorname{Re} \left[\frac{ia}{\gamma+i\Omega} - 2i\psi_{\gamma+i\Omega} \right]}} + O((\gamma\Lambda_1)^{-3/2}) \quad (3.8)$$

is very close to unity. Hence there is no “recoherence” in ALZP’s sense in this regime, where the quantum state of the combined system is far from being a direct product of the state of the detector and that of the field.

A large linear entropy at late times also shows up at the ultrahigh acceleration (or Unruh temperature) limit ($a \gg \gamma\Lambda_1, \Omega$),

$$S_L|_{\gamma\eta \gg 1} \approx 1 - \frac{\pi\Omega_r}{2a} + O(a^{-2}), \quad (3.9)$$

(see Figure 2). When the coupling is weak enough, the energy eigenstates can still form a good basis in which the RDM describes the detector in an approximate thermal state, i.e., all off-diagonal elements are negligible. Note that the ultrahigh temperature limit is still in a Markovian regime, so we see that strong entanglement does not imply a non-Markovian process.

4. Discussion

Our results show that, as long as the coupling between the detector and the field is on, the detector and the field are separately in a mixed state of its own, while the combined system remains in a pure state. The detector in a mixed state carries information of the initial state until $\langle Q^2 \rangle_a$, $\langle P^2 \rangle_a$ and $\langle P, Q \rangle_a$ all decay away as the detector reaches a steady state. At late times, while the energy eigenstates of the free detector cannot form a good basis, the RDM of the detector can be diagonalized to a Boltzmann distribution from which one can read off the same effective temperature as those reported in [23]. In this sense the final mixed state of the detector is a thermal state containing no initial information. This is true for both inertial and uniformly accelerating detectors, the former case might be a surprise.

Similarly, when the black hole is radiating, the black hole itself and the field outside the black hole are each in a mixed state. While the Markovian regime is where one can define the Einstein A and B coefficients of black holes [16], only in the ultraweak coupling limit (where the temperature of the environment is sufficiently small) can they restore most of their purity at late times. In the more prevalent non-Markovian regime, we expect that the area eigenstates of the black hole cannot form a good basis, and the entanglement between the black hole and the field is always large. Nevertheless, in this scenario the quantum state of the combined system is always pure due to the unitarity assumed from the start.

The existence of Einstein A and B coefficient for black holes suggests that, if the field is initially in a vacuum state, the information about the black hole would be encoded in its spontaneous emission, namely, its emitted radiation which is not exactly thermal⁴. All initial information in the black hole will eventually go to the field at late times, while the final state of the black hole is sustained by the vacuum fluctuations of the quantum field. This is consistent with the “no-hiding theorem” of Braunstein and Pati (with their ancilla as our quantum field) [24]: no information is hidden in the correlations between the field and the black hole.

In our model the difference between the degrees of freedom of the detector and those of the field is put in by hand and never disappears, so we cannot address whether the ground state corresponds to a black hole remnant, an ordinary localized mass, or nothing. However, in Bekenstein’s atom analog highly excited eigenstates correspond to large-area black holes. Our results indicate that in the rather general non-Markovian regime, the combined system would evolve to a highly entangled state between the black hole and the quantum field, and the final state of the black

⁴Note that as different from atoms, for a real black hole its event horizon would have an effect on such emission.

hole would be a mixed state distributed widely from the ground state to the highly excited area-eigenstates. So at late times in the broad ranged non-Markovian regime the black hole could end up as a remnant with a large mean area and with all its initial information already leaked out and dispersed into the quantum field.

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