PROCEEDINGS OF SCIENCE

Opening Remarks

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Beginning in 2004, much progress has been made towards the determination of $|V_{us}|$ using new measurements of branching ratios, lifetimes and form factor parameters in semileptonic kaon decays. My discussion here concentrates on some presently unsatisfactory aspects about these measurements, especially the attempts to determine strongly correlated parameters that leads to strong fluctuations. There are large discrepancies between the results from different experiments, especially for the parameter λ_0', which have been combined, using the dubious procedure of introducing scale factors. This poor practice accepts clearly incorrect results which leads to incorrect central values that, after just enlarging the errors, still remain incorrect. Yet, when compared to the situation of only a few years ago, there has indeed been *large* progress in the knowledge of $|V_{us}|$ and the verification of lepton universality. Means for further improving the determination of several parameters are indicated.

KAON International Conference
May 21-25 2007
Laboratori Nazionali dvi Frascati dell'INFIN, Rome, Italy

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1. Welcome from KLOE

The Laboratori Nazionali di Frascati, LNF, is the home of the KLOE experiment. All members of the KLOE collaboration who designed, built and use the KLOE detector, are very proud that the 2007 edition of the International Kaon Conference is hosted by LNF and joins in welcoming the participants. KLOE is at present hibernating and the KLOE collaboration is quite busy analyzing the data collected prior to the run end in early 2006. The detector is parked in its assembly hall and remains fully operational. This fall, a crucial accelerator experiment will be carried out on DAΦNE. There are good reasons to expect a significant increase in luminosity and improved background conditions. If the experiment is a success, KLOE will be back on the beam in 2009. All this will be presented Thursday afternoon during the panel discussion.

2. 60 years of kaon physics

Kaons were discovered 60 years ago, [1]. 1963 was an important year for kaon physics. That year Nicola Cabibbo [2], proposed Universality as a way of avoiding introducing additional couplings in the weak interactions. Extending the idea of Cabibbo's angle, through GIM [3] and then Kobayashi and Maskawa [4], we arrived to the flavor mixing CKM matrix that can accommodate CP violation. $\not{C}\not{P}$ was also discovered in 1963. While the official publication of Cronin, Christenson, Fitch, and Turlay is dated 1964 [5], the result was known before the end of '63, at least in Brookhaven. It took a long time to get to prove the existence of direct $\not{C}\not{P}$ and even longer to arrive to an accurate verification of Cabibbo unitarity. In 2004, KTeV presented the first good measurements of the K_L semileptonic branching ratios in this hall, [6]. The subsequent two years have seen quite a consolidation of our knowledge of $|V_{us}|$, essentially $\sin\theta_C$, [6, 7, 8, 9].

There are still some unsatisfactory points with the $|V_{us}|$ business, especially some wild discrepancies in the value of the form factor parameters. I will briefly comment about some of this, also with respect to two questions raised at the last kaon meeting, Kaon 05, by Vincenzo Cirigliano [10] and Giancarlo D'Ambrosio [11].

Before continuing I would like to recall that unitarity of the CKM flavor mixing matrix translates into 6 "unitary triangles", as many as the zeros of the Kronecker's δ_{ij} in three dimensions, [12]. The "kaon", J_{12}, and "B", J_{13}, triangles are shown in figure 1.

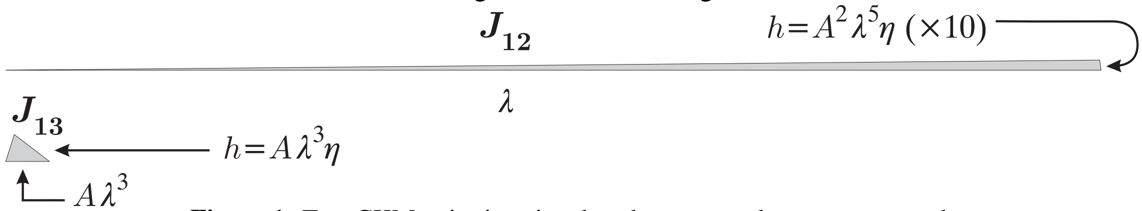

Figure 1: Two CKM unitarity triangles, drawn to scale except as noted.

3. Measuring the $K\pi$ current form factor parameters

3.1 Definitions and error estimates

The matrix element of the hadronic current between a kaon and a pion has the general form

$$\langle\pi(p)|\bar{u}\gamma_\alpha s|K(P)\rangle = f(0)\left((P+p)_\alpha\tilde{f}_+(t) + (P-p)_\alpha\left(\tilde{f}_0(t)\frac{\Delta}{t} - \tilde{f}_+(t)\frac{\Delta}{t}\right)\right) \tag{3.1}$$

where P and p are the kaon and pion 4-momenta, $t = (P-p)^2 = M^2 + m^2 - 2ME_\pi$, $\tilde{f}_{+,0}(0) = 1$ and $\Delta = M^2 - m^2$, with $M = m(K)$ and $m = m(\pi)$. $f(0)$ accounts for $SU(3)$ breaking effects and must be obtained from prime principles. The spectrum obtained from the amplitude above is distorted by radiative corrections. Because I am interested in understanding the errors in the determination of the FF parameters, I will ignore radiative corrections which are relevant to values of the parameters but do not affect the statistical errors.

The shape of the $\tilde{f}$ form factors, necessary for computing the phase space integrals in $K_{\ell 3}$ decays is obtained experimentally from the pion energy spectrum. The form factors can be parameterized as polynomials in t:

$$\tilde{f}_i(t) = 1 + \lambda_i' \frac{t}{m^2} + \frac{\lambda_i''}{2} \frac{t^2}{m^4} \tag{3.2}$$

with i= + or 0, or as poles in the S-wave and P-wave $\pi - K$ scattering amplitude:

$$\tilde{f}_i(t) = \frac{M_j^2}{M_j^2 - t} \tag{3.3}$$

where $i = +,\ 0$ and $j = j(i) = V,\ S$ for $i = +,\ 0$. The expressions in equation 3.2 and 3.3 are trivially related. Expanding the pole form:

$$\frac{M_j^2}{M_j^2 - t} = 1 + \frac{t}{M_j^2} + \frac{t^2}{M_j^4} \cdots$$

from which λ_+'=m^2/M_V^2, λ_+''=$2\times\lambda_+'^{\,2}$. Let $F(\mathbf{p},x)$ be a probability density function, PDF, where $\mathbf{p}$ is some parameter vector, which we want to determine and x is a running variable, like t. The inverse of the covariance matrix for the maximum likelihood estimate of the parameters is given by [13] ($\mathrm{d}\upsilon$ is the appropriate volume element):

$$(\mathbf{G}^{-1})_{ij} = -\frac{\partial^2 \ln L}{\partial p_i \partial p_j}, \qquad \text{from which, for } N \text{ events,} \qquad \left(\mathbf{G}^{-1}\right)_{ij} = N \int \frac{1}{F} \frac{\partial F}{\partial p_i} \frac{\partial F}{\partial p_j} \mathrm{d}\upsilon$$

3.2 Errors for the λ_+ parameters from K_{e3} decays

Only the term in $(P+p)_\alpha$ of equation 3.1 contributes to K_{e3} decays. Since the form factor, FF, is a function of $t = M^2 + m^2 - 2ME_\pi$, the form factor parameters are obtained from a fit to the pion spectrum, after integration over the electron (or neutrino) momentum. Using the form factor from equation 3.2, the covariance matrix obtained from a fit to the pion energy is given by:

$$\mathbf{G} = \begin{pmatrix} \overline{\delta\lambda_+'^{\,2}} & \overline{\delta\lambda_+'\delta\lambda_+''} \\ \overline{\delta\lambda_+''\delta\lambda_+'} & \overline{\delta\lambda_+''^{2}} \end{pmatrix} = \frac{1}{N} \begin{pmatrix} 1.25854^2 & -0.606064 \\ -0.606064 & 0.509406^2 \end{pmatrix}.$$

I have used λ_+'=0.025 and λ_+''=0.00125 to obtain the result above which however is not very sensitive to the λ values. For example, from a 4% change of λ_+', the changes in $\delta\lambda_+'$, $\delta\lambda_+''$ and the correlation are, respectively, 0.25%, 0.17% and 0.01%. The same result holds for fits to the pion momentum as well as to the pion transverse momentum distribution. *No information is lost* using the transverse component of the pion momentum. For 10^6 events the statistical errors and the correlation, $\rho(a,b) = \overline{\delta a \delta b}/(\delta a \times \delta b)$, are:

$$\begin{array}{l}\delta\lambda'_+ = 0.00126 \\ \delta\lambda''_+ = 0.00051\end{array}, \quad \rho(\lambda'_+, \lambda''_+) = -94.5\%.$$

The unpleasant (but to be expected) "surprise" is the large, negative correlation between the two parameters of the FF expansion, which allows a trading between λ'_+ and λ''_+, resulting in almost tripling the error for the coefficient of t with respect to assuming a linear t dependence. This is quite evident when comparing results from fits ignoring λ''_+, [6, 7, 8, 9]. One finds λ'_+∼0.029±0.0003, while the inclusion of λ''_+ gives λ'_+=0.025.., λ''_+∼0.0012... The approximate rule is that ignoring λ''_+ in the fit increases λ'_+ by $\sim 3\times\lambda''_+$.

Another disadvantage of using a FF of the form of equation 3.2 is that small statistical fluctuation can lead to wrong results for the phase space integral. This is illustrated in figure 2 which shows the results from fitting 100 pion spectra of 10^6 events each. KTeV was the first experiment to experience this problem. They noticed [6] that the pole fit was inconsistent with the quadratic fit. The phase space integral computed from the quadratic FF fit is therefore different from that from a pole fit. For some reason they chose to prefer the value from the former, adding as systematic uncertainty the difference between the two values, underestimating however the value of the integral. The KTeV result fluctuated to the extreme upper left corner of figure 2. We know today that their pole answer was correct.

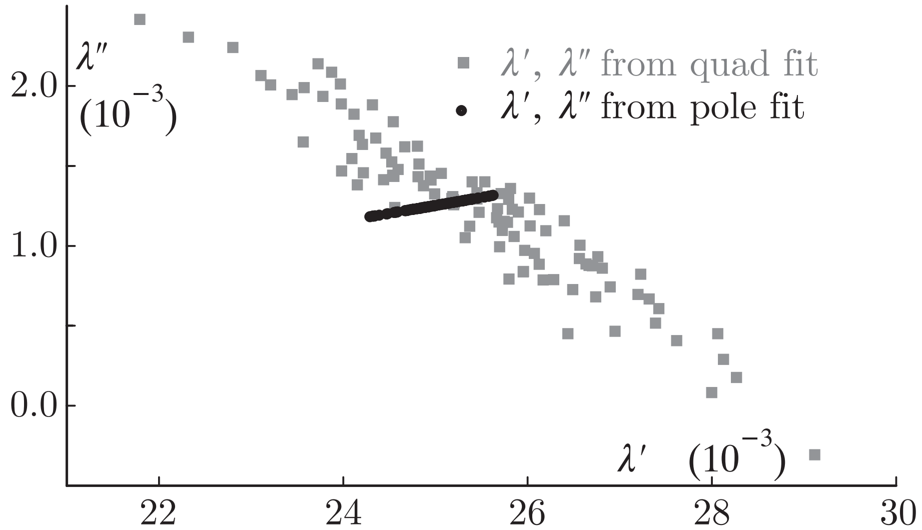

Figure 2: Spread of the λ'_+ and λ''_+ values from fits to 100 pion spectra with quadratic and pole FF.

Results reported by KTeV, ISTRA+, NA48 and KLOE [6, 7, 8, 9] are quite consistent and are given elsewhere [14]. The averaged values for λ'_+ and λ''_+ from the K_{Le3} measurements [6, 8, 9], λ'_+=0.02486±0.00113, λ'_+=0.00153±0.00046 are in perfect agreement with the values computed from the average of the pole fits to the same data, λ'_+=0.02542±0.00003 and λ''_+=0.00129±0.000032. Fit with pole FF have a slightly higher CL, 40% vs 30% for the quadratic fits. The validity of the pole form is confirmed by at least two phenomenological analysis, [15, 16]. It is ironic that KTeV chose to give a value for the phase space integral which was 0.6% low with an adjusted fractional error of 0.68%, while their pole fit gave the correct answer, with an error of only 0.15%. At the present level of accuracy, pole fits are preferable to quadratic fits. The answer to D'Ambrosio [11] is therefore that we should use the pole as standard, at least until improved experimental accuracy allows checking whether small corrections to a simple pole form are needed [17].

3.3 Errors for the $\lambda_{+,0}$ parameters from $K_{\mu 3}$ decays, I

While measurements of the vector FF are satisfactory, that is not the case for the scalar FF. The pion spectrum depends on both $\tilde{f}_+$ and $\tilde{f}_0$. If a quadratic form is assumed for both FF we have to determine four parameters: λ'_+, λ''_+, λ'_0 and λ''_0. There is a folk-lore that a measurement of λ''_0 might help improve the value of $f(0)$ although it is not clear by how much [10]. It is however a moot point: λ''_0 is not measurable. The covariance matrix for the four parameters is

$$\begin{pmatrix} \overline{\delta\lambda_0'^2} & \overline{\delta\lambda_0'\delta\lambda_0''} & \overline{\delta\lambda_0'\delta\lambda_+'} & \overline{\delta\lambda_0'\delta\lambda_+''} \\ \overline{\delta\lambda_0''\delta\lambda_0'} & \overline{\delta\lambda_0''^2} & \overline{\delta\lambda_0''\delta\lambda_+'} & \overline{\delta\lambda_0''\delta\lambda_+''} \\ \overline{\delta\lambda_+'\delta\lambda_0'} & \overline{\delta\lambda_+'\delta\lambda_0''} & \overline{\delta\lambda_+'^2} & \overline{\delta\lambda_+'\delta\lambda_+''} \\ \overline{\delta\lambda_+''\delta\lambda_0'} & \overline{\delta\lambda_+''\delta\lambda_0''} & \overline{\delta\lambda_+''\delta\lambda_+'} & \overline{\delta\lambda_+''^2} \end{pmatrix} = \frac{1}{N}\begin{pmatrix} 63.9^2 & -1200 & -923 & 197 \\ -1200 & 18.8^2 & 272 & -59 \\ -923 & 272 & 14.8^2 & -49 \\ 197 & -59 & -48 & 3.4^2 \end{pmatrix}$$

and the normalized correlations are

$$\begin{pmatrix} -0.9996 & -0.974 & 0.91 \\ & 0.978 & -0.919 \\ & & -0.976 \end{pmatrix}.$$

The latter are all very close to ± 1, note $\rho(\lambda'_0, \lambda''_0)$=−99.96%, which results in $\delta\lambda'_0$ and $\delta\lambda''_0$ growing out of control. In particular $\delta\lambda''_0$=0.0188 for 10^6 events or $\delta\lambda''_0$=0.00188 for 10^8 events. Since we expect λ''_0 to be of $\mathcal{O}$(0.0005), 10^8 events provide a ∼400% measurement of $\delta\lambda''_0$, which is no measurement at all. It is therefore not possible to measure the "curvature" of the scalar FF and contribute to knowledge about $f(0)$. The answer to Vincenzo Cirigliano is "no" [10]. In practice a χ^2 fit will not even converge.

3.4 Errors for the $\lambda_{+,0}$ parameters from $K_{\mu 3}$ decays, II

Ignoring λ''_0 the covariance matrix for λ'_0, λ'_+ and λ''_+ is

$$\frac{1}{N}\begin{pmatrix} 1.75^2 & 3.32 & -1.88 \\ 3.32 & 3.09^2 & -3.87 \\ -1.88 & -3.87 & 1.34^2 \end{pmatrix}$$

which shows that λ'_0 can be reasonably measured. The problem here is that the form factor does contain higher powers of t. Ignoring such terms however results in a systematic shift in the value of λ'_0 and ultimately the estimate of the phase space integral necessary to obtain $|V_{us}|$ from the $K_{\ell 3}$ decay widths. It is easy to get an estimate of this shift. The scalar form factor is dominantly a pole,

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which gives the relation λ_0''=$2\lambda_0'^2$. With this condition, the result of a fit with a linear scalar FF gives a result for λ_0' systematically higher by$\sim$$3\times\lambda_0''$as illustrated in figure 3.

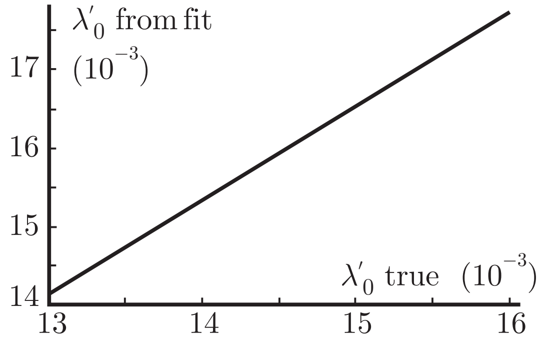

Figure 3: λ_0' from fit vs λ_0' true, due to λ_0'-λ_0'' correlation.

We therefore conclude that all results reported for the value of λ_0' are systematically higher than the correct value.

A possible way out is to simultaneously fit K_{e3} and $K_{\mu3}$ or appropriately combine the results from separate fits. We note however that even if λ_+' and λ_+'' were known without errors, λ_0'' would be determined to ±500% with 1 million events or ±50% with 100 million events, statistical error only of course.

The systematically higher results from a fit with a scalar FF linear in t gives a higher value for the phase space integral. The integral, given by 0.363426+ 0.822071λ_0'+1.442195($\lambda_0'^2$+λ_0'')+$\ldots$, changes by 0.15% if λ_0'=0.015 and the presence of a quadratic term is ignored. This of course is not much, when compared to the present experimental situation, with λ_0' spanning the interval 0.0095 (NA48) to 0.0128 (KTeV), 0.0156 (KLOE) and 0.0171, ISTRA+. The latter is clearly an unacceptable situation, especially when all the parameter's values are averaged. Palutan [14] showed a fit to all slope and "curvature" results, with a CL of 10^{-6}. There is no question in my mind that some results are just plainly wrong. Moreover the errors quoted are also at times incorrect. The covariance matrices that I have given are of course ideal lower limits to the errors. Resolution, full coverage and limited Monte Carlo statistics all contribute to enlarging the statistical errors.

It is clear that since a linear fit for $\tilde{f}_0(t)$ is incorrect and a quadratic fit is impossible we must find a better way of doing things. Stern and collaborators [17] have developed a dispersive approach that is approximately equivalent to setting λ_0''=$\lambda_0'^2$+0.000416 in the scalar FF. This procedure reduce to one the number of parameters in $\tilde{f}_0(t)$ making the fit possible. Note that for λ_0'=0.014, $\lambda_0''$$\sim$0.0002+0.0004$\sim$3 $\lambda_0'^2$ and all λ_0' values from a linear $\tilde{f}_0(t)$ fit are even more wrong than what is indicated in figure 3.

3.5 Systematic errors

In addition to what has been discussed so far there are imponderables which we all like to add on and call systematics errors. In the end, it all comes to a doubling (or more) of the errors with respect to the values I have derived, which is quite reasonable. The ISTRA+ group is always very optimistic in this respect and I also believe they made a mistake when they rewrote their original K_{e3} paper in order to claim observation of a quadratic term in their data. They give the same systematic uncertainty for λ'_+ in the linear and the quadratic fit. The statistical error approximately triples and the same should happen for the systematic error. Doing this aligns their one sigma contour with all others, see figure 4.

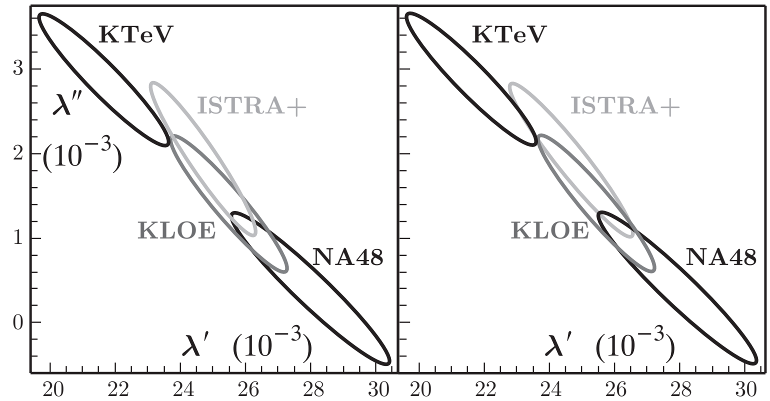

Figure 4: 1 σ contours. Left: original ISTRA+ errors. Right ISTRA+ systematic error tripled .

KTeV is quite conservative with their systematics but I believe they got confused on how to fit the data and also did not recognize that their pole solution was the right one. NA48 is over cautious with their K_{e3} analysis, they give the largest errors with the largest data sample of all. They are on the contrary sloppy with their $K_{\mu 3}$ analysis. KLOE is certainly very conservative, that is not so good either. One last comment. All fits to a quadratic form in t must give the same error ratios and correlation, *i.e.* one and the same shape. I have distorted the pion spectrum from K_{e3} decays multiplying it with the factors $1 \pm 0.05\,z$ and $1 \pm 0.05\,z^2$, where $z = (E(\pi) - E_{min}(\pi))/(E_{max}(\pi) - E_{min}(\pi))$. Fitting the distorted spectra, I obtain the results in figure 5. Note that while λ'_+ and λ''_+ move around a lot, the errors $\delta\lambda'_+$, $\delta\lambda''_+$ and their correlation hardly change at all.

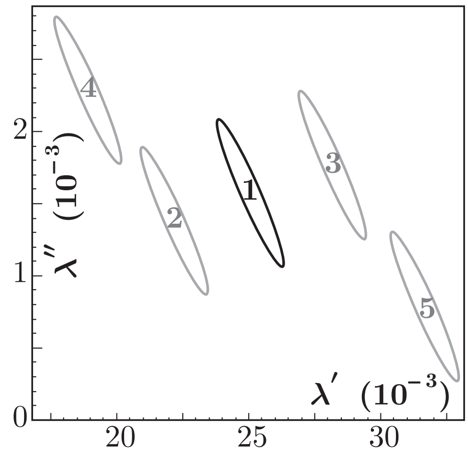

Figure 5: 1 σ contours. Original spectrum, 1 and distorted spectra 2-5.

This proves that results showing strangely different contours, other than an overall scale due to the number of events and the estimates of systematic effects, are suspect.

The accuracy achieved at present is barely adequate for obtaining $|V_{us}|$ at the 0.2% level. In fact we can do better, if we were to reduce inconsistencies between experiments and among modes. Consistency is poor everywhere, branching ratios and form factor parameters. Where it really is terrible is in the measurements of the scalar FF. This is due in part to the intrinsical difficulty of

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measuring strongly correlated quantities. However the probability that the four measured values fluctuate the way they do is of $\mathcal{O}(10^{-8})$, ignoring correlations.

4. A better parameterization for the form factor

As discussed above, trying to fit for two highly correlated parameters results in large fluctuation which could be avoided. Both experiment and phenomenology suggest that a good parameterization of the vector FF is:

$$\tilde{f}_+(t) = 1 + \lambda_+ \frac{t}{m^2} + \lambda_+^2 \frac{t^2}{m^4}$$

which includes the established t^2 dependence of the FF. For the scalar FF a dispersive approach has been developed [17]. In its approximate form it suggests the relation:

$$\tilde{f}_0(t) = 1 + \lambda_0 \frac{t}{m^2} + \frac{\lambda_0^2 + 0.000416}{2} \frac{t^2}{m^4}.$$

The total number of parameters is thus reduced to 2 from 4 while allowing for t^2 terms in the FFs. The Hill parameterization for the FF [19], on the other hand, does not solve the problem of too many parameters. As applied by KTeV to their K_{e3} decay data [20], leads to a FF with an even larger t^2 term than their original quadratic fit, see figure 2 in [20], in clear contradiction with the average of all available data.

5. Conclusions

Progress since 2004 is outstanding! The product $f(0)\,V_{us}$ is known to 0.2% and $|V_{us}|$ itself to 0.84%, where the error is mostly from the theoretical uncertainty on $f(0)$. Unitarity of the CKM mixing matrix is satisfied, for its first row, to $\sim 1.2\sigma$ or $\sim$1 part in 1000, with $|V_{us}|$ and $|V_{ud}|$ contributing approximately equally to the uncertainty. Lepton universality, as verified with the $K_{\ell 3}$ modes, is reaching the accuracy obtained in pion decays and could go well beyond with better analysis and improved data statistics.

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