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Have we tested Lorentz invariance enough?

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Motivated by ideas from quantum gravity, Lorentz invariance has undergone many stringent tests over the past decade and passed every one. Since there is no conclusive reason from quantum gravity that the symmetry *must* be violated at some point we should ask the questions: a) are the existing tests sufficient that the symmetry is already likely exact at the Planck scale? b) Are further tests simply blind searches for new physics without reasonable expectation of a positive signal? Here we argue that the existing tests are not quite sufficient and describe some theoretically interesting areas of existing parameterizations for Lorentz violation in the infrared that are not yet ruled out but are accessible (or almost accessible) by current experiments. We illustrate this point using a vector field model for Lorentz violation containing operators up to mass dimension six and analyzing how terrestrial experiments, neutrino observatories, and Auger results on ultra-high energy cosmic rays limit this model.

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1. Introduction

Over the last century one of the most important problems in theoretical physics has been quantum gravity and, despite efforts by many luminary physicists, no complete model yet exists. A primary reason why quantum gravity has remained elusive is the lack of observational input. Relevant observations are difficult to obtain because of the enormous difference between terrestrial energies and the natural quantum gravity scale - the Planck energy. The Planck energy of $E_{Pl} = 1.22 \cdot 10^{19}$ GeV is 16 orders of magnitude larger than current accelerator energies, precluding direct probes of Planck scale physics. However, recently it has been recognized that some knowledge about quantum gravity can be indirectly gleaned from low energy processes. Various models/ideas about quantum gravity suggest that the fundamental symmetry of special relativity, Lorentz invariance, may not be an exact symmetry but instead violated at the Planck scale. At low energies physics would then presumably show tiny deviations from Lorentz invariance as well. The exact size of these deviations is dependent on the underlying quantum gravity model. Apart from any quantum gravity motivations, tests of Lorentz invariance have historically been important because of the fundamental role Lorentz invariance plays in quantum field theory and general relativity.

Incredibly precise and sensitive tests of Lorentz symmetry have been performed by numerous researchers over the past two decades. An issue arises when considering the relevance of these tests for models of quantum gravity, however. While quantum gravity models might suggest that Lorentz symmetry is not exact, there is no firm calculated prediction from any model for the *size* of the violation. The predominant approach up to this point has been to simply search for/constrain Lorentz violation in some low energy framework that we hope is the right infrared limit for quantum gravity. The difficulty is that all of the infrared frameworks mathematically allow for infinitesimally small Lorentz violation and hence we can never rule out any framework a priori. We therefore have the unpleasant combination of incredibly precise tests of frameworks that can never be falsified, which begs the question: are there any reasonable spots where we might still see a signal of a violation of Lorentz invariance, or at this point are experimental searches simply expanding the range of validity where we know Lorentz invariant physics works without any real expectation that we might see a symmetry violation?[1]

The answer is that there are still reasonable spots. While the infrared frameworks mathematically allow for infinitesimal amounts of Lorentz violation, there are regions of each framework that are preferred on physical grounds. Even better, experimentally we are able to investigate some of these regions of theoretical interest. Our goal in this paper is to discuss these specific experimental possibilities.

2. Lorentz violation in field theory

2.1 Lorentz violation by itself

The most common systematic approach to studying Lorentz symmetry violation (LV) is to

[1] It is not the intent of this work to argue that tests of Lorentz symmetry are ever unimportant. Even if the regions of theoretical interest discussed here (and perhaps others) are eventually excluded, tests of Lorentz symmetry are still interesting as we should always strive to extend the limits of our physical theories. However, it is much more important to explore areas where there are credible theoretical ideas to be tested.

construct a Lagrangian that contains the Lorentz violating operators of interest. The complete set of renormalizable operators that can be added to the standard model, the standard model extension (SME) [1], contains dozens of various operators made up of standard model fields and derivative operators coupled to tensor fields with non-zero vacuum expectation values. It is the presence of these non-zero tensors that beaks Lorentz invariance.[2]

Rather than deal with the entire SME, we can work with a simpler model that yields the same essential physics: matter and gauge fields couple not only to the metric, but also to a preferred frame (c.f. [2, 3, 4]). We can specify this frame by a unit timelike vector field u^α, the integral curves of which define the world lines of observers at rest in the frame. u^α must be a field with its own kinetic terms that induce couplings between the metric and u^α [3]. Here we will only concern ourselves with the possible couplings between u^α and matter fields, specifically fermions and photons. The stable, non-trivial, and renormalizable operators that couple u^α to fermions and photons for this construction are

$$\mathscr{L}_f = \overline{\psi}(i\not{D} - m)\psi - E_{Pl} b u_\mu \overline{\psi}\gamma_5\gamma^\mu\psi + \frac{1}{2} i c u_\mu u_\nu \overline{\psi}\gamma^\mu \overleftrightarrow{D^\nu}\psi + \frac{1}{2} i d u_\mu u_\nu \overline{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D^\nu}\psi \tag{2.1}$$

and

$$\mathscr{L}_\gamma = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(k_F)u_\kappa \eta_{\lambda\mu} u_\nu F^{\kappa\lambda}F^{\mu\nu} \tag{2.2}$$

respectively [1]. b, c, d and k_F are the coefficients that determine the size of any LV and each fermion species can in principle have different coefficients. We have "de-dimensionalized" the dimension three b operator by the Planck energy so that all coefficients are dimensionless. We neglect LV terms for other gauge bosons as they will not be relevant for the observations we will discuss, essentially because they a) have high masses and b) do not propagate freely over long distances.

A natural question is, why do we choose the dimensionful coefficient to be the Planck energy and not, say, the particle mass? On one hand, this is just a matter of convention. However, since we are looking for LV sourced by quantum gravity, we would expect the quantum gravity scale to be the energy scale that controls the size of various operators. We will see later that this assumption is natural but leads us into serious difficulties for an experimentally viable model for LV.

There are other terms, of course, but they are of higher mass dimension. If we stick with our prescription of determining the size of the operator by E_{Pl} all the higher dimensional operators are small and irrelevant. We still list them, however, as part of our task is to describe how these operators can become relevant again. The complete dimension five operators have been catalogued in [5], while the dimension six operators are not yet completely known. Of interest to us are only those operators that modify free particle behavior, i.e. kinetic terms. Interaction terms of higher dimension are suppressed by the Planck mass and lead only to very small modifications to particle reaction rates. [3] The known fermion operators are

[2] We concentrate here on coupling of Lorentz violating tensors to matter fields. The gravitational sector of Lorentz violating theories is much less constrained and can generate interesting and useful phenomenology. For a discussion see the talks by Robert Bluhm and Ted Jacobson in this volume.

[3] It might still be possible to see these terms if they introduced new particle decays that do not exist in the standard model. However, the number of events would be extremely small and we know of no current experiment that looks for LV in this manner.

$$\frac{1}{E_{Pl}}\overline{\psi}(\eta_L P_L+\eta_R P_R)\not{u}(u\cdot D)^2\psi+\overline{\psi}\Big[-\frac{1}{E_{Pl}}(u\cdot D)^2(\alpha_L^{(5)}P_L+\alpha_R^{(5)}P_R) \tag{2.3}$$
$$-\frac{i}{E_{Pl}^2}(u\cdot D)^3(u\cdot\gamma)(\alpha_L^{(6)}P_L+\alpha_R^{(6)}P_R)-\frac{i}{E_{Pl}^2}(u\cdot D)\Box(u\cdot\gamma)(\tilde{\alpha}_L^{(6)}P_L+\tilde{\alpha}_R^{(6)}P_R)\Big]\psi$$

where P_R and P_L are the usual right and left projection operators, $P_{R,L}=(1\pm\gamma^5)/2$, and D is again the gauge covariant derivative. Again, we have used the Planck energy to make all remaining coefficients dimensionless. The currently known photon operators are

$$\mathscr{L}_b=\frac{\xi}{E_{Pl}}u^\mu F_{\mu\nu}(u\cdot\partial)u_\alpha\tilde{F}^{\alpha\nu}-\frac{1}{2E_{Pl}^2}\beta_\gamma^{(6)}F^{\mu\nu}u_\mu u^\sigma(u\cdot\partial)^2F_{\sigma\nu}.$$

It is useful to classify the set of fourteen operators above by both mass dimension and behavior under CPT. The classification of operators is shown below in Table 2.1. An $\times$ means that no operator exists with the specified properties while a ? implies that the operators are unknown.

Table 1: Stable, nontrivial kinetic fermion and photon LV operators

Dim	CPT Odd	CPT Even
Fermions		
3	$-E_{Pl}bu_\mu\overline{\psi}\gamma_5\gamma^\mu\psi$	$\times$
4	$\times$	$\frac{1}{2}icu_\mu u_\nu\overline{\psi}\gamma^\mu\overleftrightarrow{D^\nu}\psi$ $\frac{1}{2}idu_\mu u_\nu\overline{\psi}\gamma_5\gamma^\mu\overleftrightarrow{D^\nu}\psi$
5	$\frac{1}{E_{Pl}}\overline{\psi}(\eta_L P_L+\eta_R P_R)\not{u}(u\cdot D)^2\psi$	$-\frac{1}{E_{Pl}}\overline{\psi}(u\cdot D)^2(\alpha_L^{(5)}P_L+\alpha_R^{(5)}P_R)\psi$
6	?	$-\frac{i}{E_{Pl}^2}\overline{\psi}(u\cdot D)^3(u\cdot\gamma)(\alpha_L^{(6)}P_L+\alpha_R^{(6)}P_R)\psi$ $-\frac{i}{E_{Pl}^2}\overline{\psi}(u\cdot D)\Box(u\cdot\gamma)(\tilde{\alpha}_L^{(6)}P_L+\tilde{\alpha}_R^{(6)}P_R)\psi$
Photon		
3	$\times$	$\times$
4	$\times$	$-\frac{1}{4}(k_F)u_\kappa\eta_{\lambda\mu}u_\nu F^{\kappa\lambda}F^{\mu\nu}$
5	$\frac{\xi}{E_{Pl}}u^\mu F_{\mu\nu}(u\cdot\partial)u_\alpha\tilde{F}^{\alpha\nu}$	$\times$
6	?	$-\frac{1}{2E_{Pl}^2}\beta_\gamma^{(6)}F^{\mu\nu}u_\mu u^\sigma(u\cdot\partial)^2F_{\sigma\nu}$

For the rest of this discussion we will ignore any unknown CPT odd dimension six operators and concentrate on the phenomenology and constraints on the known operators.

2.2 Constraints and the fine tuning problem

The constraints on the operators above come from a number of sources, from tabletop laboratory experiments to high energy astrophysics. Below we list the best constraints on each operator, noting for fermions which fermion species it applies to.

The bounds on the renormalizable and CPT odd dimension five operators are all very tight. To assess what these bounds mean for theory, we first need to know what the expected size of any Lorentz violation coming from quantum gravity might be. As mentioned before, there is no firm

Table 2: Direct bounds on LV operators.

Dim	CPT Odd	CPT Even
Fermions		
3	Neutron:$\|b\| < 10^{-46}$ [6]	$\times$
4	$\times$	Neutron: $\|c,d\| < 10^{-27}$ [6]
5	Electron: $\|\eta_{R,L}\| < 10^{-5}$ [7]	Proton: $O(10^{-1})$ [8]
6	$\times$	Proton (+extra assumptions): $O(10^{-2}, 10^{-4})$ [8, 10]
Photon		
3	$\times$	$\times$
4	$\times$	$\|k_F\| < 10^{-15}$ [11]
5	$\|\xi\| < 10^{-7}$ [12]	$\times$
6	None	None

prediction from any theory of quantum gravity that Lorentz invariance must be violated and hence no estimate from the fundamental theory of the size of the violation. We can, however, argue from a bottom up perspective, using the rules of effective field theory, what the expected size should be.

The essential point is the following: If Lorentz invariance is violated by quantum gravity, then why should our low energy world exhibit the symmetry to *any* degree, much less the experimental situation where Lorentz invariance is at the very least an excellent approximate symmetry? In the physical theories we know about, the dimensionless numbers that appear are usually of order one. Since we have explicitly factored out the Planck scale in our Lagrangian, which controls the scale at which quantum gravity comes into play, the LV terms (including the renormalizable ones!) are also therefore most naturally expected to have coefficients of order one.

They don't of course, and therefore there must be some reason why they are small or zero. In light of this fact many studies turned to the non-renormalizable operators, as these are already suppressed by the Planck energy and hence naturally small. Concentrating solely on these operators isn't correct though, as they will, via loop corrections integrated up to the cutoff of E_{Pl}, generate the dangerous renormalizable operators with large coefficients. The generated coefficients are O(1) because while the non-renormalizable operators are suppressed by E_{Pl}, they are involved in divergent loop corrections. These divergences are regulated by the cutoff of our EFT, which is also E_{Pl}, and the factors of E_{Pl} cancel. This is generically known to happen in non-commutative field theories [13] or in field theories with a LV regulator [14]. In light of these arguments all the CPT odd coefficients are bounded at the level of 10^{-46} and all the CPT even coefficients are bounded at the level of 10^{-27}. Hence further improvements on existing constraints of LV are likely irrelevant unless we can get around this extreme fine tuning problem.

2.3 Supersymmetry and CPT to the rescue?

The arguments in the previous section don't rely on any particular quantum gravity model, so they are generic as long as the quantum gravity corrections contain LV and are describable by EFT. There is a hole in the argument, however - it assumes there is no new physics between experimentally accessible energies and the Planck scale as we integrate loop integrals with known physics. This is a large and dangerous assumption as over the history of physics we have encoun-

tered new physics every few orders of magnitude in energy. How would new physics affect the above argument? Assume for the moment, that there exists a combination of symmetries other than Lorentz invariance that is incompatible with all the LV renormalizable operators and CPT odd nonrenormalizable ones. Loop corrections involving the higher dimension operators, instead of generating dangerous terms, would instead cancel or be zero. In such a case, our field theory would be more experimentally feasible, as we would have only the CPT even higher dimensional operators to consider which are much less tightly constrained.

If we found such a symmetry, where is it? We don't see such an extra symmetry at low energies, which means it should be broken at some scale Λ_b above 1 TeV. Below Λ_b this symmetry is nonexistent, so the same EFT terms as before will exist. However, now if we have a non-renormalizable term suppressed by E_{Pl} with an O(1) coefficient it does *not* generate large renormalizable terms. The loop integrals will only contribute up to the new symmetry breaking scale Λ_b, which leads to dimension three renormalizable terms of size Λ_b^2/E_{Pl} and dimension four terms of size Λ_b^2/E_{Pl}^2. In terms of our original parameterization in Table 2.1, the b,c,d coefficients are now naturally of size Λ_b^2/E_{Pl}^2! The price to pay is the introduction of an entirely new symmetry and a symmetry breaking mechanism.

Roughly, Λ_b can be as low as 1 TeV and still not be seen in direct accelerator tests on the standard model. This would get us to a size for b,c,d of 10^{-32}. Current limits on b are well beyond this, however, we can set b identically zero if we assume CPT. Lorentz invariance is usually assumed in proofs of the CPT theorem [15] and so if we wish to create a viable LV model, CPT must be an assumption instead. This assumption is, of course, experimentally compatible with known physics.

While the above construction logically works, is there any symmetry that can actually do the magic above? Wonderfully enough, supersymmetry, which has been considered for a number of other reasons, has (almost) exactly the necessary behavior [17]. The underlying reason is that SUSY can be thought of as a field transformation symmetry, which means that different fields can't, for example, propagate at different limiting speeds. The same effect occurs in two-component condensed matter analog models for spacetime, where the limit in which the low energy quasiparticles have the same speed is the same as the limit where there exists a field transformation symmetry between the quasiparticles [16]. Supersymmetry forbids renormalizable LV operators while allowing dimension five and six operators [17]. Note, however, that in order to be compatible with current limits, naively the SUSY breaking scale must be (roughly) below 1 PeV. This leads to an interesting method to test for the presence of SUSY in a LV theory and a nice interplay between low energy LV searches and high energy collider experiments. If low energy searches for LV see a signal, that implies not only that Lorentz invariance is violated, but also that there exists another symmetry, which we assume is SUSY for the sake of argument, that must exist at much lower energies. Furthermore, if we assume that the energy scale of LV is E_{Pl} the size of c,d give a prediction of the SUSY breaking scale. Finally, note that if we can improve the bounds on c,d by six orders of magnitude, to 10^{-33} or so, there will be no room for the construction above to work. Admittedly this may be a tall order, but if experiments reach this sensitivity we can begin to rule out these split scenarios where LV occurs at high energies but a low energy custodial symmetry protects against experimental signatures. Improvement by even three to four orders of magnitude will drop the required SUSY scale to under 10 TeV, putting it close to searches for SUSY at the LHC.

There is a significant downside for astrophysical searches for LV in the supersymmetric case. Supersymmetric LV theories do not significantly modify the free field equations for high energy particles [17], i.e. O(1) $\alpha^{(5)}_{R,L}$ and $\alpha^{(6)}_{R,L}$ terms in Table 2.1 are not present in known supersymmetric Lagrangians. Therefore experiments searching for LV with high energy neutrinos or the Griesen-Zatsupin-Kuzmin (GZK) cutoff for ultra-high energy cosmic rays are not able to probe this scenario.

3. Interesting regions in LV EFT

The first area of interest is that mentioned above, improving the constraints on the dimension four operators by at least a few orders of magnitude. The second hot spot is establishing better direct constraints on the CPT even dimension five and six operators to limit them well below O(1). This is important to do, because as the above discussion shows, new low energy physics can change the hierarchy of LV operators. While SUSY won't allow for the above dimension six operators, there may be some other physics which has a similar effect. Since we are able to simply constrain directly the CPT even dimension five and six operators to be less than O(1), it makes sense to spend the relatively minimal effort to do so even if we don't know what the "new physics" might be. What does not make sense is to continue much further after that as the next order operators are well beyond our experimental reach for the foreseeable future.

3.1 Dimension six operators

We first need the field equations for the CPT even dimension five and six operators. For fermions, the Hamiltonian corresponding to (2.3) commutes with the helicity operator, hence the eigenspinors of the modified Dirac equation will also be helicity eigenspinors. We now solve the free field equations for the positive frequency eigenspinor ψ. Assume the eigenspinor is of the form $\psi_s e^{-ip\cdot x}$ where ψ_s is a constant four spinor and $s = \pm 1$ denotes positive and negative helicity. Then the Dirac equation becomes the matrix equation

$$\begin{pmatrix} -m-\alpha^{(5)}_L \frac{E^2}{E_{Pl}} & E-sp-\alpha^{(6)}_R \frac{E^3}{E^2_{Pl}} \\ E+sp-\alpha^{(6)}_L \frac{E^3}{E^2_{Pl}} & -m-\alpha^{(5)}_R \frac{E^2}{E_{Pl}} \end{pmatrix} \psi_s = 0. \tag{3.1}$$

We have dropped the $\tilde{\alpha}^{(6)}_{R,L}$ terms as the $\Box$ operator present in these terms makes the correction to the equations of motion proportional to m^2 and hence tiny. The dispersion relation, given by the determinant of (3.1), is

$$E^2 - \frac{m}{E_{Pl}}(\alpha^{(5)}_L + \alpha^{(5)}_R)E^2 - \alpha^{(5)}_L \alpha^{(5)}_R \frac{E^4}{E^2_{Pl}} - (\alpha^{(6)}_R E^3)(E+sp) \\ -(\alpha^{(6)}_L E^3)(E-sp) = p^2 + m^2 \tag{3.2}$$

where we have dropped terms quadratic in $\alpha^{(6)}_{R,L}$ as they are small relative to the first order corrections for those terms. Terms quadratic in $\alpha^{(5)}_{R,L}$ must be kept, as the particle mass suppresses the linear term.

At $E >> m$ the helicity states are almost chiral, with mixing due to the particle mass and the dimension five operators. Since we will be interested in high energy states, for notational ease

we re-label LV coefficients by helicity, i.e. $\alpha_+^{(d)} = \alpha_R^{(d)}, \alpha_-^{(d)} = \alpha_L^{(d)}$. The resulting high energy dispersion relation for positive and negative helicity particles can easily be seen from (3.2) to involve only the appropriate $\alpha_+^{(d)}$ or $\alpha_-^{(d)}$ terms. For compactness, we denote the helicity based dispersion terms by $\alpha_\pm^{(d)}$. The difference between positive and negative helicity particle dispersion may seem counterintuitive since the original Lagrangian is CPT invariant. However, the helicity dependent terms are odd in E and p and therefore break both parity and time reversal, leaving the combination CPT invariant. Note also that at energies $E >> m$, we can replace E by p at lowest order, which yields the approximate dispersion relation

$$E^2 = p^2 + m^2 + f_\pm^{(4)} p^2 + f_\pm^{(6)} \frac{p^4}{E_{Pl}^2} \tag{3.3}$$

where $f_\pm^{(4)} = \frac{m}{E_{Pl}}(\alpha_-^{(5)} + \alpha_+^{(5)})$ and $f_\pm^{(6)} = 2\alpha_\pm^{(6)} + \alpha_-^{(5)}\alpha_+^{(5)}$. Note that positive coefficients correspond to superluminal propagation, i.e. $\partial E/\partial p > 1$, while negative coefficients give subluminal propagation.

We now turn to photon dispersion. In Lorentz gauge, $\partial^\mu A_\mu = 0$, the free field equation of motion for A_μ in the preferred frame with the dimension six LV operator is

$$(1 - \frac{\beta^{(6)}}{E_{Pl}^2}\partial_0^2)\Box A_0 = 0 \tag{3.4}$$

$$(\Box + \frac{\beta^{(6)}}{E_{Pl}^2}\partial_0^4)A_i = 0 \tag{3.5}$$

where $i = 1,2,3$. With (3.4) and the assumption that LV is small, we can use the residual gauge freedom of the Lorentz gauge to set $A_0 = 0$ as long as A_μ is assumed to not contain any Planckian frequencies. For a plane wave $A_\mu = \varepsilon_\mu e^{-ik\cdot x}$, there are hence the usual two transverse physical polarizations with dispersion

$$\omega^2 = k^2 + \beta^{(6)} \frac{k^4}{E_{Pl}^2}. \tag{3.6}$$

where we have substituted the lowest order dispersion $\omega = k$.

3.2 Constraints

There are a number of direct constraints that can be placed on these dispersion relations. The best constraints to date are those in [8], which place limits around $O(10^{-2})$ on the various fermion and gauge boson parameters from the existence of ultra high energy cosmic ray (UHECR) protons. In short, the existence of UHECR protons implies that 10^{10} GeV protons are long-lived on astrophysical scales. If, however, protons travel faster than the low energy speed of light, they can emit photons via the vacuum Cerenkov effect, the rate for which is exceedingly fast (see the appendix of [9] for a discussion). Similarly, depending on the LV coefficients of various fermion species, protons can be come unstable to electron/positron pair emission, conversion to neutrons via positron/neutrino emission, etc. All of these processes must be forbidden if we see UHECR protons. With some mild assumptions on the behavior of the LV coefficients (for example that all gauge bosons have the same LV coefficients), two sided bounds can be derived via these decay

processes and the parton distribution functions (PDF's) of the UHECR protons and constituent decay products. The PDF's are required to calculate the net LV behavior of the composite particles in the reactions.

With a single source species, i.e. protons, this is about the best we can do, as all we know is that the source species must be stable. The problem is, we already know that there is likely some other symmetry present, so therefore there will other particles (for example superpartners) involved in the PDF's evolved up to 10^{10} GeV. The constraints depend on the PDF's and so will change depending on the symmetry, although perhaps only slightly. Furthermore, the two sided constraints are only available with some assumptions about how the LV coefficients behave. As a complementary approach, we would like to be able to construct strong limits treating each particle as fundamental without needing assumptions about the parton makeup or structure of the LV coefficients.

3.2.1 Ultra high energy cosmic ray limits

We can construct such limits if we have two source species or if we have a more stringent constraint than just "this one particle species is stable". The Pierre Auger Observatory is able to provide us with both possibilities. First, Auger will be able to confirm the location of the Greisen-Zatsepin-Kuzmin (GZK) cutoff [18, 19] and the detailed UHECR spectrum around it. The GZK cutoff is an expected cutoff in the UHECR spectrum at 5×10^{19} eV due to pion production from UHECR proton scattering off the cosmic microwave background, $p + \gamma \to p + \pi^0$. Recent results from Auger have already confirmed the existence of the cutoff [20] near the expected value, which has a number of consequences for the LV coefficients of the CPT even dimension five and six operators.

As an example of the sensitivity of GZK physics to these operators, we can perform a basic threshold analysis with a simplified model, similar to what was done in [11]. We first assume that parity is a good approximate symmetry and that left and right handed protons/pions have the same LV coefficients. Without LV, a UHECR proton can scatter of a CMB photon with energy ω_0 and produce a pion when $E_{th} > m_\pi(2m_p + m_\pi)/4\omega_0$, where the threshold energy E_{th} is the necessary proton energy. LV operators change the effective mass of the proton and pion and so will change E_{th}. In Lorentz invariant physics, a 5×10^{19} eV proton is at threshold with a 1.3 meV CMB photon. If LV is such that a lower energy proton is able to produce pions off the same region of frequency space in the CMB, one would expect to photopion production process to be enhanced and the GZK cutoff be lowered. Similarly, if the necessary proton energy was raised, it would it turn raise the location of the cutoff. Hence we can get an estimate of the size and structure of GZK constraints by asking how the proton threshold energies for photopion production with a 1.3 meV photon vary in LV parameter space. Expressed in terms of the $f_p^{(6)}$, $f_\pi^{(6)}$ parameters in (3.3) this yields Figure 1 for how the cutoff location deviates from the Lorentz invariant value.

We see from Figure 1 that this simple requirement removes most of parameter space. However, we warn the reader that while the sensitivity of the cutoff to LV parameters is evident, the situation is more complicated than in our simple model. The independent coefficients for different chiralities can mask any effect, in that opposite sign coefficients for fermions will have canceling effects. In addition, with different fermion coefficients new effects must be considered during propagation such as proton helicity decay [9] or electron-positron pair production in the photon component [21]. An additional, and more important, problem is disentangling any LV source effects from the reac-

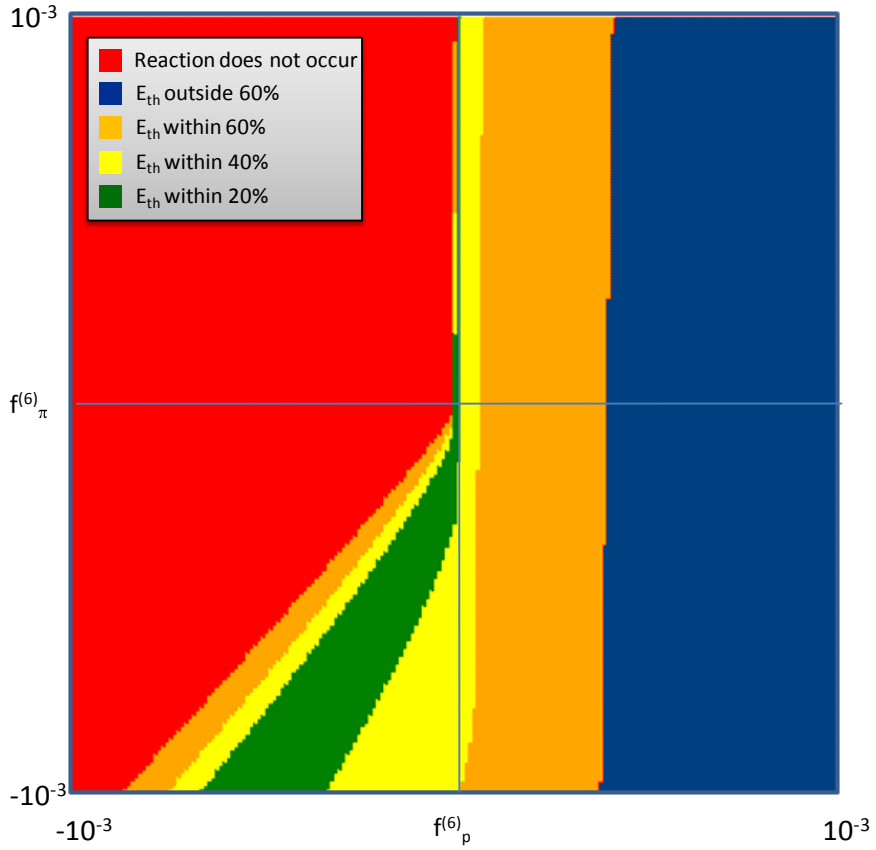

Figure 1: Deviation of GZK cutoff E_{th} from 5×10^{19}eV as a function of Lorentz violating parameters. To the right of the green region E_{th} is below 5×10^{19}eV. To the left E_{th} is either greater or the reaction is forbidden.

tion kinematics. The GZK cutoff is a deviation away from an initial power law source spectrum. Various source models predict different spectral indices and different composition by species [22], many of which match the Auger data. Unfortunately, there is no analysis for any source model on how LV effects modify the spectrum. Since LV contains an energy scale near the GZK energy at which it becomes important, it is conceivable that the source spectrum could be a power law below GZK energies (matching existing cosmic ray data) and change drastically above it. It is therefore difficult to establish concrete constraints just from the existence of the cutoff, independent of any knowledge of LV at the source. Due to these issues, it is therefore unknown at this time what the actual constraints on the parameter space will look like.

We can get around many of these problems if we consider a different way of using Auger data. Auger can discriminate between different cosmic ray primaries and, depending on the source dynamics, Auger may see both proton and photon primaries [23]. This allows us to drastically simplify our LV physics, as we no longer need to consider source dynamics or multiple new reactions. All we need to require is that the photon and one chirality of proton is stable. With LV

physics, either of these particles can become energetically unstable, in that above the threshold energy E_{th} (determined again by the LV coefficients and the mass), a proton can emit photons via vacuum Cerenkov or a photon can decay into a proton/antiproton pair. Stable protons and photons forbids both these reactions. For an idea of the strength of the constraints, let us assume again that each chirality of proton has the same LV coefficient. The threshold energy for these reactions to begin to occur for O(1) $\beta^{(6)}$, $f^{(6)}_{R,L}$ is approximately $E_{th} \approx (E_{Pl}^2 m_p^2)^{1/4}$, which puts $E_{th} \approx 5 \cdot 10^{18}$ eV, well below the GZK cutoff. The timescale for either a GZK proton to lose most of its energy or a GZK photon to decay rapidly approaches E_{Pl}^2/E^3 [9] once the energy is above E_{th} which is approximately 10^{-16} seconds for a 10^{19} eV particle. Hence the LV coefficients must be such that neither reaction is kinematically allowed. The parameter space with different threshold energies is shown in Figure 2. As we can see, positive identification of different species at GZK (or even

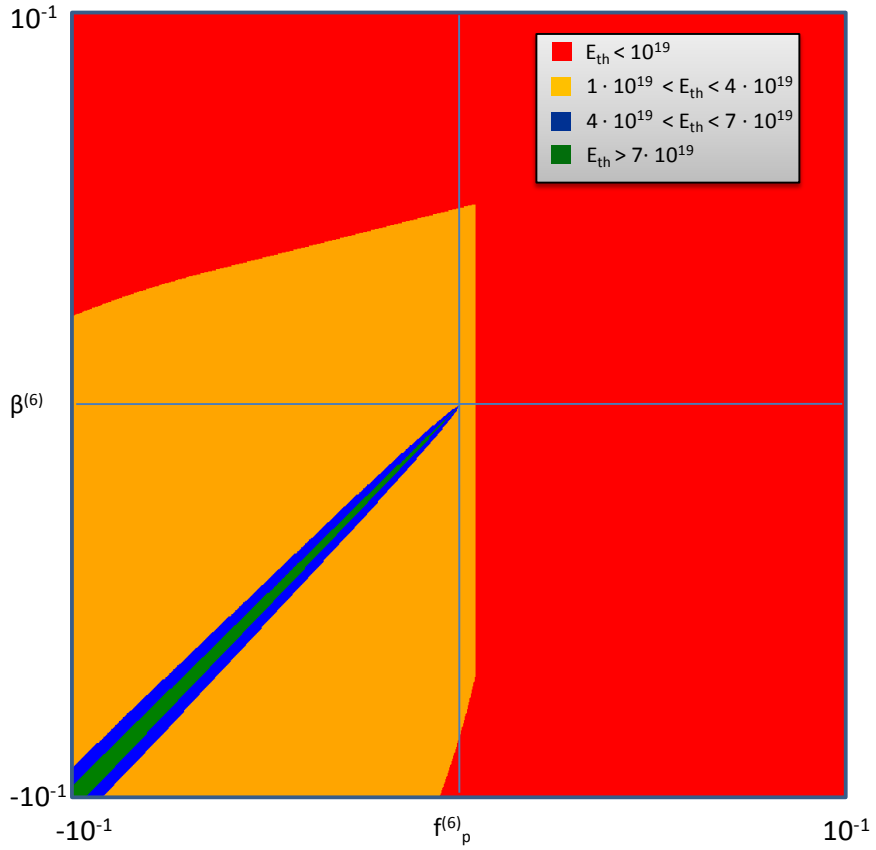

Figure 2: Threshold energies E_{th} for vacuum Cerenkov and photon decay as a function of Lorentz violating parameters. Shaded regions are where E_{th} for at least one of the reactions is in the marked energy range. The green area, which includes the usual Lorentz invariant origin, contains the region where both particles are stable.

lower) energies, can significantly restrict the allowed LV parameter space to be very small.

3.3 Neutrino constraints

Due to their small mass, neutrinos provide another sensitive probe of dimension six operators. Remember from above that the threshold energy for photon decay or vacuum Cerenkov involving protons was given by $E_{th} \approx (E_{Pl}^2 m_p^2)^{1/4}$. If we consider a neutrino of characteristic mass 0.1 eV the LV terms become equivalent to the mass term at $E_{th} \approx (E_{Pl}^2 m_\nu^2)^{1/4} \approx 32\text{TeV}$! This is within the observed range of existing neutrino telescopes such as AMANDA [24], and high statistics for this energy range will come with next generation detectors such as ICECUBE [25]

Existing precision data for neutrinos is currently unable to strongly constrain the dimension five and six CPT even operators. For example, some of the best constraints on renormalizable operators are provided by a combination of Super-Kamiokande atmospheric and K2K data [26]. To translate these constraints into (rough) constraints on the higher dimension operators we first need the formalism for LV neutrino oscillations [27]. Let us consider Dirac neutrinos, so we can use (3.1). In this case, the energy eigenstates are also the mass eigenstates. Now consider a neutrino produced via a particle reaction in a definite flavor eigenstate I with momentum p. We denote the amplitude for this neutrino to be in a particular energy eigenstate i by the matrix U_{Ii} where $\sum_i U_{Ji}^\dagger U_{Ii} = \delta_{IJ}$. The amplitude for the neutrino to be observed in another flavor eigenstate J at some distance L, T from the source is then

$$A_{IJ} = \sum_i U_{Ji}^\dagger e^{-i(ET-pL)} U_{Ii} \approx \sum_i U_{Ji}^\dagger e^{-i(2E)^{-1}(m_i^2 + f_{-,\nu_i}^{(4)} p^2 + f_{-,\nu_i}^{(6)} \frac{p^4}{E_{Pl}^2})L} U_{Ii} \tag{3.7}$$

for relativistic neutrinos. If we define an "effective mass" N_i as

$$N_i^2 = m_i^2 + f_{-,\nu_i}^{(4)} p^2 + f_{-,\nu_i}^{(6)} \frac{p^4}{E_{Pl}^2} \tag{3.8}$$

then the probability $P_{IJ} = |A_{IJ}|^2$ can be written as,

$$P_{IJ} = \delta_{IJ} - \sum_{i,j>i} 4F_{IJij} sin^2\left(\frac{\delta N_{ij}^2 L}{4E}\right) + 2G_{IJij} sin\left(\frac{\delta N_{ij}^2 L}{2E}\right) \tag{3.9}$$

where $\delta N_{ij} = N_i^2 - N_j^2$ and F_{IJij}, G_{IJij} are functions of the U matrices. For maximal mixing between flavor and energy eigenstates G_{IJij} vanishes and the F_{IJij} term in (3.9) reduces to

$$P_{IJ} = \delta_{IJ} - sin^2\left(\frac{\delta N_{ij}^2 L}{4E}\right) \tag{3.10}$$

Note that above we have dropped the positive helicity terms. We are dealing with Dirac neutrinos so there are right-handed (positive helicity) particles. However we can ignore these as any signal will be dominated by the left-handed neutrinos produced and interacting via the usual standard model interactions. Therefore we are only concerned with $f_-^{(4)} = \frac{m}{E_{Pl}}(\alpha_-^{(5)} + \alpha_+^{(5)})$ and $f_-^{(6)} = 2\alpha_-^{(6)} + \alpha_-^{(5)}\alpha_+^{(5)}$ in (3.8).

The absolute value of the difference between $f_-^{(4)}$ terms for muon and τ neutrinos, which acts as a change in the terminal velocity away from c, is limited from the atmospheric oscillation data collected by Super-K and K2K [28] to be less than 10^{-24} in general and 10^{-27} for the maximal

mixing case here [26]. This translates into the constraint $|(\alpha^{(5)}_{\nu_\mu -} + \alpha^{(5)}_{\nu_\mu +}) - (\alpha^{(5)}_{\nu_\tau -} + \alpha^{(5)}_{\nu_\tau +})| < 10^2$ (or larger if the neutrino mass in question is less than 0.1 eV), which is not particularly strong. As for the $f^{(6)}_-$ dispersion correction, the K2K beam has an average energy of 1.3 GeV, while the atmospheric neutrino spectrum of Super-K has an average energy of roughly 100 GeV [28]. The results in [26] use neutrinos in this energy band, so while the direct limits on these terms have not been calculated, we can easily overestimate the constraints on $f^{(6)}_-$ by using an energy of 100 GeV for the neutrinos (i.e. maximizing the size of the LV corrections). The deviation at this energy is equivalent to an $f^{(4)}_-$ term of size $f^{(6)}_- E^2/E^2_{Pl} = f^{(6)}_- \cdot 10^{-34}$. Hence the constraints from Super-K and K2K on $f^{(6)}_-$ are worse than 10^7 and not very meaningful.

Fortunately, new detectors such as ICECUBE will dramatically raise the neutrino energies for which there are a large observed population of atmospheric neutrinos. ICECUBE will see a population of upgoing events from atmospheric neutrinos traveling through the earth up to energies of roughly a PeV, where the earth becomes opaque to neutrinos. In [25] Gonzalez-Garcia et. al. consider muon and τ neutrinos and construct an observable of the number of muon events vs. zenith angle with different values of $f^{(4)}_{\nu_\mu} - f^{(4)}_{\nu_\tau} = 2\delta c/c$, which is the notation used in [25]). Atmospheric neutrinos propagating through the earth have different path lengths as a function of zenith angle and the variation is on the order of the diameter of the earth, $\approx 10^7$ m. There will be a variation in the number of muon events with zenith angle as long as the oscillation length between ν_μ and ν_τ is also of this order, i.e. when

$$\frac{\delta N^2_{ij} 10^7 \mathrm{m}}{4E} \approx 1 \tag{3.11}$$

which we rearrange to

$$\delta N^2_{ij} \approx 4 \cdot 10^{-23} \frac{E}{1\mathrm{GeV}} \mathrm{GeV}^2. \tag{3.12}$$

The actual variation, taking into account attenuation and regeneration can be found in [25]. For energies around 100 TeV, the limit is $\delta N^2_{ij} \approx 10^{-18}\mathrm{GeV}^2$, which puts a limit $|f^{(4)}_{\nu_\mu} - f^{(4)}_{\nu_\tau}| < 10^{-28}$. The corresponding constraints on the $\alpha^{(5)}_{\nu_{\mu,\tau},-}$ coefficients are therefore still at best only of $O(10)$. For the $f^{(6)}_-$ term, the constraint becomes $|f^{(6)}_{\mu,-} - f^{(6)}_{\tau,-}| \approx 1$ and we finally achieve order unity constraints. Since the $f^{(6)}$ terms scale strongly with energy, pushing the neutrino energy higher will rapidly increase the size of the constraint. Unfortunately, we can only push the energy up to near a PeV, where the earth becomes opaque to neutrinos. At this energies, the constraints would be of $O(10^{-2})$. Due to the earth's opacity above a PeV it does not appear that we will be able to go beyond this limit with a detector such as ICECUBE.

4. Time of flight

4.1 Time of flight in EFT

Unfortunately, the cosmic ray and neutrino constraints above, while tight, don't actually limit the absolute value of any coefficient but instead generically limit differences of coefficients (or functions thereof). This is obvious in the neutrino constraints and manifests itself in Figures 1 and 2 via the qualitative form of the constraints - open wedges in parameter space. With bounds on only differences, Lorentz violation doesn't need to be small but only similar for different particle species.

When considering simple changes in the terminal velocity of particles, as would be generated by dimension four operators, this isn't an issue since the Lorentz group doesn't specify the magnitude of the speed of light, only the fact that some speed is invariant. Therefore constraining differences between the terminal speeds for different species means one is completely constraining all forms of LV at this order. The situation is different for the higher dimension operators. Here, even though the dimension five and six coefficients can be constrained to be almost equal, LV can still be very large at high energies. Therefore one would very much like to constrain the absolute value of the operators, not just their differences.

It is of course possible to establish tight two sided bounds on higher dimension operators. CPT odd dimension five operators have been tightly constrained on both sides from studies of the Crab Nebula [29, 7] and a combination of synchrotron radiation, TeV γ-ray annihilation off the IR background, and existence of TeV photons [30]. Both these constraints rely on experimental confirmation that at least three different reactions involving electrons and photons are unaffected by LV to construct their constraints, however. At the very high energies needed to probe the dimension five and six CPT even operators we no longer have this luxury and so it is difficult to derive two-sided bounds with the threshold type reactions considered previously.

The other method for deriving two sided bounds is to pick an observation that only involves one LV parameter. The simplest way to do this is by comparing the arrival times of high energy particles or gamma rays versus low energy gamma rays, where all particles are emitted from the same event. Since LV with higher dimension operators scales with energy, the LV terms for the low energy γ-rays are irrelevant and the arrival time delay is effectively a function of only one parameter. The delay ΔT between a low energy photon and a fermion ψ with (high) energy E traveling over a time T is

$$\Delta T = -\frac{3 f_\psi^{(6)} E^2}{2 E_{Pl}^2} T \tag{4.1}$$

where we have neglected the $f_\psi^{(4)}$ term as it is irrelevant.

If we maximize T and consider cosmogenic neutrinos at distances of 1 Gpc, we immediately see that for O(1) $f^{(6)}$ to even reach a delay ΔT of one second requires energies at 10^{20} eV. While such energies will likely be observed in the future by neutrino observatories such as ANITA [31], time of flight observations are not likely to be possible. First, one needs to identify the sources for the flux. For high energy neutrinos produced as secondaries from interactions of cosmic rays with the CMB [32], Z-burst [33] or other decay scenarios [34, 35], this is impossible. GRB's where the source can be established and the secondary low energy signal seen have neutrino energies far too low to constrain the LV we are considering. Hence clear two-sided bounds on the higher dimension operators seem out of reach without imposing additional assumptions.

4.2 Time of flight outside of EFT

While time of flight isn't particularly useful in constraining higher dimension operators in EFT they are useful for proposals for LV that do not fit within EFT and constitute our last hot spot. There are two proposals in particular, one based on non-critical string theory [36], and one coming from "doubly special relativity" (see [37] for a discussion of DSR phenomenology). Both have as one of the primary testable phenomenological features a modified dispersion for photons of the form

$\omega^2 = k^2 + \xi|w|^3/E_{Pl}$, where ξ is a coefficient universal for all photons. There is hence a deviation from the low energy speed of light that scales linearly with energy. Even though this dispersion is rotationally invariant, it is *not* a dispersion law that can be constructed with a vector field in an operator expansion. Rotationally invariant dispersion relations where the dispersion correction is suppressed by a single power of E_{Pl} have been constructed in an EFT context, they require the CPT violating dimension five operator [38] in (2.4)

$$\frac{\xi}{E_{Pl}} u^\mu F_{\mu\alpha}(u\cdot\partial)(u_\nu \tilde{F}^{\nu\alpha}). \tag{4.2}$$

This operator yields a dispersion of the form $\omega^2 = k^2 \pm \xi k^3/E_{Pl}$, where the $\pm\xi$ corresponds to right and left circularly polarized photons. Hence there is birefringence in vacuum for photons in addition to time of flight delays. From the absence of birefringence for polarized photons from afterglow of GRB's [12], we know that $|\xi| < 10^{-7}$.

Recently, the MAGIC collaboration has reported a four minute delay in the arrival times for photons from a flare of Markarian 501 that is compatible with a $\xi \approx -3$ [39]. In an EFT context this is meaningless, as we already have bounds 10^7 times stronger and therefore the delay must be caused by source effects. Conversely, if source effects are ruled out and/or a time of flight delay of this size is confirmed for other flares, then the conclusions will be startling. Not only would a positive result mean that LV exists, but more drastically it would imply that standard EFT is unable to describe at least one low energy correction from quantum gravity! Since there are proposals for LV that don't fit within our usual EFT framework, confirming or conclusively ruling out this type of dispersion is an important question. Fortunately current experiments have the necessary sensitivity and so this question should be answered soon.

5. Conclusions

LV, as illustrated by our simplified model of a LV vector field, is very tightly constrained by current experiments. Without custodial symmetries LV is so tightly constrained at all orders that it seems very likely that Lorentz invariance is an exact symmetry. However, a combination of CPT and supersymmetry can protect Lorentz invariance enough that one can still have LV at the quantum gravity scale and be compatible with experiment as long as supersymmetry is broken at scales less than around 1 PeV. Therefore it is still of interest to study LV, although the models we should primarily be interested in now must consist of both LV at the Planck and new low energy physics. We have illuminated here three regions that are still of theoretical interest in studies of LV. The first is that of tests of the CPT even dimension four operators by low energy experiments. As these experiments improve, they will force the SUSY breaking scale in a LV theory to decrease until finally it reaches the TeV scale. At this point SUSY will be reachable by accelerators and we can rule out this type of split scenario.

The second region of interest is the CPT even dimension six operators. While supersymmetry predicts these operators to be small, the lesson of supersymmetry is that we should not naively trust the simple hierarchy for sizes of operators at each mass dimension. Since we are able to directly constrain these operators below O(1) by cosmic ray experiments such as Auger and upcoming neutrino observatories such as ICECUBE it makes sense to do so. The final region of theoretical

interest is in time of flight delays for high energy particles. In EFT, time of flight observations cannot provide us with better constraints than other experiments. However, there are proposals that sit outside the framework of EFT for which time of flight delays are the only currently experimentally testable signature. While we personally find it implausible that the corrections to physics at TeV energies from quantum gravity do not fit within EFT, nature has surprised us in the past with radically new physics. [4] Time of flight observations of high energy photons from GRB's recorded by MAGIC and other GRB observatories are able to probe these proposals and hence this is one final region of theoretical interest.

If we test all of these regions of parameter space and find no signal of LV, does this mean we should consider LV an exact symmetry? No, there are of course other forms of LV besides vector fields and they may have different behavior. However, LV from quantum gravity must be either more exotic (i.e. outside the realm of effective field theory?) or carefully, and possibly unnaturally, hidden if we do not find a signal in these regions.

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