PROCEEDINGS OF SCIENCE

Quantization of the Myers-Pospelov model: a progress report

C. M. Reyes, L. Urrutia* **and J. D. Vergara**
Universidad Nacional Autónoma de México-MEXICO
E-mail: carlos.reyes@nucleares.unam.mx, urrutia@nucleares.unam.mx, vergara@nucleares.unam.mx

The Myers-Pospelov (MP) model is an effective field theory (EFT), including dimension five operators, which describes the phenomenology of active Lorentz invariance violation produced by a preferred reference frame. We concentrate here in the case of the modified electrodynamics. The point of view taken in this work is that the Lorentz violating part of the action in the MP model, which includes higher order time derivative (HOTD) operators, is to be considered as a perturbation over the dynamics described by standard Electrodynamics, particularly in the quantum case. HOTD theories, besides incorporating additional degrees of freedom, suffer from well known difficulties in their quantization, among which one finds Hamiltonians which are not bounded from below. Thus, in order to cope with these challenges it will be necessary to deal with a modified perturbation theory which is well described in the literature. We apply such methods to this specific model providing a quantization of the free sector of the theory by identifying the modified normal mode expansions of the fields together with the modified propagators and the interaction terms. The calculation of interacting processes, together with radiative corrections, is beyond the scope of the present article and will be deferred for future publications.

From Quantum to Emergent Gravity: Theory and Phenomenology
June 11-15 2007
Trieste, Italy

*Speaker.

 http://pos.sissa.it/

PoS(QG-Ph)040

1. Introduction

The MP model [1] is an EFT theory that incorporates scalars, fermions and photons in an observer (active) Lorentz violating theory using dimension five operators together with the presence of a fixed direction selecting a preferred frame. It has recently been generalized to a non-abelian model including interactions arising from the fields associated to the Standard Model [2], thus providing a dimension-five-operator generalization of the original Standard Model Extension [3].

In this work we will concentrate upon the simpler version of Ref.[1], particularly upon the proposed modified electrodynamics. We present here the first steps to our final goal, which is to provide a quantum version of MP electrodynamics considered as a perturbation of standard QED, in the precise sense that when making zero the parameters encoding the corrections we must recover the well known results for any physical process in Lorentz covariant QED. This requirement is motivated by the fact that all experimental and observational evidence point to negligible Lorentz invariance violation at standard model energies. The amazingly precise experimental predictions of standard QED can be obtained as the result of perturbative calculations, which now will require the incorporation of the extra perturbation arising from LIV into the scheme. An additional challenge arises because the presence of dimension five operators encoding the corrections to QED in the MP model make the theory of the HOTD type. At least at the perturbative level, it is well known that HOTD theories, besides the obvious property of having additional degrees of freedom with respect to the lower order ones, give rise to Hamiltonians which are not positive definite, irrespectively of the interaction terms [4, 5]. In fact, a perturbation of electrodynamics should not introduce additional degrees of freedom, so that a careful strategy is required to define an adequate perturbative procedure in the LIV parameters. Fortunately, a systematic approach to carry out this task, which also provides a positive definite zeroth order Hamiltonian, already exists in the literature [7, 8] and here we adhere to it.

In view of the above considerations we will proceed in the following way to define the quantum field theory extension of the MP model: (i) as usual, our starting point will be the classical version of it given in Ref.[1] and which has been thoroughly studied in relation to synchrotron radiation in Refs. [6]. (ii) next we apply the procedure of Ref.[8] to the classical HOTD MP model and reduce it to a modified effective theory of the same time derivative character as classical electrodynamics. The procedure leads to field redefinitions plus additional contributions to the interactions. (iii) finally we take this resulting classical theory as the correct starting point for quantization, which we carry along the standard lines. The resulting quantum theory provides the basis for the calculation of interacting processes using the standard perturbative scheme of quantum field theory (QFT). In this sense it is clear that we are not producing a quantum version of the full MP model, but only one which is adapted to our basic requirement of describing the LIV corrections as perturbations to QED.

Perhaps we should emphasize at this stage that we are dealing with two different classes of perturbations: the first one concerns only the LIV parameters, occurs at the classical level and serves to define the correct starting point for quantization. Once the resulting theory is quantized, the usual QFT interacting processes can be calculated, corresponding to the second class of perturbations. Both approximations should be made consistent when predicting a result to a given order in any of the LIV parameters.

PoS(QG-Ph)040

Furthermore, in this work we only deal with the calculation of the free propagators, which nevertheless incorporate modified dispersion relations to lowest order in the LIV parameters. In other words, we identify the corresponding free propagating excitations which will subsequently subjected to interactions. The calculation of interacting processes either at the tree or at the one loop level are deferred to future publications. Of particular interest to us will be the calculation of self-energies which can be used as first indications of the fine-tuning problems arising in some LIV theories [9, 10].

2. The MP electrodynamics

The corresponding free field Lagrangian densities are given by

$$\mathscr{L}_{photon} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\xi}{M}E_i\partial_0 B_i, \tag{2.1}$$

$$\mathscr{L}_{fermion} = \bar{\Psi}i\gamma^{\mu}\left(\partial_{\mu} - m\right)\Psi + \frac{1}{M}\bar{\Psi}\gamma^{0}\left(\eta_1 + \eta_2\gamma_5\right)\partial_0^2\Psi, \tag{2.2}$$

in the particular frame where the Lorentz symmetry is broken in the direction $n^{\mu} = (1, \mathbf{0})$ and in standard notation with metric $(1,-1,-1,-1)$. We interpret the scale M, together with the dimensionless parameters ξ, η_1 and η_2, as the effective low energy imprints upon standard particle dynamics produced by a fundamental quantum gravity theory, which has induced an spontaneous LIV characterized by the vacuum expectation value n^{μ}. The choice $M = M_{Planck}$ leads to the observational/experimental bounds $|\xi| < 10^{-7}$ and $|\eta_{\pm} = \eta_1 \pm \eta_2| < 10^{-5}$ [11]. In the limit of such parameters going to zero we demand that the standard Lorentz covariant quantum results for electrodynamics are recovered. After some basic features of the theory are revealed we present a more detailed discussion of the relevant energy scales that define the effective model in the last section.

In order to assess the real character of a HOTD contribution it is convenient to look at the contribution of the Lorentz violating terms to the equations of motion. The fermion modification will add a second-order time derivative to the standard first-order equation of motion. Additional degrees of freedom will appear in this case. On the contrary, the photon contribution only incorporates a modification to the standard second-order term in electrodynamics and no additional degrees of freedom are present. Nevertheless, such modifications will still require some field redefinitions in order to exhibit a positive definite Hamiltonian consistent with the field operators commutation relations.

3. The perturbative expansion

The general method for dealing with the canonical description of HOTD theories was given a long time ago in Ref. [12]. In order to highlight some general features of these theories we briefly review their basic properties in the context of a scalar field theory satisfying the non-degeneracy condition $\left(\frac{\partial^2\mathscr{L}}{\partial\phi^{(k)}\partial\phi^{(k)}}\right) \neq 0$, where the fields depend upon the space-time coordinates. The generalizations incorporating spinor and vector fields they have been analyzed in Refs. [7], [13].

If the highest time derivative in the Lagrangian density $\mathscr{L} = \mathscr{L}(\phi(t,\mathbf{x}),...,\phi^{(k)}(t,\mathbf{x}))$, is of order k, with the notation [1]

$$\phi^{(k)}(t,\mathbf{x}) = \frac{\partial^k \phi(t,\mathbf{x})}{\partial t^k}, \tag{3.1}$$

the corresponding phase space will be of dimension $2k$ per space point, been characterized by k fields : $Q_0 = \phi(t,\mathbf{x}),\ Q_1 = \phi^{(1)}(t,\mathbf{x}), ...,\ Q_{k-1} = \phi^{(k-1)}(t,\mathbf{x})$ together with k momenta

$$P_i(t,\mathbf{x}) = \frac{\partial \mathscr{L}}{\partial \phi^{(i+1)}} + \sum_{j=1}^{k-i-1} \left(-\frac{\partial}{\partial t}\right)^j \frac{\partial \mathscr{L}}{\partial \phi^{(j+1)}}, \quad i = 0, \ldots, (k-1). \tag{3.2}$$

Here and in the sequel, as far as no confusion arises, we avoid writing the explicit space-time dependence in the fields. The equation of motion for ϕ will be of order $2k$ in the time derivatives, requiring the fixing of $2k$ initial conditions, which is consistent with the existence of $2k$ degrees of freedom phase space. The Hamiltonian is

$$H = \int d^3x \left(\sum_{i=0}^{k-1} P_i Q_i - \mathscr{L}(Q_0, \ldots, Q_{k-1},\ \phi^{(k)}(P_{k-1}, Q_0, ..., Q_{k-1})) \right), \tag{3.3}$$

where we have used the non-degeneracy condition which implies that we can solve $\phi^{(k)}$ as a function of $Q_0, ..., Q_{k-1}, P_{k-1}$. The above expression is linear in the momenta $P_i,\ i = 0, ..., k-2$, thus making the Hamiltonian (3.3) unbounded from below, independently of the interaction terms included in the Lagrangian.

Since we are interested in dealing with HOTD corrections in the action as perturbations upon standard theories we must rely on a procedure which (i) retains the original number of degrees of freedom and (ii) produce free Hamiltonians bounded from below as adequate starting points for quantization. Such a method has been already developed in Refs. [7, 8] and we present here a brief summary of it adapted to the case of a scalar field theory. In order to point out some its basic features let us consider the simplest framework of a non-covariant Lagrangian density, analogous to those described in Eqs. (2.1), (2.2), depending upon accelerations and where the HOTD contribution is only present as a perturbation characterized by the small parameter g

$$\mathscr{L}(\phi, \partial_\mu \phi, \ddot{\phi}) = \mathscr{L}(\phi, \partial_\mu \phi) + \frac{1}{2} g \ddot{\phi}^2. \tag{3.4}$$

In this case the quantities $\phi, \dot{\phi}$ play the role of coordinate fields. The standard procedure of extremizing the action leads to

$$\begin{aligned} \delta S = \delta \int d^4x \mathscr{L}(\phi, \partial_\mu \phi, \ddot{\phi}) &= \int d^4x\ \partial_\mu \left[\left(\frac{\partial \mathscr{L}}{\partial \partial_\mu \phi} - \partial_\nu \left(\frac{\partial \mathscr{L}}{\partial \partial_\mu \partial_\nu \phi} \right) \right) \delta\phi \right. \\ &\left. + \frac{\partial \mathscr{L}}{\partial \partial_\mu \partial_\nu \phi} \delta \partial_\nu \phi \right] + \int d^4x\, E\left([\phi]\right) \delta\phi, \end{aligned} \tag{3.5}$$

where

$$E\left([\phi]\right) = \partial_\mu \partial_\nu \left(\frac{\partial \mathscr{L}}{\partial \partial_\mu \partial_\nu \phi} \right) - \partial_\mu \left(\frac{\partial \mathscr{L}}{\partial \partial_\mu \phi} \right) + \frac{\partial \mathscr{L}}{\partial \phi} = 0. \tag{3.6}$$

[1] For the lower order time derivatives we might use also the standard dots notation

The first term in the right hand side of Eq. (3.6) gives the fourth order time derivative contribution $\phi^{(4)}(t,\mathbf{x})$ to the equation of motion. The space-time dependent momenta, associated to ϕ and $\dot{\phi}$ respectively, can be directly read off from the surface boundary term in Eq. (3.5)

$$P_0 = \frac{\partial \mathscr{L}}{\partial \dot{\phi}} - \frac{\partial}{\partial t}\left(\frac{\partial \mathscr{L}}{\partial \ddot{\phi}}\right), \quad P_1 = \frac{\partial \mathscr{L}}{\partial \ddot{\phi}}, \tag{3.7}$$

in accordance with the general expression in Eq. (3.2). From the simple form assumed for the HTOD term, which satisfies the non-degeneracy condition, it is clear that both velocities can be solved in terms of the momenta: $\ddot{\phi}$ can be expressed in terms of P_1, and $\phi^{(3)}$ in terms of P_0. Nevertheless, notice that both substitutions carry the non-analytical factor $1/g$. This is precisely what makes non-trivial a perturbative expansion around $g=0$.

The full Hamiltonian H and symplectic form Ω are defined according to the Ostrogradski procedure as

$$H = \int d^3x \left(P_0\dot{\phi} + P_1\ddot{\phi} - \mathscr{L}\right), \qquad \Omega = \int d^3x\, d^3y \left(dP_0(t,\mathbf{x}) \wedge d\phi(t,\mathbf{y}) + dP_1(t,\mathbf{x}) \wedge d\dot{\phi}(t,\mathbf{y})\right). \tag{3.8}$$

The dangerous contributions to the Hamiltonian arise from the non-analytic term $P_1^2/2g$ together with the unbounded piece $P_0\,\dot{\phi}$.

Let us summarize now the general perturbative procedure for the non-degenerate case according to the method in Ref.[8], in the framework of a system having a Lagrangian density of the form

$$\mathscr{L} = \mathscr{L}_0(\phi\,,\partial_\mu\phi) + g\mathscr{L}_1(\phi\,,\partial_\mu\phi\,,\phi^{(2)},\ldots,\phi^{(n)}), \tag{3.9}$$

where g is a small parameter. The steps are the following: (i) in order to obtain the appropriate Hamiltonian to order g^k, one starts by iteratively solving the equations of motion to order $g^{(k-1)}$. (ii) next express all time derivatives $\phi^{(k)}(t,\mathbf{x})$ for $k>2$ in terms of the lowest time-derivative fields describing the unperturbed system, which are $\phi(t,\mathbf{x})$ and $\dot{\phi}(t,\mathbf{x})$ in our example. This will introduce further contributions in powers of the perturbation parameters which need to be maintained only up to the required order. (iii) then rewrite the Hamiltonian together with the symplectic form obtained from the Ostrogradski method by substituting the momenta together with all $\phi^{(k)}(t,\mathbf{x})$, $k>2$ in terms of these basic variables.

$$\mathscr{H} = \mathscr{H}(\phi,\dot{\phi},\nabla\phi), \qquad \Omega = \int d^3x\, d^3y\, \omega(\phi(t,\mathbf{x}),\dot{\phi}(t,\mathbf{y}))\, d\dot{\phi}(t,\mathbf{y}) \wedge d\phi(t,\mathbf{x}), \tag{3.10}$$

from where we can read the bracket $\{\phi(t,\mathbf{x})\,,\dot{\phi}(t,\mathbf{y})\}$. (iv) finally find an invertible change of variables $\phi(t,\mathbf{x})$, $\dot{\phi}(t,\mathbf{x}) \to Q(\phi,\dot{\phi},\ldots)$, $P(\phi,\dot{\phi},\ldots)$ in such a way that the corresponding Poisson bracket $\{Q(t,\mathbf{x})\,,P(t,\mathbf{y})\}$ is canonical to the order considered. That is to say

$$\begin{aligned}&\delta^3(\mathbf{x}-\mathbf{y}) + O(g^{k+1}) = \{Q(t,\mathbf{x})\,,P(t,\mathbf{y})\}\\ &= \int d^3z\;\, d^3z' \left(\frac{\delta Q(t,\mathbf{x})}{\delta\phi(t,\mathbf{z})}\frac{\delta P(t,\mathbf{y})}{\delta\dot{\phi}(t,\mathbf{z}')} - \frac{\delta Q(t,\mathbf{x})}{\delta\dot{\phi}(t,\mathbf{z}')}\frac{\delta P(t,\mathbf{y})}{\delta\phi(t,\mathbf{z})}\right)\{\phi(t,\mathbf{z})\,,\dot{\phi}(t,\mathbf{z}')\}.\end{aligned} \tag{3.11}$$

At last, the Hamiltonian density $\tilde{\mathscr{H}}(Q,P,\ldots) = \mathscr{H}(\phi(Q,P,\ldots)\,,\dot{\phi}(Q,P,\ldots))$ together with the Poisson bracket $\{Q(t,\mathbf{x}),P(t,\mathbf{y})\} = \delta^3(\mathbf{x}-\mathbf{y})$ define the physical approximation of the system to the order considered. This Hamiltonian will be bounded from below provided the initial one obtained from $\mathscr{L}_0$ is. The effective Lagrangian density is given by $\tilde{\mathscr{L}}(Q,\dot{Q},\ldots) = P\dot{Q} - \tilde{\mathscr{H}}$ and the quantization is straightforward since it is first order.

A proof of self-consistency to all orders, in a mechanical setting[2], is provided in Ref.[8]. It consists in showing that to each order in the expansion parameter, the Lagrangian constructed according to the summary described in the above paragraph reproduces exactly the corresponding equations obtained by the iteration procedure stating from the exact HOTD ones.

An illuminating example of the relevance of the above procedure to our case is also given in Ref. [8]. The authors consider a system of two one-dimensional oscillators coupled in the presence of a constant gravitational field. Each oscillator has natural free frequencies $\Omega_0 = \sqrt{K/M}$ and $\omega_0 = \sqrt{k/m}$, respectively. The full normal modes frequencies Ω and ω can be exactly calculated. On the other hand, the equations of motion consisting in two coupled second order differential equations can be uncoupled through a fourth order differential equation for one coordinate, which can be obtained from an acceleration dependent Lagrangian. Next, they look at the situation in terms of a perturbative scheme starting from this acceleration dependent Lagrangian. The expansion parameter is taken to be $g = k/K \ll 1$ and for the sake of the discussion the two masses are considered of the same order, i.e. $m \sim M$. When probing energies of the order of ω_0 one verifies that the acceleration corrections in the Lagrangian become negligible with respect to the velocity and coordinate dependents ones, which coefficients depend upon all the parameters. The most immediate possibility to proceed along the lines of maintaining the original number of degrees of freedom $x, \dot{x}$, is to simply neglect the acceleration term and compute the corrections to ω to first order in g. Nevertheless, this result does not coincide with the first order expansion of the exact frequency ω to that order. On the other hand, the application of the modified perturbative method proposed in [8] does indeed leads to the correct expression to that order. Moreover, in this simple case, the corrections can be calculated to all orders in g and the sum of this perturbation series can also be performed, leading to the exact expression for ω. The high frequency mode Ω, which is non-analytical when written in terms of the parameter g, is not seen by the procedure, meaning that the results are valid only for energies much lower than Ω. One could summarize the above description by saying that the physical meaning of reducing the enlarged original configuration $(x,\dot{x})$ - velocity $(\ddot{x},\dddot{x})$ space to that generated by x and $\dot{x}$ in the proposed perturbative formulation is to allow the calculation of further corrections to the excitations of the low energy modes already present in the zeroth order system, in a way consistent with the exact evolution. We refer the reader to the original paper for further details.

4. The perturbative expansion in the Myers-Pospelov model

Here we describe the main ingredients and results of the application of the method described in the previous section to the quantization of the free sector in MP electrodynamics. Our general strategy will be the following: (i) since the fermions acquire corrections of the HOTD type, we start by constructing the corresponding effective Lagrangian and Hamiltonian densities to a desired order in $g = 1/M$. Then we quantize this effective theory and calculate the modified free propagator. (ii) next we introduce the photons via minimal coupling in this effective fermion Lagrangian density

[2] Field theory in $1+\mathbf{0}$ dimensions

and also add the contributions of Eq. (2.1), identifying the resulting free and interaction terms. (iii) subsequently we find the correct effective Hamiltonian formulation for the free photon field and proceed to its quantization, obtaining also the modified propagator. In both cases the normal modes of the free sector correspond to particles with propagation properties which are different from the usual ones described by the limit $\eta_1/M, \eta_2/M \to 0$.

4.1 The fermionic sector

Here we consider corrections to order $g = 1/M > 0$ and start from the HOTD Lagrangian density

$$\mathscr{L} = \bar{\psi}(i\gamma^{\mu}\overrightarrow{\partial_{\mu}} - m)\psi + g\,\bar{\psi}\Sigma_0\ddot{\psi}, \quad \Sigma_0 = \gamma^0\,(\eta_1 + \eta_2\gamma_5)\,, \tag{4.1}$$

which produces the equation of motion

$$\dot{\psi} = \overrightarrow{\alpha}\,\psi + ig\chi\ddot{\psi}, \quad \overrightarrow{\alpha} = -\gamma_0(\gamma^i\overrightarrow{\partial_i} + im). \tag{4.2}$$

In the approximation $\dot{\psi} = \overrightarrow{\alpha}\,\psi$, $\ddot{\psi} = \overrightarrow{\alpha}\,\dot{\psi}$, the canonical momenta, the Hamiltonian density, and the symplectic form, respectively, are

$$\Pi_{0\psi} = \bar{\psi}i\gamma^0 + g\bar{\psi}\overleftarrow{\alpha}\,\Sigma_0, \quad \Pi_{1\psi} = g\bar{\psi}\Sigma_0, \quad \Pi_{0\bar{\psi}} = 0 = \Pi_{1\bar{\psi}}, \tag{4.3}$$

$$\mathscr{H} = -i\bar{\psi}\gamma^k\partial_k\psi + m\bar{\psi}\psi + g\bar{\psi}\overleftarrow{\alpha}\,\Sigma_0\,\overrightarrow{\alpha}\,\psi, \tag{4.4}$$

$$\Omega = \int d^3x\left\{id\psi^{\dagger}\left[1 - ig\gamma^0\overleftarrow{\alpha}\,\Sigma_0 - ig\gamma^0\Sigma_0\,\overrightarrow{\alpha}\right]\wedge d\psi\right\}. \tag{4.5}$$

The required change of variables $\psi \to \tilde{\psi}$ to recover the standard symplectic structure

$$\{\tilde{\psi}_A(t,\mathbf{x}), \tilde{\psi}_B^{\dagger}(t,\mathbf{y})\} = \delta_{AB}\delta^3(\mathbf{x}-\mathbf{y}), \tag{4.6}$$

where the labels $A, B = 1,2,3,4$ denote the four-spinor components, is

$$\tilde{\psi} = (1 - ig\gamma_0\Sigma_0\,\overrightarrow{\alpha}\,)\,\psi, \qquad \bar{\tilde{\psi}} = \bar{\psi}\,(1 - ig\overleftarrow{\alpha}\,\Sigma_0\gamma^0). \tag{4.7}$$

This leads to the following effective Lagrangian and Hamiltonian densities

$$\tilde{\mathscr{L}} = \bar{\tilde{\psi}}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\tilde{\psi} + g\bar{\tilde{\psi}}\gamma_0\left(\overrightarrow{\alpha}\right)\gamma^0\Sigma_0\left(\overrightarrow{\alpha}\right)\tilde{\psi}, \tag{4.8}$$

$$\tilde{\mathscr{H}} = \tilde{\psi}^{\dagger}\left[-i\overrightarrow{\alpha}\,\tilde{\psi} - g(\gamma_0\,\overrightarrow{\alpha}\,\chi\,\overrightarrow{\alpha}\,)\tilde{\psi}\right] = \tilde{\psi}^{\dagger}i\frac{\partial\tilde{\psi}}{\partial t}. \tag{4.9}$$

Next we proceed to quantization by introducing fermionic creation (annihilation) operators for particles $b_{\lambda}^{\dagger}(\mathbf{k}), (b_{\lambda}(\mathbf{k}))$ and for antiparticles $d_{\lambda}^{\dagger}(\mathbf{k}), (d_{\lambda}(\mathbf{k}))$, with standard anticommutation relations. The label $\lambda = \pm 1$ denotes the helicity quantum number. The normal modes (physically propagating particles) corresponding to the full Dirac equation derived from Eq.(4.8) are characterized by positive energies $E_u^{\lambda}(\mathbf{k}), E_v^{\lambda}(\mathbf{k})$ according to the modified dispersion relations

$$E_u^{\lambda}(\mathbf{k}) = E_0 + g(\eta_1 E_0^2 + \lambda\,|\mathbf{k}|\,\eta_2 E_0), \quad E_v^{\lambda}(\mathbf{k}) = E_0 - g(\eta_1 E_0^2 + \lambda\,|\mathbf{k}|\,\eta_2 E_0), \tag{4.10}$$

where the notation is $E_0 = +\sqrt{\mathbf{k}^2 + m^2}$. Here we can appreciate one indication that we are dealing with an EFT: it is clear that for any choice of the parameters η_1, η_2, in the high momentum limit there will be at least one value of the above energies which will be negative, thus producing a

Hamiltonian which is not positive-definite. To avoid this possibility we must impose the restriction $|\mathbf{k}| < M/|\eta_1 + \lambda\,\eta_2|$. Further discussion of this issue is given in the last section.

In terms of the above normal modes, the fermionic field can be expanded as

$$\tilde{\psi}_A(x) = \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3}} \sum_{\lambda=\pm} \sqrt{\frac{m}{E_u^\lambda(\mathbf{k})}}\, b_\lambda(\mathbf{k})\, U_{A\lambda}(\mathbf{k}) e^{-ik_u^\lambda\cdot x} + \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3}} \sum_{\lambda=\pm} \sqrt{\frac{m}{E_v^\lambda(\mathbf{k})}} d_\lambda^\dagger(\mathbf{k})\, V_{A\lambda}(\mathbf{k}) e^{ik_v^\lambda\cdot x}. \tag{4.11}$$

Here the notation is $k_u^\lambda \cdot x = E_u^\lambda t - \mathbf{k}\cdot\mathbf{x}$ and analogously $k_v^\lambda \cdot x = E_v^\lambda t - \mathbf{k}\cdot\mathbf{x}$. Also, $U_{A\lambda}(\mathbf{k})$ and $V_{A\lambda}(\mathbf{k})$ are the corresponding eigenspinors of the one particle Hamiltonian associated to (4.9). Such spinors can be explicitly written to order g and satisfy the following properties

$$V_\lambda(g,\mathbf{k}) = \gamma_5 U_\lambda(-g,\mathbf{k}), \qquad V_\lambda^\dagger(\mathbf{k}) U_{\lambda'}(-\mathbf{k}) = 0, \tag{4.12}$$

$$U_\lambda^\dagger(\mathbf{k}) U_{\lambda'}(\mathbf{k}) = \delta^{\lambda\lambda'} \frac{E_u^\lambda(\mathbf{k})}{m}, \qquad V_\lambda^\dagger(\mathbf{k}) V_{\lambda'}(\mathbf{k}) = \delta^{\lambda\lambda'} \frac{E_v^\lambda(\mathbf{k})}{m}, \tag{4.13}$$

$$\frac{m}{E_u^\lambda} U_\lambda(\mathbf{k}) \bar{U}_\lambda(\mathbf{k}) = \frac{1}{2E_0} (\gamma_0 E_0 - \gamma\cdot\mathbf{k} + m - igm\eta_2\gamma_5\alpha_{-\mathbf{k}}) P^\lambda \tag{4.14}$$

$$\frac{m}{E_v^\lambda} V_\lambda(\mathbf{k}) \bar{V}_\lambda(\mathbf{k}) = \frac{1}{2E_0} (\gamma_0 E_0 - \gamma\cdot\mathbf{k} - m + igm\eta_2\gamma_5\alpha_{\mathbf{k}}) P^\lambda. \tag{4.15}$$

Here $\alpha_{\mathbf{k}}$ is the momentum representation of the operator $\overrightarrow{\alpha}$ defined in Eq. (4.2) and P^λ is the helicity projector

$$P^\lambda = \frac{1}{2}\left(I + \lambda \frac{\Sigma\cdot\mathbf{k}}{|\mathbf{k}|}\right), \tag{4.16}$$

where Σ is the spin operator.

Using the field expansion (4.11) together with the required properties from Eqs. (4.12), (4.13), the Hamiltonian, the momentum and the charge operators have the expected form in terms of the number operators $b_\lambda^\dagger(\mathbf{k}) b_\lambda(\mathbf{k})$ and $d_\lambda^\dagger(\mathbf{k}) d_\lambda(\mathbf{k})$. Our final task in this subsection is to write the fermion propagator

$$iS_{AB}(x-y) = \langle 0 | T(\psi_A(x)\,\bar{\psi}_B(y)) | 0 \rangle, \tag{4.17}$$

which, in momentum space, is given by

$$S(k_0,\vec{k}) = \sum_{\lambda=\pm} \left[\frac{1}{(k_0 - E_u^\lambda + i\varepsilon)} \left\{ \frac{m}{E_u^\lambda} U_\lambda(\vec{k}) \bar{U}_\lambda(\vec{k}) \right\} + \frac{1}{(k_0 + E_v^\lambda - i\varepsilon)} \left\{ \frac{m}{E_v^\lambda} V_\lambda(-\vec{k}) \bar{V}_\lambda(-\vec{k}) \right\} \right]. \tag{4.18}$$

Let us emphasize that the terms in curly brackets appearing in the above expression have been explicitly calculated in Eqs. (4.14) and (4.15). Also the poles in k_0 appear as exact functions of the normal mode energies to the order considered.

4.2 The photon sector

In order to include photon field A_μ we perform the minimal substitution $\partial_\mu \to (\partial_\mu + ieA_\mu)$ in the effective Lagrangian density (4.8), to which we add the free contributions from (2.1). The result is

$$\mathscr{L}_{QED} = \bar{\tilde{\psi}}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\tilde{\psi} + g\bar{\tilde{\psi}}\gamma_0\left(\overrightarrow{\alpha}\right)\gamma_0\Sigma_0\left(\overrightarrow{\alpha}\right)\tilde{\psi} + \frac{1}{2}\left(\dot{A}^i - \partial^i A^0\right)^2 - \frac{1}{4}F_{ij}F^{ij} + \bar{g}\varepsilon_{ijk}\dot{A}^i\partial_j\dot{A}^k$$
$$- \left(e\,\bar{\tilde{\psi}}\gamma^0\tilde{\psi}\right)A_0 + \mathscr{L}_{int}(A^i, \tilde{\psi}), \tag{4.19}$$

with the standard definition for the electromagnetic tensor $F_{\mu\nu}$. Here have introduced the new perturbation parameter $\bar{g} = \xi/M$. As we can see from (4.19) the additional photon contribution is not of the HOTD type. Nevertheless, the correction term proportional to $\bar{g}$ in Eq.(4.19) will demand some modifications in the correct quantization procedure. In the sequel we denote $e\,\bar{\tilde{\psi}}\gamma^0\tilde{\psi} = J^0$. The contribution $\mathscr{L}_{int}(A^i, \tilde{\psi})$, defining the interaction among fermions and transverse photons, has a linear and quadratic dependence upon A_i arising from the operator $\left(\overrightarrow{\alpha}\right)\gamma^0\Sigma_0\left(\overrightarrow{\alpha}\right)$ in (4.8). Its precise form is not relevant now for our purpose of quantizing the free sector of the theory.

In order to identify the proper normal modes of the photon it is convenient to proceed according to the Hamiltonian formulation, which goes along similar lines that in the standard case. Our notation here corresponds to that of a three-dimensional euclidean space where the relevant vectors are: $\mathbf{A} = (A^i)$, $\Pi = (\Pi_i)$, $\nabla = (\partial_i)$, $i = 1,2,3$, and $\varepsilon_{123} = 1$. The corresponding canonically conjugated momenta are

$$\Pi_0 = 0, \quad \Pi_i = \left(\delta_{ik} + 2\bar{g}\varepsilon_{ijk}\partial_j\right)\dot{A}^k + \partial_i A^0. \tag{4.20}$$

The photon sector of the Hamiltonian density, calculated to order $\bar{g}$, is

$$\mathscr{H}_{QED,\gamma} = \frac{1}{2}(\Pi_i)^2 + \frac{1}{4}F_{ij}F^{ij} + \left(\partial_i\Pi_i + J^0\right)A^0 - \bar{g}\varepsilon_{ijk}\Pi_i\partial_j\Pi_k - \mathscr{L}_{int}(A^i, \tilde{\psi}). \tag{4.21}$$

The evolution of the constraint $\Pi_0 = 0$ produces the Gauss law $\partial_i\Pi_i + J^0 = 0$ as a secondary constraint. Its further evolution must be consistent with current conservation via the equations of motion, so that we recover the standard two first class constraints of electrodynamics. A first gauge fixing is provided by choosing

$$\Pi_0 = 0, \quad A^0 = -\frac{1}{\nabla^2}(J^0 + \partial_0\partial_i A^i). \tag{4.22}$$

To complete the gauge fixing it is convenient to separate the photon fields into longitudinal and transverse components

$$\Pi_i^T = \dot{A}_T^i + 2\bar{g}\varepsilon_{ijk}\partial_j\dot{A}_T^k, \quad \Pi_i^L = \dot{A}_L^i + \partial_i A^0, \quad A_L^i = \frac{1}{\nabla^2}\partial_i(\partial_k A^k), \tag{4.23}$$

and to demand the Coulomb gauge $\partial_i A^i = 0$, which is equivalent to the requirement of $A_L^i = 0$. In this way the Gauss law turns out to be identically satisfied in virtue of the properties $\partial_i A^i = \partial_i A_L^i$, $\partial_i\Pi_i = \partial_i\Pi_i^L$ together with choice (4.22) for A^0. The longitudinal fields are fixed according to

$$A_L^i = 0, \qquad \Pi_i^L = -\frac{1}{\nabla^2}\partial_i J^0. \tag{4.24}$$

In this way the remaining independent degrees of freedom are the transverse fields

$$A^i = A_T^i, \qquad \Pi_i^T = \dot{A}_T^i + 2\bar{g}\varepsilon_{ijk}\partial_j\dot{A}_T^k, \tag{4.25}$$

satisfying the Poisson brackets

$$\left\{A_T^i(t,\mathbf{x}),\ \Pi_m^T(t,\mathbf{y})\right\} = \left(\delta_{im} - \frac{\partial_i\partial_m}{\nabla^2}\right)\delta^3(\mathbf{x}-\mathbf{y}), \tag{4.26}$$

with the remaining been zero.

In order to allow for consistency among: (i) the equal time commutators for the transverse fields arising from Eq.(4.26), (ii) the expansion of them in term of frequency modes and (iii) the standard bosonic creation-annihilation operator commutation relations, we need to introduce a canonical transformation (to order $\bar{g}$) which guaranties that $\tilde{\Pi}_i^T = \partial_0 \tilde{A}_T^i$, as opposed to the second equation (4.25). Such transformation is

$$\tilde{\Pi}_i^T = (\delta_{ik} - \bar{g}\,\varepsilon_{isk}\partial_s)\,\Pi_k^T, \qquad \tilde{A}_T^i = (\delta_{ik} + \bar{g}\,\varepsilon_{isk}\partial_s)\,A_T^k, \tag{4.27}$$

which defines the physical fields of the theory. The normal mode expansion of the photon field is

$$\tilde{A}_T^i(x) = \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3}} \sum_{\lambda=\pm} \sqrt{\frac{1}{2\omega_\lambda(\mathbf{k})}} \left[a_\lambda(\mathbf{k})\,\varepsilon^i(\lambda,\mathbf{k})e^{-ik^\lambda\cdot x} + h.c.\right]. \tag{4.28}$$

Here $a_\lambda(\mathbf{k}), a_\lambda^\dagger(\mathbf{k})$ are standard creation-annihilation operators and the complex numbers $\varepsilon^i(\lambda,\mathbf{k})$ define a circularly polarized basis with helicity λ. The notation is $k^\lambda \cdot x = \omega_\lambda(\mathbf{k})\,t - \mathbf{k}\cdot\mathbf{x}$, where the modified positive frequencies (to first order in $\bar{g}$) are

$$\omega_\lambda(\mathbf{k}) = |\mathbf{k}|(1-\lambda\bar{g}|\mathbf{k}|). \tag{4.29}$$

Again, the effective character of the theory manifests itself in the condition $|\mathbf{k}| < 1/\bar{g} = M/\xi$, which will be further discussed in the last section.

In this way, the final photon Hamiltonian density arising from (4.21) reads

$$\mathscr{H}_{MP,\,\gamma} = \frac{1}{2}(\tilde{\Pi}^T)^2 + \left(\frac{1}{2}\tilde{\mathbf{B}}^2 - \bar{g}\tilde{\mathbf{B}}\cdot(\nabla\times\tilde{\mathbf{B}})\right) - \frac{1}{2}J^0\frac{1}{\nabla^2}J^0 - \tilde{\mathscr{L}}_{int}(\tilde{A}_T,\tilde{\psi}). \tag{4.30}$$

As usual $\tilde{\mathbf{B}} = \nabla\times\tilde{\mathbf{A}}$. Let us emphasize that the contribution $\frac{1}{2}\tilde{\mathbf{B}}^2 - \bar{g}\tilde{\mathbf{B}}\cdot(\nabla\times\tilde{\mathbf{B}}) = \frac{1}{2}\left(\tilde{\mathbf{B}} - \bar{g}\nabla\times\tilde{\mathbf{B}}\right)^2$ is positive definite to order $\bar{g}$. This can also be verified by calculating the normal ordered expression for the free sector of the Hamiltonian arising from (4.30), using the expression (4.28), which leads to the expected result

$$\tilde{H}_0 = \int d^3\mathbf{k} \sum_{\lambda=\pm} \omega_\lambda(\mathbf{k})\,a_\lambda^\dagger(\mathbf{k})\,a_\lambda(\mathbf{k}), \tag{4.31}$$

in terms of the corresponding positive frequencies (4.29), to the order considered.

The transverse photon propagator is

$$i\Delta_T^{ij}(x,y) \equiv \langle 0|\,T\left(\tilde{A}_T^i(x)\tilde{A}_T^j(y)\right)|0\rangle\,. \tag{4.32}$$

In momentum space we obtain

$$\Delta_T^{ij}(k_0,\mathbf{k}) = \sum_{\lambda=\pm}\frac{1}{2\omega^\lambda(\mathbf{k})}\left(\frac{\varepsilon^i(\lambda,\mathbf{k})\,\varepsilon^{j*}(\lambda,\mathbf{k})}{\left(k_0-\omega^\lambda(\mathbf{k})+i\varepsilon\right)} - \frac{\varepsilon^j(\lambda,-\mathbf{k})\,\varepsilon^{i*}(\lambda,-\mathbf{k})}{\left(k_0+\omega^\lambda(\mathbf{k})-i\varepsilon\right)}\right). \tag{4.33}$$

The required expressions for the polarization vectors contributions are explicitly given by

$$\varepsilon^i(\lambda,\mathbf{k})\,\varepsilon^{j*}(\lambda,\mathbf{k}) = \frac{1}{2}\left[\delta_{ij} - \frac{k_ik_j}{|\mathbf{k}|^2}\right] - \lambda\frac{i}{2}\left[\varepsilon_{ijm}\frac{k_m}{|\mathbf{k}|}\right]. \tag{4.34}$$

5. Final comments

We have constructed a perturbative modification to the classical version of the photon-fermion sector of the Myers-Pospelov model, which has been subsequently quantized at the non-interacting level. The LIV terms, which in general are of the HOTD character, are assumed to represent very small perturbations over standard QED. In this way, the resulting interacting quantum theory will be required to reproduce the Lorentz covariant results of standard QED in the limit when the LIV parameters go to zero. That is to say, the quantum model should be able to smoothly interpolate between the initial Lorentz violating theory and the final Lorentz preserving one. Such a realization already exists in the literature for theories characterized by a fully dimensionless Lorentz violating parameter [14]. In our case Lorentz invariance violation is characterized by the energy scale M, so that these methods would not be directly applicable.

To perform the free field quantization we have started at the classical level using the method of Ref. [8], which allowed us to successfully deal with two of the general problems originating from the HOTD character of the terms describing the Lorentz violations in the model: (i) the increase in the number of degrees of freedom, which we must not allow in a perturbative modification of standard QED, and (ii) the appearance of Hamiltonians which are not bounded from below, which do not provide a good starting point for quantization. The calculation of interacting processes is deferred for a future publication.

Some remarks regarding the effective character of the model, together with the associated characteristic energy scales are now in order. The combinations of parameters $\xi/M, \eta_{1,2}/M$, denoted collectively by Ξ/M, appearing in Eqs. (2.1-2.2) are considered as remnants of a more fundamental quantum gravity (QG) theory, which include effects that make space no longer describable in terms of a continuum. Such parameters could arise in the process of calculating expectation values of well defined QG operators in semiclassical states that describe Minkowski space-time, for example, which would be necessary to derive the induced corrections to standard particle dynamics at low energies. Let us emphasize that what is bounded by experiments or observations is the ratio Ξ/M, so that a neat separation of the scale M and the correction coefficients Ξ, that could even be zero if no corrections arise, is not possible until a semiclassical calculation is correctly performed starting from a full quantum theory. Initially, the naive expectation was that taking $M = M_{Planck}$ will be consistent with Ξ values of order one, which is certainly not the case. Nevertheless, we should not rule out rather unexpected values of Ξ or M until the correct calculation is done.

Let us assume that we have identified the correct separation in Ξ_{QG}/M_{QG} consistent with the experimental bounds and arising from a correct semiclassical limit of the QG theory. Then we will interpret M_{QG} as the scale in which quantum effects are manifest and where space is characterized by strong fluctuations forbidding its description as a continuum.

Nevertheless, another scale $\bar{M}$ naturally arises in this approach, which is the one that separates the continuum description of space from a foamy description related to quantum effects. That is to say, for probe energies $E < \bar{M}$ we are definitely within the standard continuum description of space where EFT methods should apply. For probe energies $E > \bar{M}$ we enter the realm of quantum gravity and there we assume that any EFT has to be replaced by an alternative description. It is natural that a very large number of the basic quantum cells of space characterized by the scale $(1/M_{QG})^3$ will contribute to the much larger cells characterizing a continuum description, so that we expect $\bar{M} \ll M_{QG}$.

The maximum allowed momenta $|\mathbf{k}_{max}| \approx M_{QG}/\Xi_{QG}$ in the theory will be mathematically dictated by the positivity of the normal modes energies (4.10), (4.29) and certainly constitutes an extrapolation of the EFT that can be considered as the analogous of taking the maximum momentum equal to infinity in the standard QED case.

Nevertheless, we need to introduce and additional suppression of the excitation modes in our EFT which will be settled by the scale $\bar{M}$, thus defining the effective energy range of the model. This is required by the EFT description of excitations in space which demands that the Compton wave length $1/|\mathbf{k}|$ of the allowed excitations be larger than the scale $1/\bar{M}$ setting the onset of the continuum. The implementation of this proposal is directly related with our demand that the quantum model constructed from the MP theory be such that it produces a continuous interpolation between those physical results including Ξ corrections and those predicted by standard QED ($\Xi = 0$). Preliminary calculations of radiative corrections indicate that such suppression, together with the desired limit, can be achieved by the following prescription: (i) even though the loop integrals are all finite in the adopted MP setting, for each would be divergent integral in the limit $\Xi \to 0$ we introduce the required covariant Pauli-Villars type factor with mass parameter $\bar{M} \gg m^2$

$$\frac{1}{k^2 - m^2} \to \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \bar{M}^2} \to \frac{1}{k^2 - m^2}\left(\frac{\bar{M}^2}{\bar{M}^2 - k^2}\right). \tag{5.1}$$

Besides providing a cutoff for the excitation modes in the region $\bar{M} < |\mathbf{k}| < M_{QG}/\Xi_{QG}$, further motivation for such factor is that it would correspond to an adequate smooth regulator when $\Xi \to 0$. (ii) Since the relevant scales are such that $\bar{M} \ll M_{QG}/\Xi_{QG}$, the correct limit to standard QED will be defined by first taking $\Xi_{QG}/M_{QG} \to 0$, i.e. $|\mathbf{k}_{\max}| \to \infty$ and subsequently $\bar{M} \to \infty$. A renormalization prescription consistent with this proposal needs to be implemented at the level of the MP model.

The work of CMR, LU and JDV has been partially supported by the projects DGAPA-UNAM # IN109107, CONACYT # 47211-F and CONACYT # 55310.

References

[1] R. C. Myers and M. Pospelov, *Ultraviolet modifications of dispersion relations in effective field theory*, *Phys. Rev. Lett.* **90**, 211601 (2003) [hep-ph/0301124].

[2] P. A. Bolokov and M. Poslepov, *Classification of dimension 5 Lorentz violating interactions in the Standard Model*, hep-ph/0703291.

[3] D. Colladay and V. A. Kostelecký, *CPT violation and the Standard Model*, *Phys. Rev. D* **55**, 6760 (1997) [hep-ph/9703464]; D. Colladay and V. A. Kostelecký, *Lorentz violating extension of the Standard Model*, *Phys. Rev. D* **58**, 116002 (1998) [hep-ph/9809521].

[4] X. Jaen, J. Llosa and A. Molina, *A Reduction of order two for infinite order lagrangians*, *Phys. Rev. D* **34**, 2302 (1986).

[5] J. Z. Simon, *Higher Derivative Lagrangians, Nonlocality, Problems And Solutions*, *Phys. Rev. D* **41**, 3720 (1990).

[6] R. Montemayor and L. Urrutia, *Synchrotron radiation in Lorentz-violating electrodynamics: The Myers-Pospelov model*, *Phys. Rev. D* **72**, 045018 (2005) [hep-ph/0505135]; R. Montemayor and L. Urrutia, *Synchrotron radiation in Myers-Pospelov effective electrodynamics*, *Phys. Lett. B* **606**, 86 (2005) [hep-ph/0410143].

PoS(QG-Ph)040

[7] D. A. Eliezer and R. P. Woodard, *The Problem of Nonlocality in String Theory*, *Nucl. Phys. B* **325**, 389 (1989).

[8] T.-C. Cheng, P.-M. Ho and M.-C. Yeh, *Perturbative approach to higher derivative and nonlocal theories*, *Nucl. Phys. B* **625**, 151 (2002) [hep-th/0111160].

[9] J. Collins, A. Pérez, D. Sudarsky, L. Urrutia and H. Vucetich, *Lorentz invariance: An additional fine tunning problem*, *Phys. Rev. Lett.* ***93**, 191301 (2004* [gr-qc/0403053].

[10] P. M. Grignino and H. Vucetich, *Quantum corrections to Lorentz invariance violating theories: Fine-tuning problem*, *Phys. Lett. B* **651**, 313 (2007) [hep-th/0607214].

[11] L. Maccione, S. Liberati, A. Celotti and J. G. Kirk, *New Constraints on Planck-scale Lorentz Violating in QED from the Crab Nebula*, *JCAP*, **0710**: 013 (2007) [arXiv:0707.2673]. For additional discussion see for example: D. Mattingly, *Lorentz violating effective field theories: current limits and open questions*, these Proceedings.

[12] M. Ostrogradski, *Mémoire sur les équations différentielles relatives aux problèmes des isopérimètres*, *Mem. Acad. St. -Pétersbourg* **VI**, 385 (1850).

[13] T.-C. Cheng, P.-M. Ho and M.-C. Yeh, *Perturbative approach to higher derivative theories with fermions*, *Phys. Rev. D* **66**, 085015 (2002)[hep-th/0206077].

[14] J. Alfaro, *Quantum gravity and Lorentz invariance deformation in the standard model*, *Phys. Rev. Lett.* **94**, 221302 (2005)[hep-th/0412295]; J. Alfaro, *Quantum gravity induced Lorentz invariance violation in the Standard Model: Hadrons*, *Phys. Rev. D* **72**, 024027 (2005)[hep-th/0505228].