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QNM spectrum in (1+1)-dimensional BEC black holes

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The object of this contribution is to provide an illustration of the impact of modified dispersion relations with respect to relativistic or hydrodynamic ones. The example studied here is the quasi-normal mode (QNM) spectrum of a toy model for an acoustic black hole in a (1+1)-dimensional flow of a Bose–Einstein condensate (BEC). Acoustic black holes in the hydrodynamic limit, just like general relativistic black holes, have no quasinormal modes in 1+1 dimensions, whereas in 3+1 a discrete QNM spectrum appears. However, when using the full or modified (Bogoliubov) dispersion relation, these (1+1)-dimensional BEC black holes present a QNM spectrum consisting of a continuous region in the complex frequency plane. A straightforward speculation about similar effects in high-energy physics scenarios with modified dispersion relations will briefly be discussed.

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1. Introduction

The presence of quasinormal mode frequencies in the gravitational radiation of black holes might provide a direct experimental confirmation of the existence and main properties of black holes in the near future. The usual analysis of quasinormal modes of black holes [2, 1] is based on general relativity (GR), i.e., on a classical model. However, it could be that quantum effects become important not only for physical processes ocurring near a singularity (where they are commonly expected), but also for the physics associated with the presence of horizons, no matter how big or small the black hole in question. This would be the case if spacetime at small scales shared some of the properties found in analogue gravity models in condensed matter (see, e.g.,[3]). As suggested by these models, a possible way for implementing these quantum corrections is to consider modified dispersion relations, associated to violations of Lorentz invariance at high energies [4].

The aim of this contribution is to illustrate the impact that modifications of the dispersion relation can have on the QNM spectrum through a simple toy model [5]. The toy model consists of a (1+1)-dimensional flow of a Bose–Einstein condensate with a black hole horizon. In the hydrodynamic limit, these acoustic black holes have a quasinormal mode spectrum qualitatively very similar to the one that is found in general relativity [6]: There are no quasinormal modes at all in 1+1 dimensions, while calculations in 3+1 models yield a discrete spectrum [2, 1]. In this contribution, we will discuss the calculation of QNMs in this (1+1)-dimensional BEC model when going beyond the hydrodynamic or relativistic limit. We will use the full Bogoliubov dispersion relation, thereby in a certain sense incorporating quantum corrections originating from (a linearization of) the Gross–Pitaevskii equation. The surprising result is that such a spectrum not only exists, in spite of the (1+1)-dimensional setting, but consists of a continuous region of the complex frequency plane.

The structure of this contribution is the following. In the first section we will briefly summarise what quasinormal modes of a general relativistic black hole are and how they are calculated. We will emphasise the absence of QNMs in 1+1 dimensions, and the discrete spectrum in 3+1, and indicate that these results are also valid for acoustic black holes in the hydrodynamic limit. In the second section we will describe the toy model for an acoustic black hole in a (1+1)-dimensional BEC flow, and indicate how QNMs can be calculated in this model. We will lay particular emphasis on the difference between the hydrodynamic or relativistic dispersion relation and the full Bogoliubov dispersion relation, and also highlight the boundary condition for the QNM problem, since these are slightly different from the GR case. Finally, in the third section we will show and discuss the result of this calculation, namely that the QNM spectrum turns out to consist of a continuous region in the complex frequency plane.

2. Quasinormal modes of black holes

A tentative definition of QNMs of a gravitational black hole could simply be the following: QNMs of a black hole are the relaxation modes or energy dissipation modes that characterise the pulsations of the black hole after perturbation. Since QNMs are relaxation modes, this means

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that we will have to work with complex frequencies ω, where $\text{Re}(\omega)$ indicates the time distance between subsequent pulses and $\text{Im}(\omega)$ the exponential decay of the envelope.

The relation with gravitational radiation is the following. According to general relativity, a perturbed black hole emits gravitational radiation in three phases [2, 1]:

1. The initial pulse depends heavily on the particular form of the perturbation.

2. In the intermediate phase, a damped oscillatory phase sets in, in which a discrete set of complex frequencies ω is excited. Although the actual selection of which frequencies are excited –usually, a single frequency is clearly dominant– depends on the concrete situation or perturbation, the spectrum of possible frequencies depends only on the properties of the black hole itself.

3. Finally, a polynomial tail characterises the return to equilibrium.

Obviously, the discrete set of frequencies of the intermediate phase corresponds precisely to the QNM spectrum that we are interested in here.

2.1 QNMs in GR black holes

The way in which the QNMs of a black hole are calculated in GR is a complex numerical problem, see e.g.[2, 1]. But the essential idea is quite straightforward. The calculation of quasinormal modes u is an eigenvalue problem in which one must solve a wave equation of the type

$$\frac{\partial^2}{\partial t^2}u+\left(-\frac{\partial^2}{\partial x^2}+V_{\text{eff}}(x)\right)u=0. \tag{2.1}$$

For a Schwarschild black hole, the x coordinate should be understood as the radial tortoise coordinate r_*, and the effective potential V_{eff} would be the Regge–Wheeler or the Zerilli potential, depending on the type (axial or polar, respectively) of perturbations one is looking at. For the problem to be well-defined, boundary conditions must be imposed. Since QNMs are decay modes, i.e., energy dissipation modes, they must be "outgoing". Outgoing in this context means:

- at asymptotic infinity: directed towards the exterior of the system;
- at the horizon: directed towards the singularity.

Note that this second condition seems obvious because of the fact that horizons in GR are strict one-way membranes. As we will see, when taking modifications of the dispersion relation into account, this condition will have to be modified. In any case, both conditions can be summarised by saying that in GR, "outgoing" means: directed towards the exterior of the spacetime region connected to an asymptotic observer.

Let us illustrate this for the concrete case of the Schwarzschild black hole. If we write the general form of a QNM as $u_\omega = e^{-i\omega t}\sum_j A_j e^{ik_j r_*}$, where the sum is over all allowed wave numbers k_j for the frequency ω, then the outgoing condition for each particular mode $u_\omega(k)$ can be imposed by requiring that this mode must have an associated group velocity $v_g(k)>0$ when $r_*\to+\infty$, and $v_g(k)<0$ when $r_*\to-\infty$.

We will not go into further details on QNMs in GR, but just mention the two general results that will be important for the rest of our discussion.

1. In 1+1 dimensions, there are no QNMs. A possible way of understanding this is the following [7]. The wave equation (2.1) is conformally invariant in 1+1 dimensions. Moreover, all two-dimensional metrics are conformally equivalent, and in particular equivalent to a flat metric. Hence all solutions of eq. (2.1) will be conformally equivalent to plane waves. Since these do obviously not satisfy the requirement of being outgoing at both ends of the spacetime region connected to an asymptotic observer, there are no QNMs in 1+1 dimensions.

2. In higher dimensions, and in particular in 3+1, the QNM spectrum is discrete [2, 1] (actually, there are an infinite number of isolated QNMs). As mentioned earlier, the QNM spectrum depends only on the properties of the black hole, and so for a Schwarzschild black hole, e.g., the actual numerical values of the associated frequencies depend only on the mass of the black hole.

2.2 QNMs in acoustic black holes in the hydrodynamic limit

The results just mentioned for general relativistic black holes are also valid for acoustic black holes in the hydrodynamic limit:

- There are no QNMs in 1+1 dimensions. This can be understood in the same sense of conformality as mentioned in the GR case, or it can also explicitly be shown in terms of a connection matrix, see [5])

- In higher dimensions, again, a discrete spectrum is obtained [6]. As a matter of fact, the spectrum of a (3+1)-dimensional acoustic black hole in the spherically symmetric flow of an incompressible fluid turns out to be very similar to the Schwarzschild QNM spectrum in GR [6].

3. BEC black holes

3.1 Introduction and strategy

The basic idea of acoustic black holes [8, 9] is that acoustic perturbations in a moving perfect fluid (e.g., a fluid of condensed matter), obey d'Alembertian equations of movement that are formally identical to the ones that describe a massless scalar field in a curved spacetime:

$$\Box_g \psi \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\psi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi) = 0, \tag{3.1}$$

where the metric

$$g_{\mu\nu} \propto \begin{pmatrix} -(c^2-\vec{v}^2) & -\vec{v}^{\mathrm{T}} \\ -\vec{v} & \mathbf{1} \end{pmatrix}. \tag{3.2}$$

In other words, the acoustic perturbations see a geometry created by the background, i.e., by the collective behaviour of the condensed matter system, and the trajectories followed by these acoustic perturbations are precisely the null geodesics of the effective metric $g_{\mu\nu}$.

The analogy can be taken further by observing that condensed matter systems are quantum structures, and asking oneself: What if the (unknown) quantum structure underlying general relativity could also be described in terms of condensed matter?

Part of the answer is of course that general relativity might then be just a low-energy emergent approximation, i.e., an effective description of a collective phenomenon (see [10] and other contributions to the present proceedings). For our present purposes, let it suffice to observe that in condensed matter systems, the geometrical picture is valid only in a certain regime of approximation, the hydrodynamic approximation. But we also have a description of these condensed matter systems which is more complete than this hydrodynamic approximation and which automatically incorporates "quantum corrections". The difference between the hydrodynamic approximation and the more complete description is encoded in the difference between the corresponding dispersion relations, which will form the basic tool for the rest of our discussion.

So, to wrap up, our strategy will be the following [3, 5]:

- First of all, assume that the quantum structure underlying GR can be described as a condensed matter system.
- Take a simple model, namely a configuration with a black hole horizon in a one-dimensional flow of a Bose–Einstein condensate [11, 12, 13].
- Consider the difference between the hydrodynamic or relativistic dispersion relation, and the full Bogoliubov dispersion relation.
- The metric description emerges in the hydrodynamic approximation.
- Study deviations from this metric description through the use of the full dispersion relation.

Before moving on to a discussion of the model and to the actual application of this strategy to the QNM spectrum, let us make a small observation. QNMs are decaying dynamical modes. Looking for QNMs only makes sense in systems which are devoid of unstable (exponentially increasing) dynamical modes. The model which we will discuss has indeed been shown to be stable in this sense, i.e., it is devoid of dynamical instabilities [3].

3.2 General description

A dilute gas of weakly interacting bosons can be described, in second quantization, in terms of quantum operators $\widehat{\Psi}$. The condensation of such a gas into a Bose–Einstein condensate [14] allows a clean separation between a macroscopic wave function ψ for the condensed state and an operator $\widehat{\varphi}$ for the quantum excitations:

$$\widehat{\Psi} = \psi + \widehat{\varphi}.$$

The evolution of the macroscopic wave function ψ is well approximated by the Gross–Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}}(\mathbf{x}) + g\,|\psi(t,\mathbf{x})|^2\right)\psi(t,\mathbf{x}).$$

Use a hydrodynamic or Madelung representation

$$\psi = \sqrt{n}e^{i\theta/\hbar}e^{-i\mu t/\hbar}$$

for the wave function ψ, where n is the total condensate density and θ its phase. Then, linearise the Gross–Pitaevskii equation and consider fluctuations of the macroscopic wave function ψ due to perturbations of the background. In terms of the condensate density n and phase θ, this means that

$$n(\mathbf{x},t) = n_0(\mathbf{x}) + g^{-1}\tilde{n}_1(\mathbf{x},t), \tag{3.3}$$
$$\theta(\mathbf{x},t) = \theta_0(\mathbf{x}) + \theta_1(\mathbf{x},t), \tag{3.4}$$

where n_0 and θ_0 indicate the background values, and n_1 and θ_1 the perturbations (the coupling constant g has been inserted for dimensional simplification of the further equations). It can easily be shown that the time evolution of these perturbations can be written in terms of the hydrodynamic quantities c, the speed of sound, and v, the velocity of the fluid flow. In particular, this time evolution is described by the following differential equations:

$$\partial_t \tilde{n}_1 = -\nabla\cdot\left(\tilde{n}_1\mathbf{v} + c^2\nabla\theta_1\right), \tag{3.5a}$$
$$\partial_t \theta_1 = -\mathbf{v}\cdot\nabla\theta_1 - \tilde{n}_1 + \frac{1}{4}\xi^2\nabla\cdot\left[c^2\nabla\left(\frac{\tilde{n}_1}{c^2}\right)\right]. \tag{3.5b}$$

Note that the speed of sound and the flow velocity in a BEC are local values determined by the condensate density and the phase gradient, respectively:

$$c \equiv \sqrt{gn_0/m}; \qquad \mathbf{v} \equiv \nabla\theta_0/m.$$

Furthermore, in the differential equations (3.5), a term $\frac{1}{4}\xi^2\nabla\cdot\left[c^2\nabla\left(\tilde{n}_1/c^2\right)\right]$ appears, which is sometimes called the "quantum potential". This quantum potential contains the healing length ξ, a characteristic length scale of the condensate, which is proportional to $\hbar$:

$$\xi \equiv \frac{\hbar}{mc}.$$

Since this term appears squared in eq. (3.5b), one might assume that in most circumstances it can be neglected. The hydrodynamic approximation consists precisely of neglecting this quantum potential term. This leads to a geometric picture because the differential equations (3.5) then reduce to a continuity and an Euler equation, which can be manipulated to obtain the effective metric (3.2).

3.2.1 Hydrodynamic versus Bogoliubov dispersion relation

In this hydrodynamic limit, a relativistic dispersion relation is obtained:

$$(\omega - vk)^2 = c^2k^2.$$

When the quantum potential is taken into account, on the other hand, the full Bogoliubov dispersion relation is found:

$$(\omega - vk)^2 = c^2k^2 + \frac{1}{4}c^2\xi^2k^4,$$

which, from a relativistic point of view, is therefore a "modified" dispersion relation. Since this full dispersion relation is of the 4$^{\text{th}}$ order in k, for each mode

$$u_\omega = e^{-i\omega t}\sum_j A_j e^{ik_j x},$$

there will be four contributions ($k_j,\ j = 1\ldots4$):

- two hydrodynamic modes, given (in the hydrodynamic approximation — see conditions below) by $k_{1,2} \simeq \omega/(v \pm c)$,
- two additional, "non-hydrodynamic" modes.

When are these additional, non-hydrodynamic modes important? Starting from the full dispersion relation, one can obtain the hydrodynamic approximation by writing ω as a series expansion in terms of ξk, and truncating the higher order terms, i.e.:

$$\begin{aligned}\omega &= \left(v \pm c\sqrt{1+\frac{1}{4}\xi^2k^2} \right) k \\ &\simeq (v \pm c)k + \frac{1}{8}c\xi^2k^3 + \mathscr{O}(\xi^3k^3) \simeq (v \pm c)k. \end{aligned} \tag{3.6}$$

For the series expansion to make sense, one must have $\xi k \ll 1$, i.e., the frequencies must be small (compared to the characteristic scale of the system). The truncation moreover requires

$$\frac{c\xi^2k^2}{8(v \pm c)} \ll 1.$$

This second condition is never satisfied around a horizon ($c^2 = v^2$). Therefore, near a horizon, the non-hydrodynamic modes are always important, even at low frequencies.

This means basically that, whenever probing the configuration either with high frequencies, or near the horizon (with any frequency), the geometrical picture will diffuminate and the underlying quantum structure will appear to some degree.

3.3 The model

The model [3, 5] consists of a BEC with two homogeneous regions, separated by a transition region of length L around the horizon at $x = 0$, see fig. 1.

In the right-hand side of the condensate, the flow has a normal subsonic regime ($c > |v|$), and hence the hydrodynamic modes, which propagate at the speed of sound c, can move in any direction, including upstream. At the left-hand side, the flow has become supersonic ($c < |v|$), so the hydrodynamic modes are dragged along downstream by the background fluid. They therefore inevitably move towards the left-hand side asymptotic region. One could thus interpret $x \to -\infty$ as the singularity of the black hole.

Note that, from the metric (3.2), $g_{\rm tt}$ changes sign at $x = 0$, i.e., it is negative or timelike for $x > 0$ and positive or spacelike for $x < 0$. So $x = 0$ really represents a horizon.

3.4 Boundary conditions

We mentioned in section 2 that the boundary condition in the calculation of QNMs in GR is that the QNMs must be outgoing at asymptotic infinity and at the horizon. However, in BEC black holes with modified dispersion relations, the non-hydrodynamic modes can become "superluminal", i.e., they can have a group velocity

$$v_g \equiv \mathrm{Re}\left(\frac{d\omega}{dk}\right) > c.$$

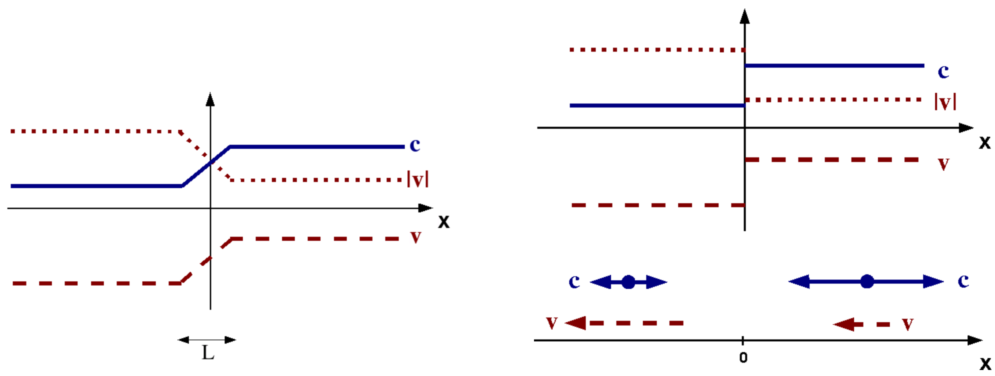

Figure 1: Flow and sound velocity profile simulating a black hole-like configuration in a BEC. The profile consists of two homogeneous regions with a transition region of width L around $x = 0$, see left-hand side figure. The solid blue line represents the speed of sound c, the dashed red line the fluid velocity v. The negative value of v indicates that the fluid is left-moving. For $x > 0$, the fluid is subsonic since $c > |v|$. At $x < 0$ it has become supersonic. At $x = 0$, there is a sonic horizon. The right-hand side figure shows the idealised profile with a step-like discontinuity at $x = 0$ (i.e., in the limit for $L \to 0$), used for actual calculations. The lower part of the right-hand side picture shows flow charts for the hydrodynamic modes. In particular, in the supersonic region ($x < 0$), it is clearly seen that, since $c < |v|$, both hydrodynamic modes will be dragged along by the background fluid towards $x \to -\infty$, i.e., towards the "singularity".

In the presence of such superluminal modes, the horizon becomes permeable. Since this permeability of the horizon is an essential consequence of modified dispersion relations, imposing an outgoing boundary condition at the horizon would be contradictory. Moreover, with such superluminal modifications, there is no causal disconnection anymore between the part of spacetime exterior to the horizon, and the part interior to the horizon. Therefore, the outgoing boundary condition must be imposed, on the one hand, in the right asymptotic region (as before), but on the other hand in the left asymptotic region instead of at the horizon.

So the outgoing boundary condition now simply becomes that a mode must have an associated group velocity $v_g(k) > 0$ when $x \to +\infty$, and $v_g(k) < 0$ when $x \to -\infty$ (i.e., the same as in section 2 but expressed in terms of the coordinate x instead of r_*).

3.5 Calculation of QNMs with the full dispersion relation

In each homogeneous region, the modes can be written in terms of plane waves. To find a global solution, the solutions for each side can be connected at the $x = 0$ discontinuity through matching conditions. Four such matching conditions are obtained from integration of the eqs. (3.5), see [3], and they are of the type

$$[\theta_1] = \theta_1|_{x=0^+} - \theta_1|_{x=0^-} = 0.$$

To these four matching conditions, a certain number of constraints shoud be added due to the boundary conditions. For each mode $u_\omega(k)$, by calculating the group velocity $v_g = \mathrm{Re}\left(\frac{d\omega}{dk}\right)$, it is easy to check whether it satisfies the boundary condition in the corresponding region. If not, it should be prohibited, and hence this would mean one (additional) constraint. So the boundary condition add a number N of constraints to the matching conditions — in this case simply equal to the number of prohibited modes — and the total number of constraints thus becomes $4+N$.

On the other hand, we have an algebraic system with 8 degrees of freedom. Indeed, the density perturbations can be written as

$$\widetilde{n}_1(x,t) = \sum_{j=1}^{4} A_j e^{i(k_j x - \omega t)}$$

at each side of the discontinuity, leading to a total of 8 free parameters A_j. Note that the same is valid for the phase perturbations:

$$\theta_1(x,t) = \sum_{j=1}^{4} B_j e^{i(k_j x - \omega t)},$$

however is is easy to show that $B_j = f(A_j)$ and so this does not add any free parameters.

We thus have an algebraic system with 8 degrees of freedom and $4+N$ constraints, leading to the following straightforward numerical algorithm.

Take a dense grid in the appropiate complex half-plane of frequencies ω connected to the origin. In particular, in our notation, QNMs correspond to modes with $\mathrm{Im}(\omega) < 0$, so we'll be looking at the lower half-plane. For each value of ω on the grid, calculate the value of a function $F(\omega)$, such that the roots of $F(\omega)$ correspond to QNMs. $F(\omega)$ can be defined as follows, depending on the corresponding value of $N(\omega)$:

- If $N = 4$, there will be a solution of the algebraic system only if the determinant of the associated 8×8 matrix Λ vanishes, i.e., $F(\omega) \equiv \det(\Lambda)$.
- If $N > 4$, one could still find solutions if the 8×8 subdeterminants of the non-square matrix Λ vanish. So one can define $F(\omega) \equiv \sum_i |\det\lambda_i|$, where λ_i are the 8×8 submatrices of Λ.
- Finally, the important case is the following.

  When $N < 4$, the algebraic system is underdetermined, and hence $F(\omega) \equiv 0$, i.e., every frequency ω for which $N(\omega) < 4$ automatically corresponds to a QNM.

4. Results and discussion

As can be readily seen from fig. 2, there is a continuous region in the complex frequency-plane where $N(\omega) = 3$. According to the algorithm just described, every frequency in this region forms part of the QNM spectrum. The appearance of such a region is independent of the concrete parameters (c, v, ξ) used in the calculation, although obviously the precise shape and location of the region might shift. So, in contrast to the absence of QNMs in 1+1 dimensions both in general relativistic black holes and acoustic black holes in the hydrodynamic limit, in BEC black holes in 1+1 dimensions with the full Bogoliubov dispersion relation, the QNM spectrum consists of a continuous region in the complex frequency-plane.

It seems that the appearance of such a continuous region does not depend much on the concrete form of the modification of the dispersion relation. These modifications in their turn are related in a quite general way to Lorentz invariance violations in high-energy physics [4]. The

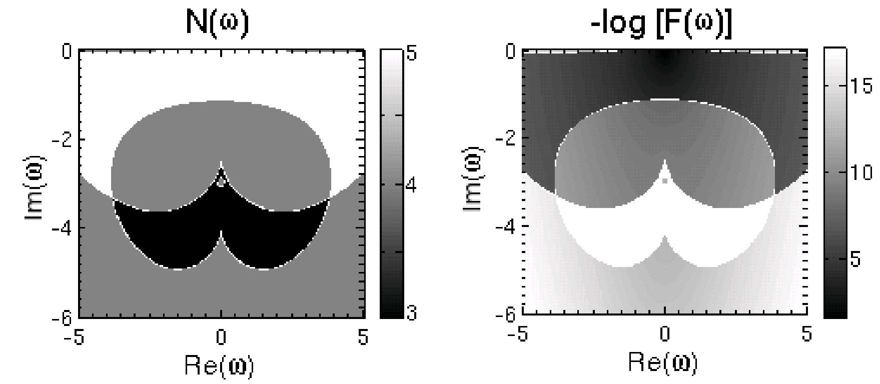

Figure 2: Plots illustrating the quasinormal mode spectrum for a one-dimensional black hole configuration in BEC. The left-hand part of the picture represents the number N of constraints following from the boundary condition that QNMs should be outgoing. The right-hand part shows the function $\log[F(\omega)]$, where $F(\omega) = 0$ corresponds to quasinormal modes. The QNM spectrum consists of the continuous region where $N < 4$. [The numerical values used for these plots, in units such that the healing length $\xi = 1$, are $c = 1; v = 0.7$ in the subsonic region and $v = 1.8$ in the supersonic region.]

essential condition for the existence of a continuous region in the QNM spectrum is the appearance of superluminal modes, which is rather usual in condensed matter systems. Therefore, the straightforward speculation mentioned in the abstract is simply that continuous regions might also appear in the QNM spectrum of gravitational black holes in scenarios for gravity with high-energy modifications of the dispersion relation.

Acknowledgements

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