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Precision tests of the Standard Model with $K_{\ell 3}$ decays

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I present a brief overview of the theoretical status of $K_{\ell 3}$ decays and their use as probes of lepton universality, light quark mass ratios, and CKM unitarity. I discuss in some detail the constraints imposed on the light quark masses by current measurements of the ratio of charged to neutral $K_{\ell 3}$ rates.

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1. $K_{\ell 3}$ decays: generalities

With the advent of precision BR and lifetime measurements [1], semileptonic Kaon decays allow one to probe the nature of weak interactions at a level which begins to compete with precision electroweak tests at LEP [2] . Moreover, since the ratio of charged and neutral $K_{\ell 3}$ decay rates is sensitive to isospin-breaking combinations of the quark mass ratios, with the increasing experimental and theoretical precision $K_{\ell 3}$ decays can be used as probes of the quark mass ratios. In this talk I will review the theoretical input needed to perform tests on the nature of weak interactions and on the quark masses.

The decay rates for all four $K_{\ell 3}$ modes ($K = K^{\pm}, K^0, \ell = \mu, e$) can be written compactly as follows:

$$\Gamma(K_{\ell 3[\gamma]}) = \frac{G_F^2 \, S_{\rm ew} \, M_K^5}{192\pi^3} C^K I^{K\ell}(\lambda_i) \times |V_{us} \times f_+^{K^0\pi^-}(0)|^2 \times \left[1 + 2\Delta_{SU(2)}^K + 2\Delta_{\rm EM}^{K\ell}\right] . \tag{1.1}$$

Here G_F is the Fermi constant as extracted from muon decay, $S_{\rm ew} = 1 + \frac{2\alpha}{\pi}\left(1 - \frac{\alpha_s}{4\pi}\right) \times \log \frac{M_Z}{M_\rho} + O(\frac{\alpha\alpha_s}{\pi^2})$ represents the short distance electroweak correction to semileptonic charged-current processes [3], C^K is a Clebsh-Gordan coefficient equal to 1 ($1/\sqrt{2}$) for neutral (charged) kaon decay, while $I^{K\ell}(\lambda_i)$ is a phase-space integral depending on slope and curvature of the form factors. The latter are defined by the QCD matrix elements

$$\langle \pi^j(p_\pi)|\bar{s}\gamma_\mu u|K^i(p_K)\rangle = f_+^{K^i\pi^j}(t)\,(p_K + p_\pi)_\mu + f_-^{K^i\pi^j}(t)\,(p_K - p_\pi)_\mu \; . \tag{1.2}$$

In the physical region these are usually parameterized as [1]

$$f_0^{K^i\pi^j}(t) \;\equiv\; f_+^{K^i\pi^j}(t) + \frac{t}{M_K^2 - M_\pi^2}\, f_-^{K^i\pi^j}(t) \;, \tag{1.3}$$

$$f_{+,0}^{K^i\pi^j}(t) \;=\; f_+^{K^i\pi^j}(0)\left(1 + \lambda'_{+,0}\,\frac{t}{M_\pi^2} + \lambda''_{+,0}\frac{t^2}{M_\pi^4} + \dots\right) \;, \tag{1.4}$$

with $t = (p_K - p_\pi)^2$. As shown explicitly in Eq. (1.1), it is convenient to normalize the form factors of all channels to $f_+^{K^0\pi^-}(0)$, which in the following will simply be denoted by $f_+(0)$. The channel-dependent terms $\Delta_{SU(2)}^K \equiv f_+^{K^+\pi^0}(0)/f_+^{K^0\pi^-}(0) - 1$ and $\Delta_{\rm EM}^{K\ell}$ represent the isospin-breaking and long-distance electromagnetic (EM) corrections, respectively.

While the decay rates and the phase space integrals are experimentally accessible, theoretical input is needed on $\Delta_{\rm EM}^{K\ell}$, $\Delta_{SU(2)}^K$, and $f_+(0)$. I will discuss three uses of $K_{\ell 3}$ decays in probing the Standard Model that require an increasing amount of theoretical input:

- Theoretical knowledge of the EM corrections allows one to predict $\Gamma_{K_{e3}}/\Gamma_{K_{\mu 3}}$ and therefore to test the lepton universality of weak interactions by comparing with the experimentally measured ratio.
- Knowledge of EM corrections and $\Delta_{SU(2)}^K$ - the $SU(2)$ breaking induced by quark masses - allows one to predict $\Gamma_{K_{\ell 3}^+}/\Gamma_{K_{\ell 3}^0}$ ($\ell = e, \mu$). Interestingly, we are reaching a precision level such that this ratio poses a non-trivial constraint on the ratios of light quark masses.

[1] See Ref. [4] for a dispersive parameterization of the scalar form factor.

- Finally, knowledge of EM, $SU(2)$, and $SU(3)$-breaking corrections (the deviation of $f_+^{K^0\pi^-}(0)$ from one) allows one to extract V_{us} and, in combination with a precision determination of V_{ud}, to test the quark-lepton universality of charged current weak interactions.

The natural framework to organize the theoretical analysis is provided by chiral perturbation theory [5, 6] (ChPT), the low energy effective theory of QCD. Physical amplitudes are systematically expanded in powers of external momenta of pseudo-Goldstone bosons (π, K, η) and quark masses. When including electromagnetic corrections, the power counting is in $(e^2)^m (p^2/\Lambda_\chi^2)^n$, with $\Lambda_\chi \sim 4\pi F_\pi$ and $p^2 \sim O(p_{\rm ext}^2, M_{K,\pi}^2) \sim O(m_q)$. To a given order in the above expansion, the effective theory contains a number of low energy couplings (LECs) unconstrained by symmetry alone. In order to retain predictive power, in the electromagnetic sector one has to resort to theoretical estimates for the LECs. I now turn to the discussion of EM, $SU(2)$, and $SU(3)$ corrections.

2. Radiative corrections and lepton universality

Long distance electromagnetic corrections were studied within ChPT to order $e^2 p^2$ in Refs. [7, 8]. To this order, both virtual and real photon corrections contribute to $\Delta_{\rm EM}^{K\ell}$. The virtual photon corrections involve (known) loops and tree level diagrams with insertion of $O(e^2 p^2)$ LECs. The relevant LECs have been estimated in [9, 10] using large-N_C techniques.

Radiation of real photons is also an important ingredient in the calculation of $\Delta_{\rm EM}^{K\ell}$, because only the inclusive sum of $K_{\ell 3}$ and $K_{\ell 3\gamma}$ rates is infrared finite to any order in α. Moreover, the correction factor depends on the precise definition of inclusive rate. In Table 1 we collect results for the fully photon-inclusive rate. ChPT power counting implies that to order $e^2 p^2$ one has to treat K and π as point-like (and with constant weak form factors) in the calculation of the radiative rate, while structure dependent effects enter at higher order in the chiral expansion [11].

Radiative corrections to $K_{\ell 3}$ decays have been recently calculated also outside the ChPT framework [12, 13]. Within these schemes, the UV divergences of loops are regulated with a cutoff (chosen to be around 1 GeV). In addition, the treatment of radiative decays includes part of the structure dependent effects, introduced by the use of form factors in the weak vertices.

In Table 1 I report the current results on radiative corrections, both in ChPT [7, 8] and in the model of Ref. [12]. The asterisk on Refs. [7, 8] indicates that the results of those references have been updated by including the finite values of the LECs as estimated in Ref. [10]. The double asterisk indicates *preliminary results* obtained in ChPT by H. Neufeld. The quoted uncertainty is meant to be a conservative estimate of neglected higher order terms in the chiral expansion and possible correlations are at the moment neglected (they tend to reduce the uncertainty of particular combinations of $\Delta_{\rm EM}^{K\ell}$).

Constructing the ratio of effective μ and e weak couplings from neutral or charged kaon decays

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{\Gamma_{K_{\mu 3}}}{\Gamma_{K_{e3}}} \cdot \frac{I^{Ke}}{I^{K\mu}} \left[1 + 2\,\Delta_{\rm EM}^{Ke} - 2\,\Delta_{\rm EM}^{K\mu}\right] , \tag{2.1}$$

one can see that radiative corrections are the only theoretical input needed to test lepton universality. For a discussion of the current status of lepton universality tests, I refer to the contribution to this conference proceedings by R. Wanke [14].

	$\Delta_{\rm EM}^{K\ell}(\%)$
K_{e3}^{+}	+0.08 ± 0.15 [7] *
K_{e3}^{0}	+0.57 ± 0.15 [8] *
	+0.65 ± 0.15 [12]
$K_{\mu 3}^{+}$	-0.12 ± 0.15 **
$K_{\mu 3}^{0}$	+0.80 ± 0.15 **
	+0.95 ± 0.15 [12]

Table 1: Summary of radiative correction for various $K_{\ell 3}$ decay modes. The asterisk on Refs. [7, 8] indicates that the results of those references have been updated by including the finite values of the LECs as estimated in Ref. [10]. The double asterisk indicates *preliminary results* obtained in ChPT by H. Neufeld. The quoted uncertainty is a conservative estimate of neglected higher order terms in the chiral expansion.

3. $SU(2)$ breaking and ratios of light quark masses

Strong isospin breaking effects $O(m_u - m_d)$ were first studied up to $O(p^4)$ in Ref. [15]. To $O(p^2)$ the isospin-breaking parameter $\Delta_{SU(2)}^{K}$ arises entirely from π^0-η mixing and reads

$$\Delta_{SU(2)}^{K} = \frac{3}{4}\frac{1}{R} \qquad \text{with} \qquad R = \frac{m_s - \hat{m}}{m_d - m_u} \tag{3.1}$$

and $\hat{m} = 1/2(m_u + m_d)$. To next-to-leading order in the chiral expansion $\Delta_{SU(2)}^{K}$ reads

$$\Delta_{SU(2)}^{K} = \frac{3}{4}\frac{1}{R}\left(1 + \chi_{p^4} + \frac{4}{3}\frac{M_K^2 - M_\pi^2}{M_\eta^2 - M_\pi^2}\Delta_M + O(m_q^2)\right) , \tag{3.2}$$

where $\chi_{p^4} = 0.219$ is a calculable chiral correction and Δ_M is related to ratios of quark masses by the relation:

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{m_u + m_d}\left(1 + \Delta_M + O(m_q^2)\right) . \tag{3.3}$$

The standard strategy up to this point has been to use all the known information on ratios of light quark masses to predict $\Delta_{SU(2)}^{K}$ via Eq 3.2. However, the precision on the decay rates and radiative corrections now allows one to determine phenomenologically $\Delta_{SU(2)}^{K}$, so that Eq 3.2 can be used as a constraint on ratios of light quark masses. I will now briefly review the two approaches.

3.1 Input on quark mass ratios

In Ref. [7] we have relied on the analysis of quark mass ratios performed in Ref. [16]. The starting point for the analysis of Ref. [16] is the elliptic constraint in the m_s/m_d vs m_u/m_d plane given by [6, 17],

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1 \qquad \text{with} \qquad Q^2 = \frac{M_K^2}{M_\pi^2}\frac{M_K^2 - M_\pi^2}{\left(M_{K^0}^2 - M_{K^+}^2\right)_{\rm QCD}} . \tag{3.4}$$

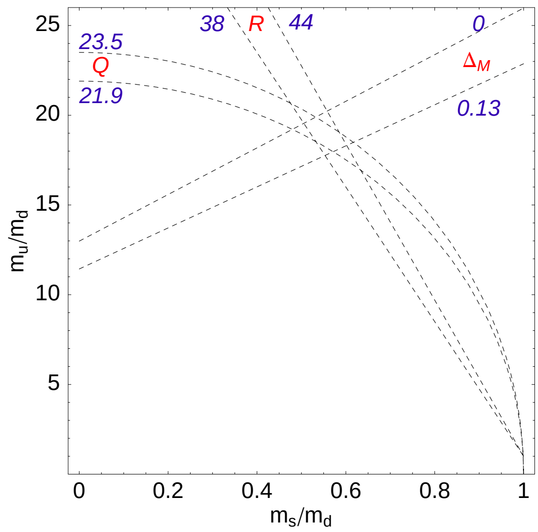

Figure 1: Constraints on light quark mass ratios as discussed in Ref. [16].

The parameter Q can be extracted from the $\eta \to \pi^+\pi^-\pi^0$ decay rate or from the QCD component of the kaon mass splitting (which can be disentangled if one knows the EM contribution to this mass splitting). Based on the information available on $\eta \to \pi^+\pi^-\pi^0$, Leutwyler concluded that $Q = 22.7 \pm 0.8$ constitutes a safe range, consistent with existing theoretical calculations of the EM kaon mass splitting [18]. This fixes an ellipse in the m_s/m_d vs m_u/m_d plane (see Fig, 1). In order to pin down a location on the ellipse, one needs additional input. Leutwyler argued that $\Delta_M > 0$ on the basis of large-N_C considerations and that $R < 44$ based on the analysis of the charmonium transitions $\Gamma(\psi' \to \psi\pi^0)/\Gamma(\psi' \to \psi\eta)$. Putting the three constraints together, one finds (see Fig. 1) $0 < \Delta_M < 0.13$ and $38 < R < 44$, leading to [7]:

$$\Delta^K_{SU(2)} \; = \; (2.36 \pm 0.22)\% \; . \tag{3.5}$$

3.2 Phenomenological determination of $\Delta^K_{SU(2)}$

In principle the quantity $\Delta^K_{SU(2)}$ can be determined phenomenologically by comparing charged and neutral $K_{\ell 3}$ decay rates for a given leptonic final state. Focusing on K_{e3} modes, we find

$$\Delta^K_{SU(2)} = \frac{\Gamma_{K^+_{e3}}}{\Gamma_{K^0_{e3}}} \cdot \frac{I^{K^0 e}}{I^{K^+ e}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - \frac{1}{2} - \left[\Delta^{K^+ e}_{\rm EM} - \Delta^{K^0 e}_{\rm EM}\right] \tag{3.6}$$

where $\Delta^{K^+ e}_{\rm EM} - \Delta^{K^0 e}_{\rm EM} = -(0.49 \pm 0.10)\%$, with uncertainty determined by one EM LEC (X_1) and an estimate of higher order corrections. Using the experimental input on $K_{\ell 3}$ rates based on results published up to March 2007, one finds [19]:

$$\left.\Delta^K_{SU(2)}\right|_{\rm pheno} \quad = (3.24 \pm 0.43)\,\% \tag{3.7}$$

which disagrees with the "canonical" theoretical prediction at the two-σ level. This slight tension has been noticed for many years at the level of central values. The improved experimental precision makes it now a two-σ effect, which is worth investigating from a theoretical point of view.

In particular, it is legitimate to ask the question: what would be the implications of $\Delta^K_{SU(2)} \simeq 3.2\%$ (vs $\Delta^K_{SU(2)} \simeq 2.4\%$) on the ratios of light quark masses? Naively, one would imagine that a larger $\Delta^K_{SU(2)}$ can be attained by moving on the ellipsis of fixed $Q = 22.7 \pm 0.8$ to the left of the "canonical" point in such a way to decrease the ratio R (see Eq 3.2 and Fig. 1). However, as one moves towards the left along the fixed-Q ellipse the ratio Δ_M decreases and effectively neutralizes the effect of increasing R, so that $\Delta^K_{SU(2)}$ is nearly a constant along the ellipses of fixed Q. This implies that it is impossible to obtain the high value $\Delta^K_{SU(2)} \simeq 3.2\%$ if $Q = 22.7 \pm 0.8$. This is illustrated in Fig. 2, where I display the behavior of $\Delta^K_{SU(2)}$ vs m_u/m_d along curves of fixed Q, for $Q = 22.7, 22.0, 19.5$. This plot also nicely illustrates the fact that higher values of $\Delta^K_{SU(2)}$ suggest smaller values of Q and, in particular, that $\Delta^K_{SU(2)} \simeq 3.2\%$ suggests $Q \simeq 19.5$. This statement remains true even allowing for chiral corrections of reasonable size in Eq. 3.2, namely $O(m_q^2) \sim 0.3 \times O(m_q)$. Hints of $Q < 20$ also emerge from the analysis of meson masses to $O(p^6)$ including isospin-breaking [20].

Another way to state the above results is the following: Eq. 3.2 provides an elliptic constraint in the m_s/m_d vs m_u/m_d plane, parameterized by $\Delta^K_{SU(2)}$. This constraint, however, turns out to be almost degenerate with the "Q constraint". So the $\Delta^K_{SU(2)}$-ellipse is practically useless to pin down the values of the quark mass ratios m_u/m_d and m_u/m_s. However, it provides a powerful independent cross check on the size of Q.

In summary, the current tension between Eq. 3.5 and Eq. 3.7 points to one of the three following scenarios:

- inconsistency in the $K_{\ell 3}$ data (if $Q \sim 22$ is robust and chiral corrections are of normal size);
- anomalously large chiral corrections (if $Q \sim 22$ is robust and $K_{\ell 3}$ data are consistent);
- $Q < 20$ (if chiral corrections are of normal size and $K_{\ell 3}$ data are consistent).

Since work is in progress on calculating the chiral corrections to Eq. 3.2 [21] and on finalizing the experimental analyses, we will be able to discriminate these possibilities in the near future.

4. $SU(3)$ breaking and determination of V_{us}

The combination $V_{us} \times f_+(0)$ can be extracted from both charged and neutral K decays and its value is dominated by K^0 modes [19]:

$$V_{us} \times f_+(0) = 0.2167 \pm 0.0005 \ . \tag{4.1}$$

So the form factor $f_+(0)$ is the missing theoretical ingredient for the extraction of V_{us}. Within ChPT we can break up the form factor according to its expansion in quark masses:

$$f_+(0) = 1 + f_{p^4} + f_{p^6} + \dots \ . \tag{4.2}$$

Deviations from unity (the octet symmetry limit) are of second order in SU(3) breaking [22]. The first correction arises to $O(p^4)$ in ChPT: a finite one-loop contribution [15, 23] determines

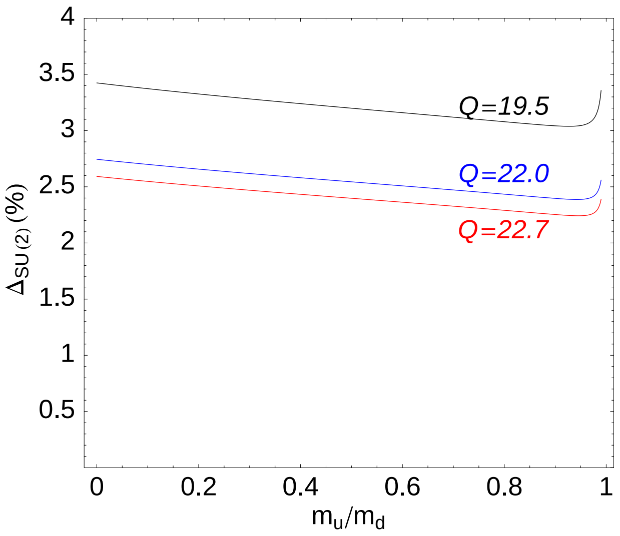

Figure 2: Behavior of $\Delta^K_{SU(2)}$ vs m_u/m_d along ellipses of fixed Q, for $Q = 22.7, 22.0, 19.5$.

$f_{p^4} = -0.0227$ in terms of F_π, M_K and M_π, with essentially no uncertainty. The p^6 term was first estimated by Leutwyler Roos [23] in the quark model framework, leading to

$$f_+(0)_{\rm LR} = 0.961 \pm 0.008 \ . \tag{4.3}$$

In the context of ChPT, the p^6 term receives contributions from pure two-loop diagrams, one-loop diagrams with insertion of one vertex from the p^4 effective Lagrangian, and pure tree-level diagrams with two insertions from the p^4 Lagrangian or one from the p^6 Lagrangian [24, 25]:

$$f_{p^6} = f_{p^6}^{2-{\rm loops}}(\mu) + f_{p^6}^{L_i \times {\rm loop}}(\mu) + f_{p^6}^{\rm tree}(\mu) \ . \tag{4.4}$$

Individual components depend on the chiral renormalization scale μ, their sum being scale independent. Using $\mu = M_\rho = 0.77$ GeV and the L_i from fit 10 in Ref. [20], one has [25] $f_{p^6}^{2-{\rm loops}}(M_\rho) + f_{p^6}^{L_i \times {\rm loop}}(M_\rho) = +0.0093 \pm 0.0005$. The p^6 constants appearing in $f_{p^6}^{\rm tree}$ could be determined phenomenologically [25], provided experimental errors on slope and curvature of the scalar form factor reach the level $\Delta\lambda_0 \sim 0.001$ and $\Delta\lambda_0'' \sim 0.0001$, which is unfortunately not achievable [26]. Therefore, further theoretical input on $f_+(0)$ is needed. A first possibility is to identify the Leutwyler-Roos estimate with the local ChPT amplitude [25]. Doing so one estimates:

$$f_+(0)_{\rm ChPT+LR} = 0.970 \pm 0.010 \ . \tag{4.5}$$

In Ref. [27] a (truncated) large-N_C estimate of $f_{p^6}^{\rm tree}$ was performed. It was based on matching a meromorphic approximation to the $\langle SPP \rangle$ Green function (with poles corresponding to the lowest-lying scalar and pseudoscalar resonances) onto QCD by imposing the correct large-momentum falloff, both off-shell and on one- and two-pion mass shells. The uncertainty was estimated by varying the matching scale in the range $\mu \in [M_\eta, 1{\rm GeV}]$, leading to:

$$f_+(0)_{\rm ChPT+1/N_C} = 0.984 \pm 0.012 \ . \tag{4.6}$$

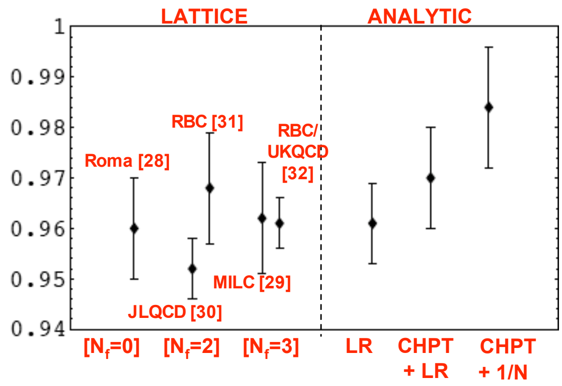

Figure 3: Comparison of results on $f_+(0)$ from lattice QCD (with different numbers of dynamical flavors N_f) and the analytic methods discussed in the text.

Finally, starting from Ref. [28] it has been realized that lattice QCD is a powerful tool to estimate $f_+(0)$ at a level of accuracy interesting for phenomenological purposes [29, 30, 31, 32]. Currently the dominant systematic uncertainty arises from the extrapolation of lattice results, obtained with unphysical quark masses, to physical light quark masses. Both quenched and unquenched results are now available and I refer to the talks by A. Juttner, T. Kaneko, and C. Sachrajda for updates on the recent activity. In Fig. 3 I report a summary of results available prior to this conference, comparing lattice and analytic results on $f_+(0)$. The lattice results tend to agree quite well with the Leutwyler-Roos estimate, while the analytic approaches tend to be higher as a consequence of including the large (~ 0.01) and positive two-loop effects [25]. In absence of a new "standard value", for current phenomenological analyses the Leutwyler-Roos result is still used as a reference value. Using experimental averages from Ref. [19] one obtains:

$$V_{us}\Big|_{K_{\ell 3}} = 0.2255\,(4)_{\rm exp}\,(20)_{\rm th} \cdot \frac{0.961}{f_+(0)} \ . \tag{4.7}$$

The implications of this result on CKM unitarity tests are discussed in the talks by W. Marciano and M. Palutan at this conference.

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