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Group field theory as the microscopic description of the quantum spacetime fluid: a new perspective on the continuum in quantum gravity

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We introduce the group field theory (GFT) formalism for non-perturbative quantum gravity, and present it as a potential unifying framework for several other quantum gravity approaches, i.e. loop quantum gravity and simplicial quantum gravity ones. We then argue in favor of and present in detail what we believe is a new GFT perspective on the emergence of continuum spacetime from discrete quantum structures, based on the idea of quantum space as a condensed matter system. We put forward a more specific, albeit still very much tentative, proposal for the relevant phase of the GFT corresponding to the continuum: a Bose-Einstein condensate of GFT quanta. Finally, we sketch how the proposal may be realised and its effective dynamics could be extracted in the GFT setting and compared with continuum gravity theories.

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1. Introduction

The purpose of this contribution to the debate on “Quantum and emergent gravity”is fourfold.

First of all, we would like to introduce the group field theory (GFT) formalism [1, 2, 3], that has recently attracted interest in the general area of non-perturbative quantum gravity, and is currently mainly used in the context of Loop Quantum Gravity [4]. We will describe the general features of the formalism, at both kinematical and dynamical level, and provide an interpretation for them.

Second, we would like to portrait a picture of group field theories as a common framework and a unifying language for several approaches to quantum gravity, in particular loop quantum gravity and simplicial quantum gravity (i.e. quantum Regge calculus and dynamical triangulations), by sketching how the basic ingredients of these various approaches can be identified within the GFT setting. We will argue that the pictures of quantum spacetime, developed in the various approaches, are compatible and can help completing each other, while acquiring a new interpretation within the GFT framework. GFTs can then represent a suitable context in which all these different approaches can inform, cross-fertilize and improve each other with the achieved results and insights into the nature of quantum geometry, and with the tools they have developed to study it. In doing so, of course, we will discuss why we think is useful to move from the contexts provided by each of these quantum gravity approaches to the GFT one.

Third, we want to stress the need to devote our research efforts to tackle the issue of the continuum approximation of the quantum discrete structures that these various approaches identify as the fundamental building blocks of spacetime. Only if we are able to show convincingly that a good continuum description of spacetime, with its dynamics governed by (some modified version of) General Relativity, emerges naturally from the formulation of quantum gravity we favor, we will have a truly convincing argument for believing this formulation. This is of course well-known by researchers working in non-perturbative quantum gravity, and in particular in the approaches we have just mentioned: loop quantum gravity (and spin foam models), quantum Regge calculus and (causal) dynamical triangulations. Indeed, many techniques and strategies have been developed, within these various approaches, to solve the continuum (and semi-classical) riddle, and many results already obtained. We will briefly discuss, and try to re-phrase, them in the GFT language. This will allow us to both understand them as providing insights about different regimes and features of the same type of models, and clarify in which sense they do not represent the most convenient or natural way to approach the continuum problem from a GFT point of view.

Last, we will argue that group field theories offer new and powerful tools to tackle the problem of the continuum in quantum gravity, together with a new perspective on the whole issue, that could prove decisive for settling it, at the same time developing further and going beyond the insights obtained from the other approaches mentioned above. The suggestion will basically be that we could try to view spacetime as a (peculiar indeed) condensed matter system, with the GFT representing the microscopic description of its “atoms”, and providing the starting point for studying both the statistical mechanics and the effective dynamics of large number of them, which we will tentatively identify with continuum physics. In particular, group field theories can offer the context and the tools to realize explicitly the intriguing idea of spacetime as a condensate of fundamental building blocks and of continuum geometry as an emergent concept. We will then put forward a proposal for this GFT condensate, suggest some concrete research directions (some of which currently pursued), and offer some speculation on how a continuum spacetime and General Relativity can emerge in this scheme, again making use (also) of the condensed matter analogy.

Given its aims, this article will contain a limited amount of technicalities, only those needed to introduce the main GFT idea and general formalism, and only references to and brief discussions of the many results obtained both in the GFT context and in the context of the other approaches to quantum gravity we will mention. At the same time, it may contain a more than average amount of speculations, especially in its last part, when we will try to forecast where the new perspective we are advocating may lead to. We will hopefully compensate for this by trying to be as precise as possible in presenting the main ideas, motivations and arguments behind this perspective, and to convince the reader that this may be an intriguing and reasonable picture of what recent results in quantum gravity research are pointing to.

2. The group field theory formalism

We now proceed to introduce the main features of the GFT formalism. We refer to the literature, in particular the reviews [1, 2, 3], for a more complete and detailed treatment and a more extensive list of references.

2.1 Kinematics: the fundamental building blocks of quantum space

We start from a field taken to be a $\mathbb{C}$-valued function of D group elements, for a generic group G, one for each of the D boundary (D-2)-faces of the (D-1)-simplex that the field ϕ represents:

$$\phi(g_1, g_2, ..., g_D) : G^{\times D} \to \mathbb{C}.$$

In models (aiming at) describing D-dimensional quantum gravity, this field is interpreted as a second quantized (D-1)-simplex, with (D-2)-faces of the same labelled by group theoretic data, interpreted as (pre-)geometric elementary quantities, or discrete quantum gravity variables. Equivalently, the same data can be associated to the links of a topologically dual graph, and the field is then seen as the second quantization of a spin network functional [4]. This means that GFTs can be seen equivalently as a second quantized formulation of spin network dynamics or as a field theory *of* simplicial geometry. We can identify the ordering of the arguments of the field with a choice of orientation for the (D-1)-simplex it represents, and we require invariance of the field under even permutations σ of its arguments and trade odd permutations with complex conjugation of the field. Other symmetry properties can also be considered. An additional symmetry that is usually imposed on the field is the invariance under diagonal action of the group G on the D arguments of the field: $\phi(g_1, ..., g_D) = \phi(g_1 g, ..., g_D g)$; but this is again model-dependent, of course, and in the models of [9, 10], for example, only invariance under a certain proper subgroup is imposed. This is the simplicial counterpart of the Lorentz gauge invariance of continuum and discrete first order gravity actions, and it has also the geometric interpretation, at the simplicial level, of requiring the D faces of a (D-1)-simplex to close.

A momentum representation for the field and its dynamics is obtained by harmonic analysis on the group manifold G. The field can be expanded in modes as:

$$\phi(g_i) = \sum_{J_i, \Lambda, k_i} \phi^{J_i \Lambda}_{k_i} \left(\prod_i D^{J_i}_{k_i l_i}(g_i) \right) C^{J_1..J_D \Lambda}_{l_1..l_D},$$

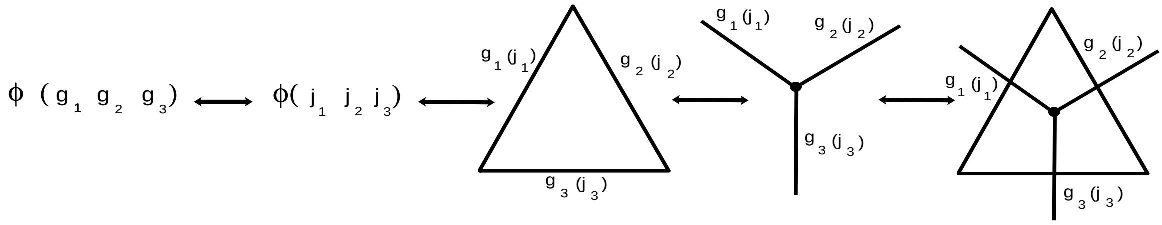

Figure 1: For the $D = 3$ case, the association of a field with a 2-simplex, or equivalently its dual vertex, and of its arguments with the 1-faces of it, or equivalently with the links incident to the vertex, together with the labelling by group-theoretic variables.

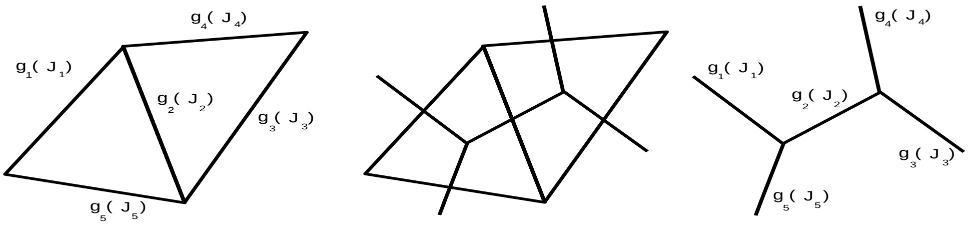

Figure 2: A '2-particle state'(again, in the D=3 example)

with the J's labelling representations of G, the k's vector indices in the representation spaces, and the C's being intertwiners of the group G. We have labelled an orthonormal basis of intertwiners by an extra parameter Λ (depending on the group chosen and on the dimension D, this may actually be a shorthand notation for *a set* of parameters). That this decomposition is possible is not guaranteed in general, but it is in fact true for all the known quantum gravity GFT models, which are based on the Lorentz group or on extensions of it. The proper geometric interpretation of the field variables can be identified by looking at the Feynman amplitudes for the GFT at hand, that either have the form of discrete path integrals for some gravity action [9, 10] or can be derived from one [1, 2, 3]. This interpretation depends of course on the specific model considered. However, generally speaking, the group variables are seen to represent parallel transport of a (gravity) connection along elementary paths dual to the (D-2)-faces, and the representations J are usually put in correspondence with the volumes of the same (D-2)-faces.

Just as one identifies a single field with a single (D-1)-simplex, a simplicial space built out of N such (D-1)-simplices is described by a suitable polynomial in the field variables, with constraints among the group or representation data, implementing the fact that some of their (D-2)-faces are identified. For example, a state describing two (D-1)-simplices glued along one common (D-2)-face would be represented by: $\phi^{J_1 J_2 .. J_D \Lambda}_{k_1 k_2 ... k_D} \phi^{\tilde{J}_1 J_2 ... \tilde{J}_D \tilde{\Lambda}}_{\tilde{k}_1 k_2 ... \tilde{k}_D}$, where the gluing is along the face labelled by the representation J_2, and effected by the contraction of the corresponding vector indices (of course, states corresponding to disjoint (D-1)-simplices are also allowed).

We see that states of the theory are then labelled, in momentum space, by *spin networks* based on the group G [4].

GFT observables are given [3] by gauge invariant functionals of the GFT field, and can be constructed in momentum space using again spin networks according to the formula:

$$O_{\Psi=(\gamma,j_e,i_v)}(\phi) = \left(\prod_{(ij)} \int dg_{ij}dg_{ji}\right) \Psi_{(\gamma,j_e,i_v)}(g_{ij}g_{ji}^{-1}) \prod_i \phi(g_{ij}),$$

where $\Psi_{(\gamma,j_e,i_v)}(g)$ identifies a spin network functional [4] for the spin network labelled by a graph γ with representations j_e associated to its edges and intertwiners i_v associated to its vertices, and g_{ij} are group elements associated to the edges (ij) of γ that meet at the vertex i.

Thus, *group field theories describe a quantum space in terms of fundamental building blocks, the quanta of the GFT field, that acquire then the status of "atoms of space"in this setting, and that can be represented both as spin network vertices or as elementary (D-1)-simplices. A generic quantum state will be a "many-particle"configuration for these quanta, representing some extended discrete structure (a larger spin network or a larger (D-1)-triangulation)* characterized by both the "particle number"and by additional symmetries or constraints imposed, specifying how the fundamental building blocks are glued together. This picture can be made more precise and a Fock space characterization of the GFT state space (and thus of quantum space, in this framework) can be obtained after Hamiltonian analysis of specific GFT models [11].

2.2 Dynamics: the interaction and evolution of the atoms of space

On the basis of the above kinematical structure, one aims at defining a field theory for describing the interaction of fundamental atoms of space, and in which *a typical interaction process will be characterized by a D-dimensional simplicial complex. In the dual picture, the same will be represented as a spin foam (labelled 2-complex).* This is the straightforward generalization of the way in which 2d discretized surfaces emerge from the interaction of matrices (graphically, segments)[17], or ordinary Feynman graphs emerge from the interaction of point particles. A *discrete* spacetime emerge then from the theory as a virtual construct, a possible interaction process among the GFT quanta.

In order for this to be realized, the classical field action in group field theories has to be chosen appropriately. In this choice lies the main peculiarity of GFTs with respect to ordinary field theories. This action, in configuration space, has the general structure:

$$\begin{aligned} S_D(\phi,\lambda) &= \frac{1}{2}\left(\prod_{i=1}^{D}\int dg_i d\tilde{g}_i\right)\phi(g_i)\mathscr{K}(g_i\tilde{g}_i^{-1})\phi(\tilde{g}_i) + \\ &+ \frac{\lambda}{(D+1)!}\left(\prod_{i\neq j=1}^{D+1}\int dg_{ij}\right)\phi(g_{1j})...\phi(g_{D+1j})\mathscr{V}(g_{ij}g_{ji}^{-1}) \ , \end{aligned} \quad (2.1)$$

and it is of course the choice of kinetic and interaction functions $\mathscr{K}$ and $\mathscr{V}$ that define the specific model considered. Obviously, the same action can be written in momentum space after harmonic decomposition on the group manifold. The interaction term describes the interaction of D+1 (D-1)-simplices to form a D-simplex ('a fundamental virtual spacetime event') by gluing along their (D-2)-faces (arguments of the fields), that are *pairwise* linked by the interaction vertex. The nature of this interaction is specified by the choice of function $\mathscr{V}$. The kinetic term involves two fields each representing a given (D-1)-simplex seen from one of the two D-simplices (interaction vertices) sharing it, so that the choice of kinetic functions $\mathcal{K}$ specifies how the information and therefore the geometric degrees of freedom corresponding to their D (D-2)-faces are propagated from one vertex of interaction to another. One can consider generalizations of the above combinatorial structure, corresponding to the gluing of (D-1)-simplices to form different sorts of D-dimensional complexes (e.g. hypercubes etc).

Some examples of GFT actions are: 1) those corresponding to the kinetic and vertex functions:

$$\mathcal{K}(g_i,\tilde{g}_i) = \prod_{i=1}^{D}\delta(g_i\tilde{g}_i^{-1}), \qquad \mathcal{V}(g_{ij},g_{ji}) = \prod_{i<j=1}^{D+1}\delta(g_{ij}g_{ji}^{-1}), \tag{2.2}$$

which produce a perturbative quantum dynamics that can be related to topological BF theories in any dimension, for internal gauge group G; 2) models in which suitably defined additional constraints on the same BF-type kinetic and/or vertex terms are imposed, and which aim at representing the GFT equivalent of the constraint reducing BF theory to gravity in a Plebanski-like formulation of the same [14, 15, 16]; 3) extended models based on more than Lorentz group variables and characterized by a proper differential operator playing the role of kinetic term, one example of which is the class of models in [10], using a complex field on $(G\times X)^D$, with G being the Lorentz group and X a metric space isomorphic to the Lie algebra of G, and based on the kinetic and vertex terms:

$$\mathcal{K}(g_i,x_i,\tilde{g}_i,\tilde{x}_i) = \prod_i (\triangle_i + \Box_i)\,\delta(g_i\tilde{g}_i^{-1})\delta(x_i-\tilde{x}_i^{-1}) \qquad \mathcal{V}(g_{ij},x_{ij}) = \prod_{i\neq j}\delta(g_{ij}g_{ji}^{-1})\delta(x_{ij}-x_{ji}) \tag{2.3}$$

where $g_i\in G$, $x_i\in X$, $\triangle$ is the Laplace-Beltrami on X and $\Box$ is the Laplace-Beltrami on G; these last models produce Feynman amplitudes with the interpretation of simplicial path integrals for 1st order gravity actions [10].

Let us now turn to the quantum dynamics. Most of the research in this area has concerned the perturbative aspects of this dynamics around the no-particle state, the complete vacuum, and the main guide for model building have been, up to now, only the properties of the resulting Feynman amplitudes:

$$Z = \int \mathcal{D}\phi\, e^{iS[\phi]} = \sum_\Gamma \frac{\lambda^{N_v(\Gamma)}}{sym[\Gamma]}\,Z(\Gamma),$$

where N_v is the number of interaction vertices v in the Feynman diagram Γ, $sym[\Gamma]$ is the number of automorphisms of Γ and $Z(\Gamma)$ the corresponding Feynman amplitude. Each edge of the Feynman graph is made of D strands, one for each argument of the field and each one is then re-routed at the interaction vertex, with the combinatorial structure of an D-simplex, following the pairing of field arguments in the vertex operator.

Each strand in an edge of the Feynman diagram goes through several vertices, coming back where it started, for closed Feynman diagrams, and therefore identifies a 2-cell (for open graphs, it may end up on the boundary, but still identifies a 2-cell). Each Feynman diagram Γ is then a collection of 2-cells, edges and vertices, i.e. a 2-complex, that, because of the chosen combinatorics for the arguments of the field in the action, is topologically dual to a D-dimensional simplicial

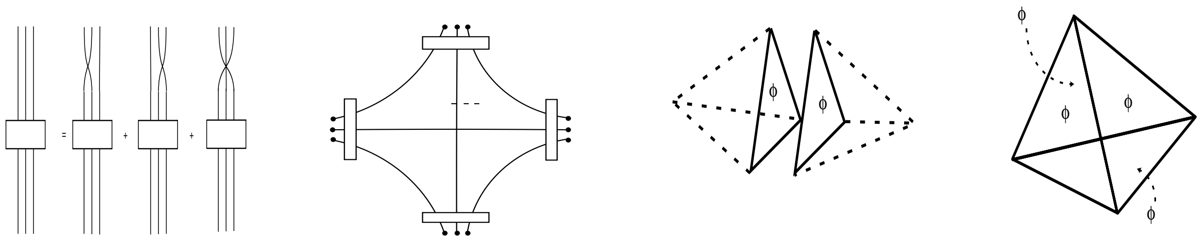

Figure 3: The basic building blocks of the GFT Feynman diagrams (for $D = 3$).

complex. Notice that the resulting 2-cells can be glued (i.e. can share edges) in all sorts of ways, forming for example “bubbles”, i.e. closed 3-cells.

No restriction on the topology of the diagram/complex is imposed, a priori, in the construction, so the resulting complexes/triangulations can have arbitrary topology. Each of them corresponds to a particular *scattering process* of the fundamental building blocks of space, i.e. (D-1)-simplices/spin network vertices. Each line of propagation, made as we said out of D strands, is labelled, on top of the group/representation data, by a permutation of $(1,..,D)$, representing the labelling of the field variables, and all these data are summed over in the construction of the Feynman expansion. The sum over permutations affects directly the combinatorics of the allowed gluings of vertices with propagators[8].

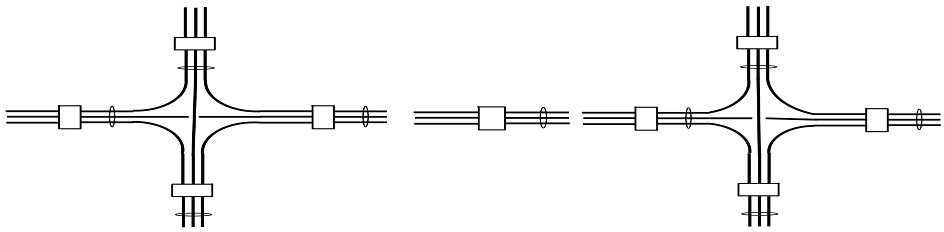

Figure 4: The gluing of vertices of interaction through propagators, again in the D=3 example. The rectangles represent the additional integrations imposing gauge invariance under the action of G, while the ellipses represent the implicit sum over permutations of the (labels of the) strands to be glued.

As said, each strand in a propagation line carries a field variable, e.g. a group element in configuration space or a representation label in momentum space. After the closure of the strand to form a 2-cell in a closed diagram, the same representation label ends up being associated to this 2-cell. Therefore in momentum space each Feynman graph is given by a spin foam (a 2-complex with faces labelled by representation variables), and each Feynman amplitude (a complex function of the representation labels, obtained by contracting vertex amplitudes with propagator functions) by a so-called spin foam model [12] (in the models [9, 10] the labelling of the spin foam 2-complex is slightly more involved). The inverse is also true: any local spin foam model can be obtained from a GFT perturbative expansion [13, 3]. The sum over Feynman graphs gives then a sum over spin foams, and equivalently a sum over triangulations, augmented by a sum over algebraic data (group elements or representations) with a geometric interpretation, assigned to each triangulation. This perturbative expansion of the partition function also allows for a perturbative evaluation of expectation values of GFT observables, as in ordinary QFT. In particular, the transition amplitude (probability amplitude for a certain scattering process) between certain boundary data represented by two spin networks, of arbitrary combinatorial complexity, can be expressed as the expectation value of the field operators having the same combinatorial structure of the two spin networks [3, 1].

$$\langle \Psi_1 \mid \Psi_2 \rangle = \int \mathscr{D}\phi \, O_{\Psi_1} \, O_{\Psi_2} \, e^{iS(\phi)} = \sum_{\Gamma/\partial\Gamma = \gamma_{\Psi_1} \cup \gamma_{\Psi_2}} \frac{\lambda^N}{sym[\Gamma]} \, Z(\Gamma)$$

where the sum involves only 2-complexes (spin foams) with boundary given by the two spin networks chosen.

The above perturbative expansion involves thus two types of sums: one is the sum over geometric data (group elements or representations of G) entering the definition of the Feynman amplitudes as the GFT analogue of the integral over momenta or positions of usual QFT; the other is the overall sum over Feynman diagrams. We stress again that, in absence of additional restrictions being imposed on the GFT, the last sum includes a sum over all triangulations for a given topology and a sum over all topologies.

2.3 A peculiar quantum field theory (still, a proper field theory!)

In the end, *GFTs are a peculiar type of quantum field theories*, defined on specifically chosen group manifolds. The main reasons why they are rather peculiar, from a purely field-theoretic perspective, are:

- the way in which field arguments are paired in the interaction term, which makes them a sort of *combinatorially non-local field theories*;
- the resulting combinatorial structure of Feynman diagrams, given, as we discussed by fat graphs dual to simplicial complexes, but also presenting no true vertex of interaction, in the usual QFT sense of simultaneous identification of more than two configuration variables, and constituted only by 'loops '(closed lines of propagation of the individual field arguments) and 'bubbles'(3-cells bounded by several such loops);
- the fact that all the arguments of the field are naturally treated on equal footing; if a specific time parameter can be identified among the group coordinates, still there would be one such parameter for each argument of the field, thus D in total, leading to a sort of 'multi-time dynamics'; in the Hamiltonian analysis of GFTs [11], this implies the need for a polysymplectic canonical formulation and has several interesting consequences;
- the fact that, for GFTs characterized by kinetic functions formed by differential operators, there is then naturally one such operator for each argument of the field, and a product structure of the full kinetic term, reproducing again this independent propagation of field arguments, but also producing technical complications.

However, as for the rest, we have an almost ordinary field theory, in that we can rely on a fixed background metric structure, given by the invariant Killing-Cartan metric on the group manifold (or extensions of it), a fixed topology, given again by the topology of the group manifold, the usual splitting between kinetic (quadratic) and interaction (higher order) term in the action, and the usual conjugate pictures of configuration and momentum space. This allows us to use all usual QFT techniques and language in the analysis of GFTs, and thus of quantum gravity, even though we remain in a background independent (in the physical sense of 'spacetime independent') context. The importance of this, in a non-perturbative quantum gravity framework, should not be underestimated, we think, and it is at the roots of the strategy we will propose later on to tackle the issue of the continuum and semi-classical approximation.

3. Group field theory as a common framework for discrete quantum gravity

GFTs can potentially represent a common framework for different current approaches to quantum gravity, in particular canonical loop quantum gravity[4] and simplicial quantum gravity formalisms, namely quantum Regge calculus [5] and (causal) dynamical triangulations [6], because the same mathematical structures that characterize these approaches also enter necessarily and in very similar fashion in the GFT framework. *We believe in the need to learn from all of them in order to solve the remaining challenges towards a complete theory of quantum gravity, and the GFT formalism may be the most suitable framework in which the many lessons we can draw from all of them can be brought together and to fruition.*

3.1 Convergence of formalisms, structures and languages

Historically, GFTs can be understood as being born as a generalisation of matrix models [17] for 2-dimensional quantum gravity. This generalisation is obtained in two steps: 1) by passing to generic tensors, instead of matrices, as fundamental variables, thus obtaining a generating functional for the sum over D-dimensional simplicial complexes that was the essence of the dynamical triangulations approach to quantum gravity[18, 19]; 2) adding group structure defining geometric degrees of freedom. The last step is what turns a tensor model into a proper field theory. In fact, the first example of a GFT was the group-theoretic generalisation of 3d tensor models proposed by Boulatov [20], corresponding to the $D = 3$ and $G = SU(2)$ case of (2.2). Already at this initial stage, group field theories allowed a direct contact between simplicial quantum gravity and what we now call spin foam models [12], as the Boulatov model produces Feynman amplitudes given by the so-called Ponzano-Regge spin foam model. As we have discussed above, we now know that this is just one example of a very general result [13]: the equivalence between (local) spin foam models and GFT Feynman amplitudes. In turn, spin foam models [12] have been a very active area of quantum gravity research in the past ten years, for two main reasons. First, one obtains a spin foam model when considering a path integral quantization of discrete gravity formulated as a gauge theory. Second, spin foams arise naturally when considering the dynamics of the kinematical quantum states of geometry as identified by canonical loop quantum gravity [4]. Indeed, from the LQG perspective, spin foams represents the histories of spin networks and are thus the crucial ingredient of any path integral or covariant formulation of the quantum gravity dynamics in LQG. From both the simplicial and canonical perspective, a sum over spin foams/triangulations, weighted by appropriate amplitudes, is a crucial ingredient in defining the dynamics of the gravitational field: in simplicial quantum gravity because such sum can compensate the truncation of geometric degrees of freedom that the restriction to a given lattice imposes; in LQG, because a complete path integral formulation of the dynamics needs, in general, a sum over all the histories between given spin network states. At present, group field theories are the only known tool to define uniquely and completely such sum over spin foams. Now let us give a closer look at how the various ingredients of these various approaches, that all have historically contributed, with hindsight, to the development of the group field theory formalism, can be identified and re-interpreted within the formalism itself.

3.2 Loop quantum gravity and group field theory

We have mentioned already the first and most basic link between the group field theory formalism and loop quantum gravity: *boundary states of generic GFTs are spin networks*, i.e. what has been identified by the canonical loop quantization programme as the kinematical quantum states of geometry [4]. *The GFT field itself*, as we have seen, *is interpreted as the result of a 2nd quantization of a spin network wave function.* This correspondence can be made more precise, and one can in fact show [21] that a generic spin network wave function can be re-expressed as a direct analogue of a multi-particle wave function, with the particle degrees of freedom being associated to the spin network vertices; a standard second quantization procedure applied to these multi-particle wave functions, then, leads to a field defined on the same group manifold from which spin network data are taken, and that can be straightforwardly identified with the GFT field. *GFTs therefore define possible dynamics for these quantum states of geometry, in a 2nd quantized formulation,* and in a way that identifies the basic dynamical degrees of freedom as those associated to the vertices of the spin networks themselves, that in turn have been shown in LQG to correspond to elementary chunks of space volume. From these kinematical considerations, it immediately follows that any quantum operator that can be defined in the 1st quantized LQG setting has a 2nd quantized GFT counterpart, that can be, at least in principle, identified. More importantly, this suggests that the LQG *dynamics* can be embedded and studied within the GFT setting. There are two equivalent ways in which this can be done. First of all, as in any QFT, the GFT classical action should encode the full 1st quantized dynamics, and the classical equations of motion should correspond to the full dynamical equations of the 1st quantized wave function. Solving the GFT classical equations, then, means identifying non-trivial quantum gravity wave functions satisfying *all* the quantum gravity constraints, an important and still unachieved goal of canonical loop quantum gravity, except in some simplified situations. The same classical equations of motion can be solved, implicitly, also at the level of the perturbative Feynman expansion: one could consider the restriction of the GFT perturbative expansion given above to *tree level*, for given boundary spin network observables [3]. This is the GFT definition of the canonical inner product between two spin network states. The definition is well posed, because at tree level every single amplitude $Z(\Gamma)$ is finite whatever the model considered due to the absence of infinite summation (unless it presents divergences at specific values of the configuration/momentum variables). Moreover, it possesses all the properties one expects from a canonical inner product [3]. This means that the physical Hilbert space for canonical spin network states can be constructed starting from the above definition of the inner product. This shows a concrete example of how the dynamics of spin network states can be encoded covariantly in a sum over spin foams, in the same sense in which the dynamics of canonical gravity in ADM variables can be formulated, in principle, as a covariant path integral over geometries (see [22] for more details on this perspective on spin foam models).

There are of course many open issues regarding the exact connection between the LQG and the GFT frameworks. One concerns, for example, the role of spatial topology change. Its status within LQG is not obvious at present: on the one hand, LQG being the result of a canonical quantization on a globally hyperbolic manifold, one would expect spatial topology change to be ruled out almost by definition; on the other hand, the resulting quantum states of geometry unavoidably describe also quantum spatial geometries with degeneracy points, and thus seem to admit the possibility of branchings of space at those points). In GFT, as we have seen, non-trivial topologies appear in perturbative expansion as soon as one goes beyond tree level, and there is no known mechanism to either suppress or avoid them. Another open issue is the interpretation, from the quantum gravity point of view in general, and within LQG in particular, of the GFT coupling constant; for some proposals on this, we refer to the literature [1]. One more unsettled point is whether one should expect a direct link between the GFT and the LQG dynamics, i.e. between the GFT action and the LQG Hamiltonian constraint already at the level of the, supposedly, microscopic definition of the GFT itself, or at the level of some macroscopic, effective QFT action defined starting from the microscopic GFT dynamics. After all, the Hamiltonian constraint operator of LQG is obtained by a direct quantization of continuum (and possibly effective) General Relativistic dynamics, and while one can be lucky enough to capture some kinematical properties of the microscopic description of a system, in general one should not expect to capture the exact microscopic *dynamics* of the same starting from some effective macroscopic description [23], although it is certainly a possibility. More specific open issues concern the exact choice of the gauge group, which is usually the full Lorentz group in GFTs and the $SU(2)$ subgroup in LQG, the need for the GFT restriction on the valence of spin network vertices, etc However, while it is clear that much more work is needed to explore and settle these issues, their presence does not spoil or modify drastically, we think, the above general picture of the GFT-LQG relation, and most importantly, all these issues can be tackled *within the GFT formalism itself*.

3.3 Simplicial quantum gravity and group field theory

The GFT Feynman diagrams, as we have seen, identify simplicial complexes to which the GFT assigns geometric data, weighted by quantum amplitudes that can be related to path integrals for simplicial gravity on the given complex, and indeed share the same interpretation. These Feynman diagrams/simplicial complexes are summed over to define the GFT partition function in perturbative expansion, and thus the full dynamics.

$$Z = \int \mathscr{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} \sum_{\{J_i\}} A_\Gamma(J_i).$$

The relation between GFTs and traditional approaches to discrete quantum gravity is therefore clear, at least in its general features. *For given Feynman diagram, and thus fixing a single triangulation as a discrete model of spacetime, the GFT provides a quantization of gravity in the spirit and language of quantum Regge calculus*, by an assignment of geometric data that are (more or less direct) analogues of the edge lengths used there, and summing over all such possible assignments. The full amplitude weighting such assignments, i.e. the specific function of the geometric data to be used, is specified uniquely b the specific GFT model one is considering. Schematically:

$$Z = \int \mathscr{D}\phi \, e^{iS[\phi]} \curvearrowright Z_{QRC} = \sum_{\{J_i\}} A_\Gamma(J_i) \approx \text{“} \int \mathscr{D}g \, e^{iS_{GR}(g)} \text{”}$$

If, instead of fixing the triangulation, i.e. considering a specific GFT Feynman diagram, one freezes GFT field degrees of freedom (thus fixing the geometric data) to some constant value, the same GFT provides a definition of the dynamics of quantum geometry via a sum over triangulations weighted by purely combinatorial amplitudes, i.e. functions of the combinatorics of the simplicial complexes only. This is a definition of quantum gravity *in the same spirit and language of the dynamical triangulations approach.* Schematically:

$$Z = \int \mathscr{D}\phi \, e^{iS[\phi]} \curvearrowright Z_{DT} = \sum_{\Gamma} \frac{1}{sym[\Gamma]} A_\Gamma(\lambda) \approx \text{“} \int Dg \, e^{iS_{GR}(g)} \text{”}$$

The quantum amplitudes weighting histories of the gravitational field are given, in both quantum Regge calculus and dynamical triangulations, by the exponential of the Regge action for discrete gravity, while in most spin foam models the connection between the quantum amplitudes and the Regge action is clear only in a particular regime and even there it is rather involved [12]. However, such relation is much clearer in the recent GFTs of [10], whose amplitudes have indeed the form of simplicial gravity path integrals, with clearly identified classical simplicial gravity actions. GFTs can then be said to incorporate both traditional simplicial quantum gravity approaches, and to do so in a nice complementary way. We do not know, however, is they also do it correctly or whether, by doing so, they extend the definition of both beyond what is useful or needed. Much more work is needed, for example, to study in greater detail the (classical and quantum) simplicial geometry corresponding, for given triangulation, to the known GFTs. And much more work is needed in order to understand what is the QFT meaning of many of the configurations, e.g. those corresponding to non-trivial spacetime topologies, or the non-manifold-like ones, appearing in the perturbative GFT sum over triangulations; how one could gain control over them is an open, important issue. Also, in the modern *causal* dynamical triangulations approach, the nice result (that we are going to discuss in the following) concerning the continuum limit of the sum over triangulations seem to depend on specific *causality* restrictions on the class of triangulations summed over [6]; whether and how one can understand and implement such restrictions from a field theory perspective and within the GFT setting is presently unclear. At the same time, there is hope that the sum over triangulations may provide a more powerful alternative to the refinement procedure of Regge calculus to lift the restriction to a fixed simplicial complex, and that the additional field-theoretic data and associated gauge symmetries and non-perturbative information of GFTs can be useful not only because they provide the theory with a well-identified space of states etc, but also for gaining control of the sum over triangulations of the dynamical triangulations approach [34]. To summarize, even given the present limited level of understanding of GFTs, it is clear that they represent a unification and a generalisation, that can perhaps turn out to be useful in the future, of both quantum Regge calculus and dynamical triangulations, together with a radical change of perspective on them: GFTs define the 2nd quantized description of the dynamics of fundamental simplicial building blocks of space, and simplicial quantum gravity path integrals arise in a perturbative definition of this dynamics around the vacuum, either when considering single virtual interaction processes, i.e. single Feynman diagrams (quantum Regge calculus), or the full perturbative Feynman sum restricted to its purely combinatorial properties (dynamical triangulations).

3.4 To whose benefit?

So far so good. All this may be interesting and indeed it is intriguing to speculate of a unifying framework for all discrete quantum gravity approaches, that encompasses loop quantum gravity structures as well as simplicial quantum gravity ones. But is it useful? Can it be helpful in solving any of the outstanding open problems that these various approaches face? Does it really offer a new perspective on them and on quantum gravity in general?

In fact, we believe it does offer such a new perspective and that because of this it can be very useful in helping to solve some of the current open problems, also by providing new technical tools for doing so. We have mentioned already some of the possibilities, e.g. the issue of the dynamics and of the definition of the physical inner product in LQG, or a possible grasp on non-trivial topologies in dynamical triangulations. However, what we have mainly in mind is the issue of the continuum limit, because it is here that the change in perspective offered by GFTs can be most relevant. We are going to discuss this issue at length in the following. Here, we limit ourselves to sketch very briefly what this change in perspective amounts to and what new tools it suggests and provides.

The change in perspective, with respect to all the other approaches we have mentioned, stems from the following consideration: *all of them, spin foam models, quantum Regge calculus, dynamical triangulations, arise in perturbative expansion around the 'no-particle fundamental vacuum', as Feynman amplitudes or Feynman diagrams sums.* This means two main things: 1) that, *from the point of view of GFTs, the discretization of spacetime used by all of these approaches in describing the dynamics of geometry, and encoded in a 2-complex (spin foams) or in a simplicial complex (simplicial quantum gravity) is not a regularization of the theory (gravity, here) in the usual lattice gauge theory sense, but corresponds to describing the physics of 'few-particles'* (be them spin network vertices or simplices) and virtual processes, with no individual meaning themselves, except in very limited and specific approximations; 2) that, at the same time, *the GFT formalism is in principle suited for going beyond this regime and describe the many-particle as well as the non-perturbative physics* of the same system, that is, unless -all- of these approaches are wrong, quantum spacetime.

Together with a change in perspective, luckily, comes therefore the possibility of using new mathematical tools and physical ideas, provided as well by the GFT formalism. This, as said, is a 2nd quantization of the same basic kinematical (space) structures used in the other approaches, and we know very well how advantageous it is to have at one's disposal a 2nd quantized and field-theoretic framework for studying the dynamics of a physical system described in terms of 'particle-like objects'. A 2nd quantized, field theory description allows: to overcome the supreme impracticability of solving the 1st quantized equations of motions involving many particles (here, very complex spin networks or extended triangulations), to deal in an easier way with the symmetries and statistics of the fundamental quanta, to have full control on quantum (e.g. self-energy) effects. Most importantly, a field theory description is the best way of: studying the properties of systems with many degrees of freedom (and, again, gravity in general, and complex spin networks or extended triangulations, are certainly examples of such systems); connecting microscopic many-particle physics and macroscopic, collective dynamics of the same, its statistical mechanics and the corresponding thermodynamical quantities. We are going to expand on this point in the following.

4. Building up a coherent picture of quantum spacetime

Once we have seen how (the basic ingredients of) different discrete approaches to quantum gravity are incorporated within the group field theory formalism, we can take a fresh look at the many important results obtained in them, regarding the classical and quantum nature of gravity and spacetime, and try to re-interpret them in the GFT language and framework. We are going to be rather brief, and possibly superficial, in our attempt to summarize in a few key points what we have learned during many years of quantum gravity research in such diverse directions, due to space (and time) constraints, as well as our limited knowledge. We apologize for this. This exercise has two purposes. 1) It may help in acquiring a new understanding of the insights the different approaches provide, and in analyzing their mutual compatibility, and possibly also suggests ways in which what we have learned from one approach can contribute to solving presently open problems of another or common to all. 2) It is needed in order to check whether a single coherent picture of quantum gravity, patching together all these various insights and results, is possible, within the GFT setting. If it turns out that, indeed, it is possible, then we believe it would be arguably the best thing to use it and develop it further.

4.1 Insights from loop quantum gravity and spin foam models

So, what have we learned about quantum gravity from loop quantum gravity [4] and spin foam models [12]? We have learned first of all that the kinematical degrees of freedom of quantum space can be captured and encoded in discrete, purely combinatorial and algebraic structures, spin network states: graphs labelled by group representations. And this applies as well to kinematical semi-classical states approximating continuum geometries. Of this space of states we have strong mathematical results concerning inner products, kinematical observables, functional properties and much more [4]. Moreover, although all this has been discovered by a direct canonical quantization of continuum classical General Relativity with Einstein-Hilbert action, we now understand this result as a very generic feature of any description of geometry based on: 1) diffeomorphism invariance and background independence, requiring a purely relational description of space, hence the purely combinatorial substratum; 2) a formulation of geometry in terms of connections (and local reference frames), i.e. a gauge-theory-like formulation of gravity, hence the use of group elements and representations to encode gravitational degrees of freedom. These are purely kinematical considerations, referring solely to the way information about space and its geometry can be encoded, to a "possible backbone"of any theory of quantum gravity, and thus may well hold regardless of specific dynamical details, e.g. choices of action, additional symmetry requirements, spacetime dimension, etc. Similar considerations apply to the dynamics of space, that can as well be represented in purely combinatorial and algebraic terms. We have learned this already from the quantization of the Hamiltonian constraint in LQG [4], but this is all the more evident in the spin foam description [12] of the dynamics of quantum space. As we have seen, we have again purely combinatorial structures (2-complexes) labelled by purely algebraic data (group representations and elements) to represent possible histories of geometry, at the quantum level. And again, this general features follow naturally from the requirements of background independence and from a description of gravity as a gauge theory, either imported from the canonical formulation, or implemented in some discrete re-formulation of lagrangian quantum gravity, or somewhat implicit in categorical quantizations of geometry, which are the main ways in which a spin foam formalism arises [12]. Recent results have confirmed that a spin foam formulation indeed is capable of describing key properties of the dynamics of quantum gravity, both in 3 and 4 dimensions, including matter coupling and graviton propagation, at least in the approximation in which the relevant spacetime geometry information can be encoded in discrete structures. For these results, we refer to the literature (see, e.g. [12, 25, 27]). And also the canonical LQG formulation of the dynamics has been shown to provide very interesting physical insights on quantum geometry, at least in the symmetry reduced context of Loop Quantum Cosmology [24].

From the overview of the GFT formalism that we have given earlier, it should be clear that all these insights are are not only compatible but also already fully incorporated in the GFT framework. In this context, they imply the following: 1) that *GFT quantum multi-particle states encode correctly quantum geometric degrees of freedom in a very precise sense, at least at a kinematical level, and satisfy the requirements of background independence*; 2) that *GFTs are also able to describe the corresponding multi-particle dynamics, at least in the approximation in which the whole perturbative series needs not be re-summed or high order Feynman diagrams can be neglected.* In particular, the results on the coupling of matter Feynman diagrams to spin foams [25] show how natural it is to treat matter Feynman diagrams on the same footing as spin foams, i.e. GFT Feynman diagrams, which is also confirmed by the corresponding GFT formulation of the same gravity+matter models [26]. And the nice results on graviton propagator in LQG/spin foams [27], using as well and in a crucial way GFT techniques, seem to us to indicate that GFTs (as LQG and spin foam models) permit first of all to re-formulate perturbative gravity questions in a fully background independent language (which it is we believe the greatest achievement, so far, of this line of work), and also that GFT perturbative particle dynamics can in fact reproduce general relativistic semi-classical dynamics in the (semi-classical, large distance and close to flat) approximation in which discrete gravity is directly applicable: GFT few particle physics, and where, in particular, GFT Feynman amplitudes reduce to semi-classical quantum Regge calculus, which indeed is at the heart of these results [27], together with LQG semi-classical kinematical states.

4.2 Insights from quantum Regge calculus

Let us then turn then our attention to what we have instead learned up to now from (quantum) Regge calculus, referring to the literature for more details [5, 28]. The main lesson, we believe, is at the classical level: Regge calculus represents a beautiful and faithful discretization of classical geometry and of its dynamics. It has been shown, in fact, that classical Regge calculus reproduces General Relativity in the continuum approximation in at least two main ways: 1) the Regge action approximates well the Einstein-Hilbert action (and the correspondence generalises to higher-derivatives extensions of the same) in the sense of measures, and 2) solutions of the linearised Regge equations converge to analytic solutions to the linearised Einstein's equations, when some appropriate conditions are met. Even more confidence in the correctness of the Regge discretization of classical geometry stems from the possibility of identifying characteristic symmetries of continuum gravity in the simplicial setting, including diffeomorphisms, when appropriately defined, as well as the related discrete Bianchi identities (but see, on this, [30]. In the GFT language, this can be re-phrased by saying that, *for GFT models that possess Feynman amplitudes of the form of simplicial path integrals for (some version of) the Regge action, or in the approximation in which such form is obtained, there is evidence that the "classical dynamics"of the GFT particles can correctly reproduce relevant features of classical gravity, including symmetries, and better and better the more GFT particles we consider.* This already hints at the relation between continuum geometry and the thermodynamic limit in GFTs (large number of particles), on which we will say more in the following.

At the quantum level, the results are also interesting [5]. In particular, in the semi-classical, large scale, and flat approximation, quantum Regge calculus reproduces very well the graviton propagator and thus Newton's law, plus quantum corrections, even for simple triangulations. It is quite natural to expect this to be the case also in GFTs with a simplicial path integral form of the Feynman amplitudes, and indeed the mentioned results on the spin foam propagator of the lattice graviton seem to confirm it, while at the same time confirming the correctness of the choice of boundary states operated in that context. Many other results concern matter coupling, quantum cosmology, etc. As for the definition of the full gravitational path integral in quantum Regge calculus, the situation is more controversial, and much debate in particular has focused on the issue of the quantum measure to be used [5]. More precisely, the object of interest is the continuum limit of the discrete path integral defined by Regge calculus, on which there are interesting but not fully conclusive results [28], and about which we will say more in the next section. As explained above, this discrete path integral is nothing more (for specific GFT models, or in special limits of the same) than the GFT Feynman amplitude for a particular interaction process of GFT quanta.

4.3 Insights from matrix models and dynamical triangulations

In matrix models for 2d quantum gravity and in their higher-dimensional extensions, i.e. tensor models, as well as in the strictly related dynamical triangulations (DT) approach [17, 6], the goal is to obtain a consistent and computable definition of the gravitational path integral, i.e. of the sum over geometries for given spacetime topology, with some results being obtained also on the limited extensions of the same to non-trivial topologies. As such, the classical simplicial geometry is of limited interest, and indeed it cannot be fully captured by the approach due to the truncation of the geometric degrees of freedom associated to the individual lattices. The classical *continuum* geometry, on the other hand, is possibly reproduced to the extent in which the DT partition function reproduces the gravitational continuum path integral. *In GFT terms* this is easily understood, as it this means that, *once one has frozen the field degrees of freedom, the classical particle dynamics* (classical simplicial gravity) *cannot be reproduced in a satisfactory manner, but at the same time the continuum field dynamics* (continuum quantum gravity) *could still in principle be reproduced, at least to the extent in which the truncated sum over Feynman diagrams, restricted to its combinatorial properties, reproduces properties of the full field partition function.* Therefore, all the many results obtained in this approach refer to the continuum approximation of the discrete gravitational path integral defined as a sum over triangulations, and we defer their discussion to the next section. Here we limit ourselves to notice that work in matrix models and dynamical triangulations has resulted in an immense amount of results and available tools, both analytical and numerical, in an almost complete understanding of 2d quantum gravity with a nice discrete-continuum correspondence, in both Riemannian and Lorentzian cases, and in important results obtained recently for higher dimensions concerning this discrete-continuum correspondence, in the Lorentzian context of so-called *causal* dynamical triangulations [6].

5. The problem of the continuum: current strategies from a GFT perspective

Given our favorite formulation of quantum gravity, using discrete structures of some sort to describe spacetime and to encode quantum geometric degrees of freedom, does it reproduce, in some controlled and well defined approximation, a smooth spacetime, and is the quantum dynamics of spacetime geometry effectively approximated, in the same regime, by continuum General Relativity, possibly modified by quantum effects? This is the problem of the continuum in quantum gravity, for how we see it. And this is, in our opinion, *the* outstanding unsolved issue that all the current approaches to quantum gravity, and certainly the ones we have mentioned, loop quantum gravity and spin foam models, quantum Regge calculus, dynamical triangulations, have to tackle hard and solve, to be considered successful. The same, of course, is true for group field theory. The importance of obtaining a satisfactory understanding of this issue cannot be overstated, we believe, as it would amount to showing that our favorite formalism, whatever it is, does indeed provide at least *one possible* quantum theory of gravity. In absence of such result the connection with gravity would remain a (more or less plausible) hypothesis, and, as stressed, for example, in [29], any interpretation of the discrete expressions one has in terms of quantum spacetime structures can be taken only as a suggestion, *before* a physically correct continuum approximation to them has been found. *The group field theory formalism, in the perspective we are proposing, can offer new tools to solve this issue* to each of the different approaches it (potentially) subsumes, and at the same time capitalise on their results and insights. However, we believe that *it also calls for a change in perspective and for a consequent new strategy.* We will be arguing in this direction in the next section; here we would like first to briefly overview the strategies currently adopted within the other approaches, all of course sensible and potentially successful, and then “translate”them in the GFT language, since this translation will make clear why a change in perspective and strategy is naturally suggested.

5.1 The loop quantum gravity/spin foam strategy

Research on the semi-classical and continuum approximation in loop quantum gravity and spin foam models has been mainly carried out, at least in the 4-dimensional setting, in the canonical formulation and is mainly confined to the kinematical setting[1] The starting point of the LQG/SF strategy (using $SU(2)$ spin networks and related observables) for recovering continuum physics is the construction of appropriate kinematical quantum states of space which approximate continuum space geometries in some sense. The first type of such semi-classical/almost continuum states are

[1]The exceptions, that may come to mind, are the many results in Loop Quantum Cosmology [24], and the recent progress on the spin foam calculation of the lattice graviton propagator. However, the first apply to symmetry reduced situations, where it is possible to encode *all* the (finite number) degrees of freedom of the continuum theory in the discrete spin network structures. The second is limited to perturbative physics *around a semi-classical space geometry*, first of all, but, more important, remains confined at the level of (justified) discrete approximations and large scale information, thus not really addressing the issue of the continuum in this framework.

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the so-called "weaves"[31, 32]. These are defined by a (directed) graph embedded in a reference compact space Σ, the links are dressed with holonomies of an $SU(2)$ connection in the representation $j = 1/2$, with appropriate intertwiners labelling the vertices of the graph. This graph is taken to be a huge collection of loops in Σ, uniformly distributed with respect to some classical 3-geometry h_{ab}. The mean spacing between the loops (akin to a sort of lattice size for the graph) is of the order of the Planck length l_P. This means that the number of loops is approximately $N = (\frac{L}{l_P})^3$ where L is the distance scale corresponding to the volume region one is interested in approximating, measured in the reference metric h_{ab}. The observables considered as a probe of the semi-classicality of our quantum state are areas of surfaces in Σ and 3-volumes of regions contained in it. The nice result is that for large enough volume regions, the areas and volumes as computed quantum mechanically on the weave state are very close to the ones measured in the classical continuum metric h_{ab}, and with very small uncertainties. In this sense, one can say that the quantum state considered has a good continuum and semi-classical approximation [31]. This type of construction can be extended to consider random weaves and averages over ensembles of graphs, using statistical techniques [32]. A different type of improvement of this construction is to change test observables [33], using for this scope the basic canonical pair of variables of loop quantum gravity, i.e. triad and holonomy operators. The resulting quantum states are then semi-classical *coherent states* providing expectation values for both of them that are close to the classical continuum values, as well as minimizing the uncertainties of both, in an appropriate sense. The resulting quantum states are then an even more satisfactory approximation of continuum 3-geometry, and many nice results can be proven for them [33] (overcompleteness, Ehrenfest properties, etc). Notice however that we are still confined at the kinematical level, while what we are really interested in reproducing, starting from out quantum gravity formalism, is the continuum *dynamics* and the *spacetime continuum*. The way to do this test in the Hamiltonian/canonical setting would be to study the action of the Hamiltonian constraint of the theory on these weave or coherent states. This is extremely complicated, due also to the intrinsic complications involved in the very definition of the Hamiltonian constraint operator, and has not been done, to the best of our knowledge. More work has been devoted recently to the spin foam formulation of the dynamics, so maybe one would want to use these weave or coherent states in that context. It has not been done, yet. The general idea however would be to use the above semi-classical/almost continuum states as boundary states for an appropriate spin foam model and compute the quantum gravity analogue of 2-point functions between two of them; the spin foam amplitudes would impose the quantum dynamics and the result should then be compared with continuum path integral calculations[2]. The calculation could be done for fixed spin foam 2-complex, but most likely should involve a sum over spin foams, that could then be truncated because of some physical requirements. One way to define such sum would be through the corresponding GFT formulation, with the GFT here used only as a auxiliary tool, devoid of physical meaning, for generating the sum over 2-complexes. All this is possible and sensible. However, notice the orders of magnitude that would be involved, generally speaking, in such calculations: if we aim at reproducing continuum physics over a scale of, say, $L = 10^{-19} cm$ (the distance scale of a quark), we would need boundary states, in our spin foam calculations, that are weaves with about $N = 10^{42}$ loops, or, which is arguably the same, spin networks with a similar number of vertices. The complexity

[2] One would also have to compute observables other than 2-point functions, but this does not alter our argument.

of the spin foam complex would go accordingly. It is not obvious that such a calculation would be doable, and at the very least we are lead to look for some alternative, more efficient procedure.

Let us look at the GFT translation of the same procedure, taking the GFT formalism to be physically meaningful in itself and not just a mathematical tool, and see how the above sounds like. In the GFT language, interpreted *realistically*, the procedure would then be the following: 1) consider a carefully chosen multi-particle state of a given GFT (a quantum field theory) corresponding to a wave function satisfying some carefully specified conditions (with respect to your favorite choice of test observables); this state should contain about $N = 10^{42}$ GFT quanta, say; more precisely you should consider two of these states, one per boundary in a typical "scattering"process; 2) construct the corresponding field observable and insert it in the GFT partition function; 3) expand the GFT partition function in perturbative expansion around the vacuum state (i.e. the state with no GFT quanta), i.e. in Feynman diagrams; these Feynman diagrams give all the possible virtual interaction processes of the 10^{42} initial and final particles, including all quantum loops, self-interactions etc.; even for the simplest diagrams (e.g. tree level and next to tree level), their complexity will be of the same order of and scale with the complexity of the boundary states; 4) compute the transition amplitude in this Feynman expansion, maybe truncating the expansion to some given order in the GFT coupling constant (notice that the needed order would be necessarily extremely high).

The strategy is not wrong, in any sense, but it definitely does not look like what one would naturally do to study the physics of such hugely populated multi-particle state in a field theory context. The basic point is that, *when we choose as our system of interest a hugely populated particle state, we put ourselves immediately in the situation in which the vacuum no-particle state and its physics is not relevant, the Feynman diagrams of the individual particles are not relevant, in a sense the microscopic dynamics itself is not relevant anymore*[3]. *In any case, the Feynman diagrammatics and the individual particle picture is not the most convenient language to describe the relevant physics of these states.* We are lead to look for an alternative.

5.2 The quantum Regge calculus strategy

In quantum Regge calculus[5, 28], the theory is defined by the Euclideanized (or statistical) discrete gravity path integral on a fixed lattice (most often hypercubic, then subdivided into simplices) T (thus also for fixed topology, usually the sphere or the torus):

$$Z_T = \prod_e \int \mathscr{D} l_e \, e^{-S_{Regge}(l_e)}, \tag{5.1}$$

where e labels the edges of the lattice, l_e are the corresponding edge lenghts, which are the fundamental variables, integrated over with some measure $\mathscr{D} l_e$, and the most studied version of the discrete action, in 4d, is the Regge one augmented by quadratic higher derivative terms (a discretization of the Riemann tensor squared):

[3]One can of course be more optimistic and hope that a smooth continuum spacetime arise, and a general relativistic description of it, holds already, say, for distances 100 times the Planck length; this will make the number of needed particles $N = 10^6$. The numbers are then vastly different but the result is the same: for this number of quanta, the direct solution of the corresponding microscopic dynamical equation for wave functions or the study of their dynamics via Feynman expansion around the vacuum are at best unpractical and possibly even conceptually mistaken. If such a lucky situation occurs, it would simply mean that already at the order of 10^6 particles, we are free to take the limit $N \to \infty$.

$$S_{Regge}(l_e) = \sum_t \left(\lambda V_t(l_e) - k A_t(l_e)\varepsilon_t(l_e) + a \frac{(A_t(l_e)\varepsilon_t(l_e))^2}{V_t(l_e)} \right), \quad (5.2)$$

where the sum runs over the triangles of the 4d simplicial complex, A_t are their areas, ε_t the associated deficit angle (discrete curvature), and V_t is the contribution of the given triangle to the total 4-volume of the lattice [5, 28, 30]. The partition function is then a function of the coupling constants λ (cosmological constant), k (inverse of Newton constant), and a. The integration over the edge lengths is usually cut-off both in the IR and UV, to ensure convergence.

Studying the continuum approximation of this theory means studying the above partition function and appropriate geometric observables (average curvature, average square curvature, etc) for very large simplicial lattices (often at fixed total 4-volume) in a scaling limit, while removing the cut-offs, as a function of the coupling constants. The aim is to show that in a region of the parameter space the above reproduces continuum spacetimes and continuum geometric observables, thus representing a good (regularised and computable) substitute of the formal continuum gravity path integral. As said, this analysis has been done exclusively for statistical path integrals over euclidean geometries, and mainly numerically. The main results are the evidence for a two-phase structure: for a certain k_c the average curvature vanishes; for $k > k_c$ (small G_N) the simplicial complex degenerates into a crumpled phase incompatible with a smooth geometry with simplices of very small volumes and large curvature; for $k < k_c$ there is instead evidence for a smooth phase, depending also on the value of a and λ, with small (and negative) curvature. See [28] for more details. There is evidence for a second order nature of the phase transition [28], which is what one needs in order to have long-range correlations, but this evidence does not seem to be considered fully conclusive by the community (see, e.g. [30]). The result seems to be rather generic, i.e. not too strongly dependent on the specific measure $\mathscr{D}l$ chosen or on the specific topology or lattice structure chosen, even if for irregular lattices the phase structure is more involved (more critical points) and singular structures seem to appear (spikes) and the choice of measure becomes more important. In the end, we cannot yet conclude whether this approach reproduces continuum physics or not, but we definitely have gained lots of insights in the properties of similar discrete gravity path integrals, and many tools to analyze them have been developed.

Once more, this does not look like the most natural procedure to adopt to study the continuum approximation of the same structures when embedded and re-interpreted in a GFT context. The discrete gravity path integral on a fixed lattice, in fact, amounts to the evaluation of a single GFT Feynman amplitude for a given interaction process of the GFT quanta, and all the lattice prescriptions used in quantum Regge calculus require a Feynman diagram with about $10^3 - 10^4$ vertices of interaction (numerical simulations have been performed with up to $16^4 \sim 6 \times 10^5$ lattice size, and the continuum approximation is expected to be only improved going to larger lattices). *Such a huge Feynman diagram computation would indeed capture some information of the many-particle physics of the corresponding GFT, which is again suggested to be the regime corresponding to continuum gravity, but the truncation to a single Feynman diagram is most likely not consistent within the GFT setting. Moreover, just as in LQG, it seems that to study the many-particle dynamics of the theory at the level of perturbative expansion around the vacuum is definitely not the most convenient thing to do.*

5.3 The dynamical triangulations strategy

In the traditional euclidean triangulations programme, the theory is defined by the partition function:

$$Z = \sum_T \frac{1}{C_T} e^{-S_{Regge}(l,k,\lambda)}, \tag{5.3}$$

i.e. by a sum over *equilateral triangulations* T, at fixed topology (usually the spherical one), with fixed edge length l (which is interpreted as a cut-off), weighted by a symmetry factor (the automorphism group of the triangulation, C_T, and a euclideanized exponential of the same Regge action usually limited to a cosmological and a curvature term, thus in the end a function of the combinatorics of the triangulation only and of the parameters l,k,λ. The continuum approximation involves again evaluating explicitly this sum $Z(\lambda,l,k)$ or, more precisely, its Legendre transform $Z(N_4,l,k)$ which corresponds to work for fixed number of 4-simplices N_4 and thus with fixed 4-volume $V \sim l^4 N_4$. Having done this, one is interested in the thermodynamic limit $N_4 \to \infty$, $l \to 0$, $V \sim$ constant. Simplifying a bit, the resulting phase structure was found to be given again by two phases separated by a critical value of k, k_c, depending ont he volume N_4. For $k < k_c$ we have a crumpled phase characterised by small curvature, high graph connectivity and very large Hausdorff dimension. For $k > k_c$ one finds an elongated phase with large and positive curvature and an effective branched-polymer geometry with effective Hausdorff dimension equal to 2. One could still hope that a continuum theory is defined at the transition point, if the transition was second order, but further analysis (again not fully conclusive) suggested that the transition is instead first order. For more details and further references, see [30].

The situation changes drastically in the modern form of this approach, the so-called *causal dynamical triangulations*, to the point that one can even make the case [6, 29, 30] for the *origin* of the troubles encountered in the euclidean dynamical triangulations, as well as, to some extent, in euclidean quantum Regge calculus, in finding a good continuum approximation to be the dominance of pathological configurations such as baby universes and other types of singular geometries. These configurations are basically unavoidable in the euclidean setting. They are not so, however, in a Lorentzian one, where instead one can indeed identify conditions on the triangulations summed over that rule out them from the start (i.e. by construction). This is what is achieved in the causal dynamical triangulations approach. Here the basic ingredients for the construction and definition of the triangulations summed over are (see [6] for more details): 1) a local light cone structure, i.e. a differentiation between spacelike and timelike edges (which have a relative proportionality factor Δ for their values, on top of the difference in the sign of their square); 2) the existence of a global discrete time function; 3) no spatial topology change allowed with respect to this 'time'structure. The triangulations are then weighted by a complex exponential of the same Regge action but now for Lorentzian simplicial geometries. The results are striking [6]. There are now three phases: a) for large k a phase characterised by 3-dimensional slices of a branched-polymer type, so not a 4d smooth geometry, once more; b) for small k and small asymmetry parameter Δ a phase with crumpled 3-dimensional slices, similar to the euclidean setting; so, again, not a smooth 4d geometry; c) for sufficiently small k and sufficiently large Δ, a stable, extended 4-geometry, with Hausdorff dimension equal approximately to 4 and a global shape of spacetime related to a simple minisuperspace model of gravity, similar to those used in quantum cosmology. This is strong and exciting evidence for a smooth geometry and thus a continuum limit, even though several features of the model itself and of the resulting dynamics of geometry are yet to be understood, such as whether the results are robust with respect to limited extensions of the ensemble of triangulations considered, how much exactly of the full dynamics of general relativity is recovered in the continuum approximation, whether there is a way to generate analytically the above sum over triangulations, that is at present constructed algorithmically and only studied numerically, etc.

How does the GFT translation of the above sound like? In the GFT language, the above corresponds to the following: 1) consider a specific GFT model, producing Feynman amplitudes with appropriate exponential form (either real or complex) for a discrete gravity action (with field theoretic data interpreted as either euclidean or lorentizian discrete geometries); 2) fix all the field theoretic data, e.g. the momenta of the GFT field to some constant value, giving then equilateral triangulations dual to the GFT Feynman diagrams (producing the parameters l and, in the causal case, Δ); this corresponds to restricting to a specific momentum regime for the GFT particles, i.e. to particles all having the same momentum; 3) restrict the perturbative sum over Feynman diagrams to only those diagrams of some given topology (and further restrictions have to be in place to recover the causal restrictions of [6]); finally, perform the *whole* sum computing in this way the corresponding restricted sector of the theory partition function, and appropriate observables. Once more, we see that *one necessarily needs to study Feynman diagrams or arbitrary combinatorial complexity, and involving huge numbers of GFT quanta, supporting further the idea that continuum physics corresponds to the many-particle physics of the theory.* Importantly, the work on dynamical triangulations provide lots of technical tools for studying it. *The CDT results*, moreover, *seem to indicate that, at least in that regime of the GFT, some continuum physics can indeed be captured in satisfactory form by this procedure*, which is exciting indeed. However, once more the above procedure seems not so convincing from the GFT perspective (that of course one is free not to take): first of all it is well possible that the DT and the CDT restrictions at the level of GFTs are not consistent from a field theory perspective. Here we are not so much concerned by the restriction on the momenta, which may well simply correspond to a particular sector of the GFT, and thus to a reasonable approximation of the full theory. Rather, what may be more problematic is the restriction to fixed topology and, in the CDT case, to fixed slicing structures of the diagrams summed over. We have a too poor understanding of the GFTs themselves [34] to specify what these restrictions amount to, from a purely field theory perspective. For example, we may run into problems in asuming these restrictions, if they do not clearly amount to a classical limit, say, as for example the large-N planar limit of matrix models, and still involve removing the GFT analogue of quantum loops or the like. Modulo these remarks, *it is clear that the (C)DT restriction does indeed amount to extract at least some non-perturbative information far beyond the physics of the GFT vacuum state, i.e. the few particle physics, so it is a sensible thing to do even within the GFT setting; in practice, in fact, amounts to solving the theory (computing the partition function) at least in a restricted sector, which may well turn out to be the one in which continuum gravitational physics lies.*

However, the same doubt put forward concerning the other approaches applies: *how convenient is it to study the non-perturbative many-particle physics, and the corresponding vacuum state and its dynamics, using what remains a perturbative expansion around the no-particle vacuum, encoding the the many-particle dynamics in hugely complicated Feynman diagrams, and then*

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re-summing all of them? Again, the GFT perspective calls for the use of different tools and for a change in strategy.

5.4 Lessons and further motivations

Let us summarize briefly the outcome of this sketchy overview. First of all, *all these strategies and approaches do teach us something about GFTs,* when embedded in it. Second, *all of them suggest or strongly support the view that continuum physics corresponds to the many-particle sector of the GFT formalism*, and most likely involve collective and non-perturbative effects. Third, *their translation in the GFT language suggests that maybe, although we have been indeed studying the relevant sector of the (GFT) theory, we have not used the most convenient set of tools* and language for doing so. Fourth, luckily enough, *the GFT formalism potentially provides us with all the non-perturbative, field-theoretic tools and concepts for trying out a different strategy.* As we had stressed, in fact, we know from condensed matter physics and statistical physics that field theory and 2nd quantization language are the most convenient ones to study many-particle physics, the corresponding phases, collective behaviour, etc.

6. Quantum spacetime as a condensed matter system

Let us now give some more specific suggestions for what this GFT perspective seems to imply. We will put forward an hypothesis for the continuum phase of a GFT, i.e. the phase or regime of the theory in which we expect continuum gravitational physics to be reproduce, and some general hints at what the strategy to check this hypothesis could be. The general idea for the above will be to take GFTs seriously for what they (formally) seem to be, at least as a working hypothesis, and consider them as the microscopic description of a very peculiar condensed matter system, which is quantum space. In other words, *we will consider the GFT quanta*, that can be pictured, as we have seen, as spin network vertices or (D-1)-simplices, *as the true "atoms of quantum space", its fundamental hypothetical constituents, and the GFT formalism as the microscopic (fundamental?) formulation of their quantum dynamics,* thus described in terms of a peculiar (non-local, etc) quantum field theory, but a quantum field theory nonetheless. Then, we will broaden the discourse a little and try to summarise some of the general insights that, once we have taken this standpoint, come to us from condensed matter physics (and from condensed matter analog gravity models).

6.1 If GFT is its microscopic description, what is the continuum and how to get there?

We have seen that all current approaches seem to suggest that continuum gravitational physics is obtained in what is, in the GFT language, the (very) many-particles sector of the theory. This perfectly match the working hypothesis of GFTs as the microscopic description of the atoms of space. In other words, we most likely need a *very large number* of them to constitute a region of space that can be governed effectively by continuum gravitational physics, and be described by a continuum space to start with.

Moreover, from results in these approaches, as well as from general physical intuition and, again, from the perspective of spacetime as a "material"of some sort, made of (GFT) constituents, we would expect these many constituents making up the continuum to be *very small,* in the appropriate sense, probably of order of the Planck length (volume). In GFT terms, generally speaking, this translates into the *GFT quanta to be in a low momentum regime*. On top of this, we expect the quantum constituents of space to be *governed, in their continuum phase/regime, by collective dynamical laws*, not anymore by the microscopic individual dynamics, simply because otherwise we would have noticed already the “true”atomic nature of space. Finally, whatever the exact phase looks like, whatever the symmetries characterizing it are, and whatever the effective dynamics governing it is, we expect our condensed matter system, i.e. quantum spacetime, modelled by our favorite GFT, to be *very close to equilibrium*. In other words, we expect a continuum description of spacetime to prove itself correct, and not only possible, when close to an *equilibrium* and stable vacuum/phase of the (GFT) system, at least to scales close to the sector of the physical world that has already probed (by us). Again, this is simply because otherwise we would have most likely already noticed a failure of the continuum description of spacetime. From this perspective, the breakdown of general relativistic theories of geometry in cosmological situations or in black hole physics can be *speculated* to be a sign of a phase transition occurring in the (fundamental?) GFT system.

Notice that none of the above implies that the continuum approximations goes necessarily in hand with the semi-classical approximation, which may be needed later on to simplify/extract a specific dynamics or for capturing some relevant features of the system in the regime we are interested in, but as far as the above reasoning is concerned, the possibility of *a continuum description may even be the result of a purely quantum property of the system*. We will give later on an explicit proposal for this.

So, *a continuum space is a very large number of very small GFT quanta very close to equilibrium, i.e. very close to some yet to be determined many-particle vacuum, to be described collectively and whose dynamics is to be given by continuum larger-scale equations*. This seems (to us, at least) just a description of a fluid (whether gaseous or liquid or what else, is to be determined by hard, technical, future work), close to equilibrium, governed indeed by hydrodynamical equations.

The picture that seems to come out of the above reasoning, then, and more indirectly (we admit that) from work in the various approaches to discrete quantum gravity we have discussed is that of *quantum spacetime as a (quantum) fluid of GFT particles, governed microscopically by the GFT partition function, but macroscopically by a suitably identified GFT effective hydrodynamics.*

As we had stressed, this is at present just a suggestion, of course, given the little we understand GFTs themselves and the (basically nihil) amount of work that has been devoted up to now to develop and test it. But we find it a very intriguing and, most important, convincing one. It immediately implies one thing: at least for a while, at least from the GFT standpoint, and only if we intend to tackle the issue of the continuum approximation and its effective dynamics, it may be convenient to partially forget about spin foams and even the simplicial gravity description of the GFT system, and focus our attention on other aspects of the formalism. This is simply because, as we have stressed, the perturbative formulation of GFTs, which is where the spin foam and the simplicial gravity descriptions appear, is very useful for the physical interpretation of the system, of its quanta and field theoretic data (indeed, we have relied exclusively on it for all of the above reasoning), but it is *technically* useful for describing the system in its few-particle regime. If we are interested in describing the many-particle behaviour of the same system we should move away from the no-particle vacuum.

In its stead, we need to develop first and then use a *statistical group field theory* formalism for identifying first and then select the different phases of the theory, i.e. the possible equilibrium configurations in which the system may find itself, hoping that some of the GFT models we have or we will construct for the scope allow for the existence of at least one with the properties that allow for a continuum geometric description. Second, we need to obtain an *effective field theory or hydrodynamic description*, coming from the fundamental GFT, for describing the dynamics close to the different phases, and probably tied to each particular phase under consideration. We will speculate more, but also try to be more specific, about how both may look like in the next section.

6.2 What can quantum gravity learn from condensed matter theory and analogue gravity models

The idea of spacetime as a condensed matter system in general, and as a fluid in particular, and of GR as an hydrodynamic effective description of it, is of course not new and has been advocated many times, and very convincingly, in the past [35, 36, 23, 37, 38], and is both motivated by and an inspiration for the many condensed matter analog gravity models [39]. What is new here is only the argument that it is the very research in non-perturbative quantum gravity carried out to date, and the many results obtained in the many approaches it is split into, that points in this direction. Also, what is new here is the hypothesis that GFTs can represent: 1) the framework in which these many approaches to quantum gravity and their insights can be seen as part of a single coherent formalism and physical picture of spacetime; 2) also because of this, a solid and motivated formalism to be used to realise concretely, in mathematical and physical terms, the suggested idea of spacetime as a condensed matter system of a peculiar type, and a concrete, if tentative only, description of its microscopic structure. This description, moreover, as we stressed repeatedly, uses a field theory language that may facilitate the application in this context of traditional condensed matter ideas and tools (probably suitably adapted).

This is probably the main contribution that GFTs can provide researchers working in condensed matter analog gravity models: *a concrete formalism and system* on which to apply their insights, if they are interested in unravelling the true microscopic structure of quantum spacetime, and not only in finding out more about its effective continuum description, once interpreted as a condensed matter system, or in using the same gravitational analogy and the general relativistic tools to discover more interesting properties of the usual condensed matter systems (Bose-Einstein condensates, etc)[4].

In other words, it is often stated in the analog gravity literature and in the condensed-matter-but-interested-in-gravity community that [36, 23, 39]: 1) quantum gravity is not so much about quantizing general relativity in a strict sense, but rather about identifying the microscopic constituents of space and provide a tentative description of their microscopic dynamics; 2) we do not know what this microscopic structure and dynamics is; 3) the current top-down approaches to quantum gravity are so different and so complicated that no coherent picture and no clear indication about the fundamental structure of space is provided by them, that could serve directly for the application of the insights coming from condensed matter theory. What we have argued is the following. The first thing is true, and *it is the very same approaches to non-perturbative quantum gravity that*

[4] Needless to say, both things are definitely worthwhile and of fundamental significance; simply, they are not quantum gravity issues.

have lead (in a rather tortuous way) to the GFT formalism which itself is not a quantization of classical GR (just look at the GFT action). The second statement is true in a sense, *we do not have a clear and complete picture of the spacetime microscopics*, but false or at least overly pessimistic in another: *we have several candidates for this microscopic structure, and one, the GFT formalism, that seem to encompass many of them*. The third statement is false: *the respective pictures that at least some of these approaches to quantum gravity* (those we have discussed) *provide are not only compatible and coherently build up a tentative picture of quantum spacetime* (the one encoded in the GFT formalism), *but also one that allows for a rather direct application of condensed matter concepts, formalisms and techniques for understanding the microscopic-macroscopic and discrete-continuum transition.*

What quantum gravity, and in particular the GFT approach, can learn from condensed matter (CDM) physics and from condensed matter analog gravity models is much more.

Concretely, the main help that condensed matter techniques can provide stems from the fact that *in that context*, as stressed in [39], *the transition from discrete microscopic physics and continuum macroscopic one is well understood* conceptually and there are many theoretical tools that can be applied to its analysis and study. As we have seen, this is the main open problem of the discrete quantum gravity approaches have to solve, even after they have provided a tentative description of microscopic spacetime. This holds for GFTs as well, and its field theory setting makes the application of CDM techniques even more straightforward. More generally, taking a CDM perspective means also a conceptual shift with respect to what we expect from our theory, and how we approach our physical challenges. We list here only some of the CDM wisdom (for more, see [36, 23, 37, 41, 39]), that is useful for approaching our quantum gravity problems, in our opinion. We should not expect a rigorous, deductive path from the microscopic dynamics to the macroscopic one, and even the kinematics (relevant variables, symmetries, etc) at the macroscopic scale or in the 'continuum phase', thus in the hydrodynamic regime, can be very loosely related to the one of the corresponding microscopic theory. In other words, even the relation between microscopic variables and collective ones is often less that direct, and the specific form of the microscopic QFT for your atoms is often not at all similar to the macroscopic effective QFT for the resulting fluid. In particular, many of the small details of the microscopic theory become irrelevant at the hydrodynamic effective level. This is governed mainly by general macroscopic symmetries and associated conservation laws, that should acquire thus a fundamental importance in our model building. It is not reasonable, in light of the above, and at least if one is first of all interested in showing that a continuum approximation exists, to demand necessarily for exact treatments or to look for exact solutions of microscopic dynamics, because this exactness will almost inevitably end up being irrelevant at a different (larger) scale. All this should apply to our future treatment of the GFT formalism, in our attempt to use it to obtain the correct macroscopic effective continuum description. Nothing revolutionary here, of course, but things that are worth keeping in mind in quantum gravity research, and in particular when one sees spacetime as a condensed matter system, because they are often neglected (by us, at least). Also, we are warned that experimental is absolutely crucial for guiding model building and for guessing what are the relevant features in the hydrodynamic regime, thus at the effective level. The recent development of quantum gravity phenomenology [40] it therefore of extreme importance, also in this condensed matter interpretation.

At the same time, as stressed very nicely in [41] (see also [37]): "The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear". This is a warning but also an encouragement because it implies richness and potential fun in unravelling it.

7. Guessing the future: several research directions, an hypothesis and some speculations

The above discussion has been very general, serving only the purpose of sketching what are further inputs to the GFT perspective on the continuum we are advocating, again, as a working hypothesis. Now we will try to be a bit more specific about how one can develop further and what may come out of this condensed matter perspective, in concrete terms, in the GFT framework. We will put forward one specific proposal for what can be the phase of the GFT, i.e. the relevant vacuum for the GFT multi-particle physics, where continuum geometry and its dynamics could be reproduced, and then explore, tentatively, some possibilities for the dynamics of the theory in this phase, and how it can relate to known formulations of classical continuum gravity.

It should be clear that, given our present understanding of the GFT formalism, any guess in this direction can be only partially based on known results, but rather speculative. The study of the GFTs in their own right, treated as peculiar but *bona fide* field theories, is in its infancy and only the first basic steps have been or are being taken [11]. Nevertheless, they already provide some hints of what may come next, and we are going to build upon these hints in the following.

Before we do so, let us mention three other directions of work that, in the perspective we are advocating, are certainly relevant (see also [34] for a more detailed discussion). One if the development and use of renormalization group techniques. The renormalization group is in fact one of the most powerful tools we have in field theory and in condensed matter physics to explore the structure and behaviour of our system at different scales. It is indeed applied routinely in condensed matter for investigating phase structures, which is exactly what we have argued we have to do in our GFTs. In particular, we believe that it would be very important, and of great direct relevance for solving the problem of the continuum, to develop the formalism of the Wilsonian Exact Renormalization Group for group field theories, with the construction of the effective action and the analysis of the corresponding flow, for specific GFT models. This would not only prove the consistency of the given models (renormalisability, etc) but also suggest what is the relevant form of the theory (action) at the scales we expect to be related to continuum physics. A second one is the study of classical solutions of the GFT equations. Of course, they encode non-perturbative information about the system, and thus are also relevant for the continuum phase. This work has started [42]. However, we would like also to stress that, from a condensed matter point of view, it may be even more important to construct *approximate* solutions to the GFT dynamics, tailored to the multi-particle situation. The third, and maybe most important, is the analysis of the GFT classical symmetries, to be done both at the lagrangian and hamiltonian level [11]; this is because, as stressed, macroscopic behaviour and hydrodynamics in particular are likely determined more by these symmetries, or their broken version, than by the exact microscopic GFT dynamics.

7.1 Geometrogenesis using GFTs

Our proposed general scheme for the emergence of continuum geometry from the dynamics of the GFT quanta can be seen as a particular possible implementation of the *geometrogenesis* idea.

This is the catchy name given in [43] to a conjectured phase transition of a combinatorial and algebraic model of quantum space described by a a labelled graph, much alike spin networks, between a high-temperature 'pre-geometric phase'in which space has the form of a complete graph, and thus no notion of locality or geometry (e.g. distance), to a 'geometric phase'in which the graph acquires a more regular, local structure, where geometric data can be identified. Furthermore, the data labelling the graph then allow for the emergence of matter degrees of freedom, having the role of qausi-particle moving on the resulting regular lattice, in the same way as the model of topological order studied by Wen et al [44] does, in terms of string condensation.

Now, the details of the model do not concern us here. We just want to note the similarity with the idea we are proposing for the emergence of the continuum in GFTs. The basic quantum states of the GFTs, as we have seen, are characterized by labelled combinatorial structures as well, of the spin network type (or, dually, of a simplicial type). It seems to us that because of this, any phase transition in a GFT setting will be described by a transition from some irregularly structured and labelled graph or from an ensemble of such graphs to a more regular and ordered one at lower temperatures, in the same spirit as the model of [43]. Further, we are suggesting that after the ground state has been identified its own effective dynamics will be described, if the scenario we are suggesting is correct, by an effective continuum field theory with a geometric interpretation, and in principle derivable (but not necessarily deducible) from the microscopic GFT. Both the hamiltonian function driving the transition, and thus the selection of the ground state lattice, and the effective hamiltonian governing the dynamics of quasi-particles around the resulting ground state, the two main ingredients of the model in [43], can in principle be derived from any given choice of GFT action, whose dynamical content is indeed the same, after appropriate simplifications. If our understanding is correct, then, the model of [43] can be interpreted as an effective simplified GFT Hamiltonian, and similar models can be constructed and inspired by the GFT formalism as well. Conversely, we believe that more work in the direction opened by the model [43] will be of importance also for the research programme we are suggesting, in that it will amount to explore models that may indeed capture relevant features of GFT phase transitions and vacua as well.

7.2 Continuum space as a GFT Bose-Einstein condensate

Our tentative proposal for a relevant vacuum of a GFT model in which a continuum approximation could be expected, i.e. a continuum and geometric phase of the model, is a simple one: a Bose-Einstein condensate. Again, here it is not so much important the idea in itself, because the similarities between continuum spacetime and condensates have been noticed long ago and a similar possibility has been advocated by several authors, and very convincingly [36, 23] and the effective (and emergent) spacetime character of real Bose-Einstein condensates (those stored in laboratories) is the basis of many condensed matter analog gravity models [39]. What is important here is the fact that the concrete realization of this scenario within a specific microscopic model of quantum spacetime, i.e. a GFT model, seems to us not only possible, but within reach. Of course, such scenario involves first of all the development of a statistical group field theory formalism, the identification of the GFT analogues of relevant thermodynamical quantities, and more, and, as we have noticed above, even basic steps in the analysis of GFTs apart from their Feynman amplitudes have been taken only recently [11]. We will now sketch, also based on these initial results, how thermodynamical quantities in a GFT setting could be defined and then how the possibility of a Bose-Einstein condensate of GFT quanta could be realized, including some likely features of the resulting vacuum state.

GFT thermodynamic quantities [45] will have to be defined in a formal way, letting ourselves be guided, at first, only by the field theory look of the GFT formalism, and only in a second stage one should try to match the definition of each of them with a corresponding physical interpretation. In turn, this physical interpretation will have to rely almost exclusively on the (pre-)geometric interpretation that the GFT variables have in the context of the Feynman expansion, i.e. in the context of simplicial gravity. This can be done more easily in an Hamiltonian setting, and in the same context we will give now a sketch of a possible concrete definition of Hamiltonian (thus of a GFT "energy") and temperature, while for other quantities we can only offer guesses, at this point, although reasonable ones, we hope.

Consider a GFT action like (we restrict here to the free theory, which sffices for our present purposes)[9]:

$$S=\left(\prod_i\int_G dg_i\int_{\mathbb{R}}ds_i\right)\phi^\dagger(g_1,s_1;...;g_D,s_D)\prod_i\left(i\partial_{s_i}+\Box_i\right)\phi(g_1,s_1;...;g_D,s_D)+h.c.$$

with $g_i\in G$, $s_i\in\mathbb{R}$, $\Box$ being the Laplace-Beltrami operator on G, for generic group G (Riemannian or Lorentzian). The kinetic term has the structure of a product of differential operators, each acting independently on one of the D (sets of) arguments of the field. Each of them is a Schroedinger-like operator with "Hamiltonian"$\Box$. This suggests that one should consider the variables s_i as "time"variables, to be used in a GFT generalization of the usual time+space splitting of the configuration space coordinates, with the group elements treated instead as "space". This implies that we have a field theory with D "times", all to be treated on equal footing. The approach chosen in [11] is to use the DeDonder-Weyl generalized Hamiltonian mechanics, as developed at both the classical and quantum level as a *polysymplectic (or polymomentum) mechanics* by Kanatchikov [46], as a starting point and to adapt it to the peculiar GFT setting.

The general idea is the following [11]. One starts from a "covariant"definition of momenta, hamiltonian density, Poisson brackets, etc treating all "time variables"on equal footing at first, i.e. when defining densities. Then one defines 'scalar' quantities referring to each 'time direction'(to be turned into operators at the quantum level), including a set of D Hamiltonians, by integration over appropriate hypersurfaces in $(G\times\mathbb{R})^{\times D}$, so that each Hamiltonian refers to a single time direction, but at the same time all time directions are treated equally but independently. A similar procedure is adopted for other canonical quantities, e.g. Poisson brackets, scalar products etc.

Let us sketch one example of such procedure, for the case $D=2$, referring to [11] for more details. We start from the naive phase space $(\phi,\phi^\dagger,\pi^i_\phi=\frac{\delta L}{\delta\partial_{s_i}\phi},\pi^i_{\phi^\dagger}=\frac{\delta L}{\delta\partial_{s_i}\phi^\dagger})$, with the product structure of the kinetic term resulting in a peculiar expression for the momenta, e.g. $\pi^1_\phi=(-i\partial_2+\Box_2)\phi^\dagger$, and define the DeDonder-Weyl Hamiltonian density (summation over repeated indices understood):

$$\mathscr{H}_{DW} = \pi^i_\phi \partial_{s_i}\phi + \pi^i_{\phi^\dagger}\partial_{s_i}\phi^\dagger - L = 2\pi^1_{\phi^\dagger}\pi^2_\phi + i\pi^1_\phi \Box_1\phi + i\pi^2_\phi\Box_2\phi + h.c..$$

One then proceeds to re-write it as a sum of two contributions, each uniquely associated to a single time parameter: $\mathscr{H}_{DW} = \mathscr{H}_1 + \mathscr{H}_2$, with $\mathscr{H}_i = \pi^1_{\phi^\dagger}\pi^2_\phi + i\pi^i_\phi\Box_i\phi + h.c.$ The Hamiltonians governing the 'time evolution' with respect to the different time directions identified by each variable s_i are then defined by integration over independent hypersurfaces, each orthogonal to a different time direction, e.g. $H_1 = \int ds_2 dg_i \mathscr{H}_1$. Each H_i results in being independent of time s_i.

One can then proceed, after suitable decomposition in modes of fields and momenta, the definition of (a GFT-adapted version of) the covariant Poisson brackets, etc, to the canonical quantization of the theory, with the definition of a Fock structure on the space of states. We refer once more to [11] for the results of this analysis.

From the above results, it is easy to guess how the notion of GFT temperature may be defined, because it simply involves following the usual QFT procedure. One could repeat the analysis above but now requiring periodicity of the fields in the s_i variables, with period β, and would then be left with a partition function in hamiltonian form:

$$Z = \int \mathscr{D}\phi\mathscr{D}\phi^*\, e^{i\sum_i \int ds_i H_i(\phi,\phi^*)}$$

with the integration over s_i restricted to the interval $(0,\beta)$, and thus obtaining, after Wick rotation in the same s_i variables:

$$Z = \int \mathscr{D}\phi\mathscr{D}\phi^*\, e^{-\beta\sum_i H_i(\phi,\phi^*)} = \int \mathscr{D}\phi\mathscr{D}\phi^*\, e^{-\beta H_{tot}(\phi,\phi^*)}$$

with $\beta = \frac{1}{kT}$ defining the GFT temperature. The notion of temperature, then, may be defined, and indeed the corresponding quantity will play the role of a temperature at least at the formal level. However, its physical interpretation will have to be studied with care (even its dimensions may not be those of a temperature). In other words, just as the variables s_i played the role of time in the formalism, and could be treated formally as such in a consistent way, but still do not have the geometric interpretation of time variables on any physical spacetime, not even at the simplicial level, similarly the GFT temperature T may be found to correspond, say, at the simplicial level, to a geometric quantity that a priori has no similar interpretation, even though the GFT sees it indeed as a temperature parameter. An even clearer example is the notion of energy in the above simple GFT. The hamiltonian in each 'time direction'is given by $\Box_i$ acting on the group manifold G for the i-th field argument, and corresponding to a particular set of field modes solutions of the GFT equations of motion. In momentum space, i.e. in representation space, it is given simply by the Casimir of the group G, and for compact groups (Riemannian models) it will have a discrete spectrum with minimal eigenvalue 0. Thus we see that the group representations J correspond to the "energy"of the GFT. However, their geometric interpretation (at least at the simplicial level) is that of (D-2)-volumes, i.e. distances, areas etc according to the dimension chosen. This is the type of procedure we were envisaging above for defining thermodynamical GFT quantities: be guided first by the field theory formalism, then look for a geometric interpretation. As a further example, as the GFTs are field theories on the group manifold $G^{\times D}$, its is (the normalisation chosen for) this group manifold and any eventual cut-off in the group integrals that will provide a definition of GFT "volume"in which the GFT quanta could be confined. From this quantities, and the partition function itself, one can proceed to define other thermodynamical quantities, standard statistical ensembles etc.

What is most relevant for us here is that within the same type of formulation, a straightforward proof of Bose-Einstein condensation seems possible, at least for the free theory, and in the case in which indeed the GFT quanta are bosons (which is not obvious [11]). Indeed, one expect to be able to even adapt to the peculiar GFT setting the standard (textbook) derivation of the Bose distribution and proceed as usual. In the model sketched above, in fact, one expects that for fixed number of particles (GFT quanta) and at low temperature T, the system will reach its ground state represented by (almost) all the GFT quanta condensed into the same state $J = 0$. Again, according to the simplicial geometry emerging from GFTs in perturbative expansion, this means having all (D-2)-volumes being of Planck size. Work on this is currently in progress [47].

The interpretation of this *vacuum state* is exciting, we think. It corresponds to *a free gas of spin network vertices or of (D-1)-simplices that has condensed in momentum space, i.e. a Bose-Einstein condensate of spin network vertices/simplices; geometrically, a Bose-Einstein condensate of the fundamental building blocks of quantum space all of Planck size*.

This also resembles, in general terms, the heuristic picture of a "semi-classical state"in LQG, with two differences: no embedding is needed for its definition, and it is selected "dynamically", in a GFT statistical setting.

From a more general perspective, there are many reasons why a condensed phase of this kind would be a very attractive possibility, in our opinion, for the vacuum relevant for the continuum limit. We have mentioned the first: it is realisable in concrete terms, and not just an hypothesis. Still at the practical level: the theory of Bose-Einstein condensates is vast and lots is known about them (see for example [48]), so in principle many tools from the condensed matter theory of BEC systems can be imported in the GFT setting to study the property of this new phase. At the theoretical and conceptual level it is also very attractive: it is *a purely quantum phenomenon*, thus a realisation of the possibility we anticipated that the emergence of a continuum spacetime from GFT structures could be considered indeed a quantum effect; it is *rather generic* [48], being robust to the presence of interactions, even strong ones, if they are repulsive, but surviving (when dealt with much care) also small attractive ones; it gives rise to a pletora of emergent phenomena [36, 23, 39]; as we will discuss in slightly more details in the following, *the approximate collective motion of the condensate admits (in mean field theory approximation) a description in term of a classical (better, 1st quantized) equation*, the Gross-Pitaevskii equation; *condensate atoms move as a whole, so that small purely quantum effects can be amplified*, and one can speculate the same to happen for this quantum gravity condensate, thus leading (we are speculating!) to observables quantum gravity effects or, more likely, to the possibility that large scale properties of spacetime (e.g. features of GR) that we are accustomed to, can be understood as originating from purely quantum features of this GFT vacuum.

To summarise, we are proposing the possibility that *GFT will produce geometrogenesis in the form of a condensation of the GFT particles in momentum space accompanied by the approach to equilibrium of the system* (otherwise, no hydrodynamic description is possible).

Let us close this section with a comment, that will be relevant for the following guesses at the effective dynamics of the condensate. GFT quanta (think of them now as open spin network vertices) are labelled by both representations of G and by corresponding vector indices in the representation spaces. It may happen (and indeed is what we would expect because of symmetry considerations at the level of the GFT action) that the GFT hamiltonian, and thus the energy of the vacuum state does not depend on these additional parameters. Now, suppose that the condensation is not complete, so that the vacuum state is actually a mixture of spin net vertices with $J = 0$ and $J = 1/2$, for $G = SU(2)$, or in general of lowest eigenvalue (which has also a single value for the vector indices) and next to lowest eigenvalue for the energy. Alternatively, suppose that the lowest eigenvalue is forbidden by some symmetry or by the quantum measure; or, more generally, the lowest allowed eigenvalue (for some group G and choice of GFT action) may have a representation space of dimension bigger than 1. What this means is that we do not necessarily expect the condensation to lead to a unique vacuum state, even in the $T \rightarrow 0$ limit. Instead, it may lead us to any of the quantum states corresponding to N spin network vertices for the lowest allowed representation parameters and some given choice for their vector indices. Now in particular, one can consider all linear combinations of such states, obtained by contracting in all possible ways the spin network vertices along their open links labelled by the vector indices. Each of these possible contractions, which is equivalent to a gluing of the dual (D-1)-simplices, corresponds to a possible choice of the topology of the corresponding quantum space, formed by the same spin networks/simplices. Of course each possible choice also corresponds to a different effective condensate wave function [48], that then carries a dependence on the resulting topology of quantum space. If on the other hand, the GFT dynamics or some additional symmetry consideration will select a specific contraction of the vector indices or the absence of any such contraction, once more this will amount to selecting one specific space topology for our quantum space in this phase.

7.3 Effective dynamics of spacetime in the condensed phase from GFT

Let us move to discuss how we could try to extract and study the effective dynamics, actually the hydrodynamics, of the GFT condensate. In discussing this issue, once more the present status of the field will force us to remain at the level of arguments, guesses, speculations. Again, we hope the reader will find them interesting.

Generally speaking, the effective collective dynamics will depend heavily on the phase the system is in, i.e. on the vacuum selected by the GFT microscopic dynamics. At this stage, even to guess it is impossible. However, we can try to forecast some general features and ask ourselves very general questions about it.

We are assuming here that a sort of Bose-Einstein condensate has formed, that the system is at equilibrium or very close to it, that we have made one specific choice of vacuum state, obtaining a specific effective vacuum wave function [48], or equivalently a classical field (the order parameter).

It is possible that a clever redefinition of the field variables will bring us collective variables with a direct geometric interpretation, say connection field or a metric, so that we could hope that the effective hydrodynamics for these collective variables is given directly by some extended gravity theory. However, we find this possibility very unlikely, for the following reasons:

- while the effective topology of the physical quantum space is probably determined by the vacuum (following the comments at the end of the previous section), nothing seems to select for us the effective topology of *spacetime*; in general, we should expect an effective theory in which spatial topology change and non-trivial spacetime topologies are included;

- in analog gravity models [39], the effective spacetime that quasi-particles see may be very different from the original spacetime on which the microscopic field theory is defined, in both geometry and topology, but the spacetime on which the *hydrodynamics* is defined is very close to the one one started from;

- in particular, the GFT we have started from has the interpretation of a discrete 3rd quantized formulation of gravity and indeed, at least in perturbative expansion, produces discrete virtual spacetimes of arbitrary topology, and moreover it was a theory on an internal group manifold and not a physical continuum spacetime; we expect neither the "formal level of quantization"nor the nature of the manifold on which the effective field is defined to change with respect to the original microscopic (group) field theory.

For all the above reasons, and some others, we expect the effective GFT dynamics for the chosen condensate vacuum to be not directly of the form of an extended gravitational theory on a fixed spacetime, but rather of the form of a continuum 3rd quantized field theory of gravity, i.e. of a quantum field theory on a continuum superspace (space of continuum geometries). This type of gravitational theories have not been much studied, beyond the original definition [49, 50], but are supposed to have the general action (schematically):

$$S = \int_{\mathscr{S}} \mathscr{D}X \Psi^*(X) \mathscr{H}(X) \Psi(X) + \Lambda \int \Psi^n(X) V(X) \tag{7.1}$$

where $\Psi(X)$ is a scalar field on the superspace $\mathscr{S}$, i.e. the space of all space geometries (not spacetime) for given space topology Σ, and X are then coordinates on this space, i.e. some geometric variables (3-metrics, connections, etc); the (non-local) interaction term $V(X)$ generates, in perturbative expansion spatial topology changing processes (producing disconnected universes) while the free kinetic term is given by a canonical Hamiltonian constraint $\mathscr{H}$. Notice that the superspace $\mathscr{S}$ is a metric space itself [52].

As we have said, for our GFT condensate, we expect the effective field, call it Ψ as well, to be determined by the vacuum state, from which would most likely inherit also the choice of space topology Σ and the topological and metric properties of the effective superspace $\mathscr{S}$, that will depend on the space topology chosen. In turn, as we have said, the properties of the vacuum state depend on the original choice of GFT field and of group manifold $G^{\times D}$. We then expect the emergent superspace to be some sort of group manifold, with an exact structure determined by the topology of space we have selected with the vacuum, and thus again parametrised by group elements or, equivalently by a (gravity) connection.

To summarise, we would probably obtain, as our effective GFT hydrodynamics of quantum space, 1st order versions of the old quantum field theories on superspace. Nothing is known (to the best of our knowledge) about how these may look like, and a detailed analysis of such possible field theories (involving the metric structure of a 1s order superspace, first of all) is called for.

In general, then, our effective GFT hydrodynamics, in the GFT analogue of the mean field approximation, will be a continuum field theory of the form:

$$S = \int_{\mathscr{S}} \mathscr{D}X \Psi^*(X)\mathscr{K}(X)\Psi(X) + \int V(\Psi, \Psi^*) \tag{7.2}$$

for some kinetic term $\mathscr{K}$ and higher order (non-local) interaction $V(\Psi, \Psi^*)$.

The corresponding equations of motion with be our hydrodynamics equations, non-linear equations for the field/wave function Ψ that will represent the GFT analogue of the Gross-Pitaevskii equation for Bose-Einstein condensates [48]. Notice that the above field theory can be easily recast in a more customary hydrodynamic form by redefining the basic variables to $\Psi(X) = \sqrt{\rho(X)}e^{i\theta(X)}$ where $\rho(X)$ is the condensate density and $v(X) = \nabla\theta(X)$ is the condensate velocity field.

Let us now see how the link with continuum GR (in some extended form, probably) can be investigated. The type of gravity theory we would have obtained will be encoded, and hopefully fully specified, by the quadratic term in the above action, that would give the effective Hamiltonian constraint of the corresponding canonical theory. Notice that all of the above (and of the following) is at the level of *classical* effective theories. We then would have to extract the quadratic part of the action, here represented by $\mathscr{K}$. However, it is clear that the split of the above action, and more generally the very form of the effective hydrodynamics action depends strongly on the specific mean field ansatz one has chosen to obtain it[5]. Anyway, assuming that, in some approximation, we have got up to here, we could then compare the kinetic term $\mathscr{K}$, which would be in general a differential operator on an effective 1st order superspace $\mathscr{S}$, and thus depending on connection variables and their conjugate variables, with the classical Hamiltonian constraints of various canonical 1st order formulations of gravity for space topology Σ, or re-interpret it as such, and study in this way what type of effective gravity theory our GFT reproduces in this phase, i.e. for this choice of condensate vacuum state[6].

Another possibility, that we mention in passing, comes from the interpretation of classical gravity as a single particle theory on superspace [51]. In our case, the continuum superspace is effective and corresponds to the effective manifold on which our GFT condensate lives. The procedure for identifying classical gravity in our hydrodynamic field theory on superspace is consistent with this interpretation. But what if classical gravity is a *"quasi-particle"* of the above theory on superspace, and not a particle? Then the effective superspace it would live in would not be given by $\mathscr{S}$, but by a space with an effective geometry function of $\rho(X)$ and $v(X)$ [39]. We are not going to expand on this, but it is clear that in this case the body of knowledge developed in condensed matter analog gravity models [39] would become even more directly relevant.

It is clear that the realm of possibilities for the structure of the vacuum and even more for the way to extract effective dynamics for it, and to find our what back to classical gravity, is enormous. This is true even if one accepts the idea of the correct vacuum being represented by a condensate of the type we suggested. And there are for sure many other plausible hypothesis that can be made at this stage. Again, condensed matter physics wisdom suggests to be cautious because condensed

[5]As they say, mean field theory, and in general the procedure of constructing effective dynamics for collective variables, is a complicated art.

[6]In principle it would be also possible to extract the corresponding lagrangian form for the same gravity theory and even the corresponding continuum path integral, i.e. the 2-point function for the corresponding free field theory on superspace. Obviously this would have only a formal meaning, and limited applicability, just as the formal quantization of hydrodynamics has, and in any case will not resembles at all the original GFT we started from, just as the quantization of hydrodynamics for ordinary quantum fluids does not reproduce at all the underlying microscopic atomic theory [23].

matter systems are rich, and always richer than we imagine. *We simply wanted to suggest one possible path from the microscopic discrete to the macroscopic continuum*: microscopic GFT → condensate → condensate hydrodynamics → effective continuum 3rd QGR → approximate free theory → classical (extended) GR.

This probably means we have been un-cautious enough already.

8. Conclusions

We have presented a brief introduction to the group field theory formalism for quantum gravity. We have then argued that GFTs may provide a common framework for several other discrete approaches to quantum gravity (loop quantum gravity, quantum Regge calculus, dynamical triangulations), and shown how the connection with these other approaches can be understood. Having done so, we have tried to sketch the elements of a single coherent picture of quantum spacetime, incorporating the insights and results achieved in all these different approaches, as seen from a GFT standpoint. We have tried to argue that the GFT formalism offers also a new perspective on the same structures.

We have then stressed the importance of solving the open problem of the continuum approximation of the discrete structures representing spacetime at the quantum level in these quantum gravity models, including GFTs, and overviewed the strategies adopted in loop and simplicial approaches to do so, and the results obtained. At the same time, we have translated these strategies in the GFT language, showing that the GFT formalism would suggest a different one instead, and then sketched what we believe is a new GFT perspective on the continuum problem in quantum gravity. This amounts to consider quantum spacetime as a condensed matter system and the GFT as the microscopic quantum field theory for its fundamental constituents. We have finally outlined a GFT strategy from tackling the problem of the emergence of the continuum, put forward an hypothesis for the relevant GFT phase, a Bose-Einstein condensate, and sketched a (rather speculative, at present) programme for realizing this idea and connecting GFT microscopics to continuum gravity and GR, obtained from the effective hydrodynamics of the GFT condensate.

We hope that, in spite of necessary conciseness of the first part of this contribution, and of the speculative nature of much of the second, we have managed to elicit interest for the ideas presented and for this, we believe, very exciting area of fundamental theoretical physics that is non-perturbative quantum gravity. The hope is also that the reader will then join the efforts of researchers working in this area, and contribute to turning the present speculations into solid results, in the conviction that most of the many impressive results already obtained in this fascinating field have been just tentative suggestions or speculations at an earlier stage.

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