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Lattice Studies of Non-Leptonic Kaon Decays

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I review recent progress in the development of the theoretical framework necessary for computing $K \to \pi\pi$ decay amplitudes in Euclidean finite volumes. The status of the theory of finite volume effects is discussed and the rôle of chiral perturbation theory and the determination of low energy constants is reviewed. A proposal to use twisted boundary conditions is explained and the status of a suggestion to investigate the rôle of the charm quark in the $\Delta I = 1/2$ rule is discussed.

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1. Introduction

Two of the most interesting challenges in Kaon physics are to explain the $\Delta I = 1/2$ rule (the enhancement by a factor of about 450 of the rates for $K \to \pi\pi$ decays into an isospin 0 final state relative to those into an $I = 2$ state) and to understand quantitatively the experimentally measured value of $\varepsilon'/\varepsilon = (17.2 \pm 1.8) \times 10^{-4}$ [1], the parameter whose non-zero value was the first evidence for direct CP-violation. In order to meet these challenges we need to be able to evaluate the non-perturbative QCD effects in the corresponding $K \to \pi\pi$ decay amplitudes. In this talk I review the status of lattice calculations of $K \to \pi\pi$ matrix elements of the $\Delta S = 1$ effective Hamiltonian and the prospects for future improvements.

The problem of handling multihadron states in Euclidean finite-volume lattice simulations is an unsolved one. The main difficulty is that, in general, there are many possible intermediate states which, after rescattering, can emerge as the required final state. For $K \to \pi\pi$ decays on the other hand, it is a good approximation to neglect inelastic contributions. We can therefore restrict ourselves to considering the single two-pion state with a given isospin (0 or 2) and are able to compute the corresponding matrix elements. For two-body non-leptonic B decays on the other hand we are unfortunately unable as yet to evaluate the amplitudes using lattice simulations. This severely limits the precision with which we can exploit the huge amount of data available from the b-factories to explore the limits of the Standard Model.

Even for $K \to \pi\pi$ decays we have further difficulties, but ones which have been solved. In a finite Euclidean volume we do not obtain the S-matrix directly, but compute matrix elements into the average of in- and out-states [2]. Moreover, as shown in the pioneering work of Lüscher [3], the propagation of two pions leads to finite-volume corrections which decrease only as powers of the volume and not exponentially as with single-hadron states. In section 2 I review the status of our understanding of finite-volume effects in extracting physical $K \to \pi\pi$ decay amplitudes from the corresponding finite-volume matrix elements computed in lattice simulations.

Chiral perturbation theory (χPT) plays a central rôle in obtaining physical information from lattice simulations, particularly in guiding the extrapolation of results obtained at unphysically large values of the u and d quark masses to the physical point. It is frequently useful to evaluate the *low energy constants* (LECs) of χPT to estimate physical quantities and this approach is used widely in $K \to \pi\pi$ decays. In 2001, the RBC and CP-PACS collaborations, for the first time were able to determine the LECs of lowest order χPT and obtained very interesting results for the $\Delta I = 1/2$ rule and for ε'/ε [4, 5], results which will provide valuable benchmarks for future calculations:

Collaboration	Re A_0/Re A_2	ε'/ε
RBC [4]	25.3 ± 1.8	$-(4.0 \pm 2.3) \times 10^{-4}$
CP-PACS [5]	9÷12	(-7 ÷ -2)$\times 10^{-4}$
Experiments [1]	22.2	$(17.2 \pm 1.8) \times 10^{-4}$

To set the scene for the developments discussed in the following sections, I would like now to make some remarks about the calculations of refs. [4, 5]:

1. At leading order in χPT it is possible to determine the LECs by computing only $K \to \pi$ and $K \to$ vacuum matrix elements. At this order it is therefore possible to avoid any computations with two-pion states.

2. The RBC and CP-PACS collaborations were able, for the first time, to control the *Ultraviolet Problem*, i.e. the subtraction of power divergences due to the mixing of the $\Delta S = 1$ operators in the weak Hamiltonian with lower dimensional ones, with sufficient precision to obtain meaningful results. This was a very important milestone.

3. The simulations were quenched and assumed the validity of lowest order χPT in the region of meson masses of approximately 400-800 MeV. We will hear from Bob Mahwhinney [6] that they are being repeated in unquenched simulations and with lighter quark masses.

   Although it may appear that the values obtained for the octet enhancement are large, it should be noted that for the quark masses at which the simulations were performed the enhancement was about a factor of 5 or 6 (with the two collaborations agreeing on the results). The remaining enhancement came from the chiral extrapolation (which led to the disagreement in the table). This highlights the importance of having a good control of the chiral extrapolation.

4. A natural extension of this calculation is to improve the precision to next-to-leading order (NLO) in the chiral expansion. This requires the evaluation of $K \to \pi\pi$ decay amplitudes directly.

5. In view of the approximations described above it is not too surprising that the results for ε'/ε even have the wrong sign. There are 10 operators in the $\Delta S = 1$ effective Hamiltonian and there is a significant partial cancellation from the $\Delta I = 1/2$ and $\Delta I = 3/2$ contributions which amplifies the relative errors?

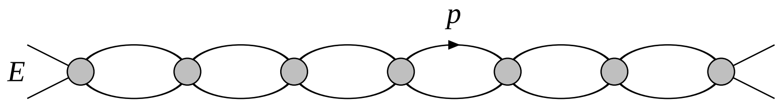
For the remainder of this talk, I will concentrate on the direct evaluation of $K \to \pi\pi$ matrix elements. In the following section I discuss the status of the finite volume effects and in section 3 I describe the strategy of using χPT at NLO. In section 4 I consider the possibility of using twisted boundary conditions to improve the momentum resolution and also report on an investigation of the role of the charm quark in the $\Delta I = 1/2$ rule. Finally in section 5 I briefly summarise and present my conclusions.

2. Finite Volume Effects

For most of the quantities being calculated in lattice simulations (such as f_K, B_K and $K_{\ell 3}$ decays discussed in this conference [7, 8]) there is at most a single hadron in the initial and final state. In $K \to \pi\pi$ decays there are two-pions and the interactions between the two pions induce finite-volume corrections which do not vanish exponentially. For the direct evaluation of the $K \to \pi\pi$ amplitudes, the theory of finite-volume effects for two-hadron states in the elastic regime is now fully understood, both in the centre-of-mass and moving frames. In this section I describe the status of our understanding based on the work of Lüscher [3] and subsequent papers [9–13]. For a discussion of two-hadron states in nucleon-nucleon systems see ref. [14] and references therein.

2.1 The Two-Pion Spectrum in a Finite Volume

Consider the two-hadron propagator represented by the diagram



where the shaded circles represent two-particle irreducible contributions in the s-channel. For simplicity let us take the two-hadron system to be in the centre-of mass frame and assume that only the s-wave phase-shift is significant (the discussion can be extended to include higher partial waves). Consider the loop integration/summation over p (see the figure). Taking the time extent of the lattice to be infinite, we can perform the p_0 integration by contours and obtain a summation over the spatial momenta of the form:

$$\frac{1}{L^3}\sum_{\vec{p}}\frac{f(p^2)}{p^2-k^2}\,, \tag{2.1}$$

where k is the relative momentum ($E^2 = 4(m^2+k^2)$, where E is the total energy), the function $f(p^2)$ is non-singular and (for periodic boundary conditions) the summation is over momenta $\vec{p} = (2\pi/L)\,\vec{n}$ where $\vec{n}$ is a vector of integers. In infinite volume the summation in eq. (2.1) is replaced by an integral and in Minkowski space the denominator has the appropriate $i\varepsilon$ prescription. It is the difference between the summation and integration which gives the finite-volume corrections. The relation between finite-volume sums and infinite-volume integrals is the *Poisson Summation Formula*, which (in 1-dimension) is:

$$\frac{1}{L}\sum_{p} g(p) = \sum_{l=-\infty}^{\infty}\int\frac{dp}{2\pi}e^{ilLp}g(p)\,. \tag{2.2}$$

If the function $g(p)$ is non-singular, the oscillating factors on the right-hand side ensure that only the term with $l=0$ contributes, up to terms which vanish exponentially with L. Hence, up to this precision, the finite-volume sum and infinite-volume integral are equal. The summand in eq. (2.1) on the other hand is singular (there is a pole at $p^2=k^2$) and this is the reason why the finite-volume corrections decrease only as powers of L. The full derivation of the formulae for the finite-volume corrections can be found in refs. [3,9–13] and is beyond the scope of this talk. Here I just sketch the key ingredients. Following ref. [12], it is convenient to start by rewriting the expression eq. (2.1) in a form without singularities so that, up to exponential precision in the volume,

$$\frac{1}{L^3}\sum_{\vec{p}}\frac{f(p^2)-f(k^2)e^{\alpha(k^2-p^2)}}{p^2-k^2} = \int\frac{d^3p}{(2\pi)^3}\frac{f(p^2)-f(k^2)e^{\alpha(k^2-p^2)}}{p^2-k^2}\,. \tag{2.3}$$

The exponential factors in eq.(2.3) are introduced to ensure ultra-violet convergence (α is the cut-off). Eq.(2.3) can then readily be rewritten in the form

$$\begin{aligned}\frac{1}{L^3}\sum_{\vec{p}}\frac{f(p^2)}{p^2-k^2} &= \int\frac{d^3p}{(2\pi)^3}\frac{f(p^2)}{p^2-k^2-i\varepsilon}\\ &-\frac{ik}{4\pi}f(k^2)+f(k^2)\left\{\frac{1}{L^3}\sum_{\vec{p}}\frac{e^{\alpha(k^2-p^2)}}{p^2-k^2}-\mathscr{P}\int\frac{d^3p}{(2\pi)^3}\,\frac{e^{\alpha(k^2-p^2)}}{p^2-k^2}\right\}\,,\end{aligned} \tag{2.4}$$

where $\mathscr{P}$ represents the principal value. The finite-volume correction exhibited above appears in every loop in diagrams such as that in the figure, and upon resummation give a geometric series.

In infinite volume there is a two-particle cut with a branch point at the two-pion threshold. In finite volume the cut is replaced by a series of poles and the positions of these poles correspond to the allowed energy levels (i.e. the Lüscher quantization condition). Note that the finite-volume corrections in eq.(2.4) depend on the function f evaluated at the external energy corresponding to k^2, which allows us to express the positions of the poles in terms of the physical amplitude (or phase-shift $\delta(k^2)$) and kinematic factors; specifically the poles occur at values of k satisfying:

$$\tan(\delta(k^2)) = -\frac{k}{4\pi}\left\{\frac{1}{L^3}\sum_{\vec{p}}\frac{e^{\alpha(k^2-p^2)}}{k^2-p^2} - \mathscr{P}\int\frac{d^3p}{(2\pi)^3}\,\frac{e^{\alpha(k^2-p^2)}}{k^2-p^2}\right\}^{-1}\,. \tag{2.5}$$

In refs. [9,12,13] the spectrum is derived also in frames in which the total momentum is not zero.

A very important practical consequence of the quantization condition, eq.(2.5), is that the phase-shifts can be derived from the spectrum, i.e. from the measured values of k corresponding to the energy eigenstates we evaluate the right-hand side of eq. (2.5) and hence determine $\delta(k^2)$. So, in spite of the fact that in Euclidean space we measure matrix elements into the average of in- and out- states, we do nevertheless have the capability of determining the phase-shifts.

2.2 Finite Volume Effects in Matrix Elements

The finite-volume corrections to the matrix elements have also been obtained both in the centre-of-mass frame [10] and in moving frames [12,13]:

$$|A|^2 = 8\pi V^2\,\frac{m_K E^2}{k^{*2}}\left\{\delta'(k^*)+\phi^{P'}(k^*)\right\}\,|M|^2 \tag{2.6}$$

where the $'$ represents the derivative with respect to k^*. k^* is the relative momentum in the centre-of-mass frame, so that if E and $\vec{P}$ are the total energy and momentum, $E^2-P^2=4(m^2+k^{*2})$. A and M are the physical $K\to\pi\pi$ amplitude and the finite-volume matrix element respectively:

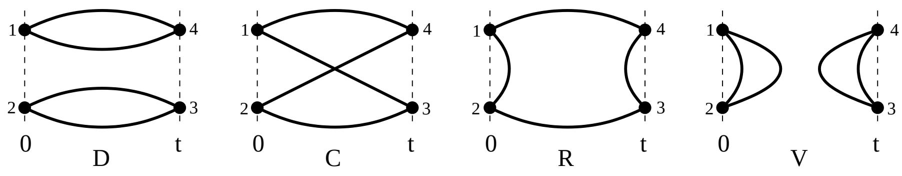
$$A = {}_\infty\langle\pi\pi;E,\vec{P}\,|\,\mathscr{H}_W(0)\,|\,K;\vec{P}\rangle_\infty \quad\text{and}\quad M = {}_V\langle\pi\pi;E,\vec{P}\,|\,\mathscr{H}_W(0)\,|\,K;\vec{P}\rangle_V\,. \tag{2.7}$$

$\phi^P(k^*)$ is a kinematical function whose explicit form can be found in eqs.(2) and (3) of ref. [12] for example.

We therefore have the necessary techniques to control the finite-volume effects in both the spectrum and in the matrix elements. Preliminary results for $\Delta I = 3/2$ $K\to\pi\pi$ decays using this technique have been presented using a quenched simulation on a course lattice ($a^{-1}=1.3\,\text{GeV}$) [15].

2.3 Summary

We have seen that finite volume effects for the two-pion spectrum and $K\to\pi\pi$ amplitudes are understood in both the rest and moving frames. For $I=2$ final states, there is now no barrier to calculating the matrix elements precisely. For $I=0$ $\pi\pi$ states we need to learn how to calculate the disconnected diagrams with sufficient precision. For example, consider the four quark-flow diagrams for two-pion propagators:



The two-pion propagators are an ingredient in the evaluation of the $K \to \pi\pi$ matrix elements. For the $I = 2$ state only the first two diagrams contribute and it appears that they are relatively straightforward to evaluate. For $I = 0$ in addition we have to evaluate diagrams R and V, and new techniques, such as the use of stochastic or all-to-all propagators, needs to be implemented to evaluate them and to establish that they can be computed precisely. Corresponding issues are present in the evaluation of the $K \to \pi\pi$ correlation functions themselves.

3. χPT at NLO

As suggested above, one approach to improving the precision of the results of refs. [4, 5] is to extend the calculation to NLO in χPT. At this order for $K \to \pi\pi$ matrix elements the generic structure is of the form:

$$\langle \pi\pi|\mathscr{O}_W|K\rangle = \text{LO} * (1+\text{Logs}) + \text{NLO counterterms}. \tag{3.1}$$

The chiral logarithms (“Logs") are calculable in one-loop χPT and we then use lattice computations of $K \to \pi\pi$ matrix elements, for a range of masses and momenta, in order to determine the LO and NLO low-energy constants. These LECs are then used to determine the physical decay amplitudes.

We have performed an exploratory quenched study with the SPQR kinematics, obtaining the matrix elements of the electroweak penguins successfully [16], obtaining

$$_{I=2}\langle\,\pi\pi\,|O_7(2\,\text{GeV})\,|\,K^0\,\rangle = (0.12\pm0.02)\,\text{GeV}^3 \text{ and } _{I=2}\langle\,\pi\pi\,|O_8(2\,\text{GeV})\,|\,K^0\,\rangle = (0.68\pm0.09)\,\text{GeV}^3 \tag{3.2}$$

On the other hand, we were unable to determine the LEC's for O_4 sufficiently well to perform the chiral extrapolation. For O_4 the chiral expansion starts at $O(p^2)$, in contrast to the electroweak penguins which start at $O(p^0)$. The positive aspects of this computation were that i) the finite-volume energy shift was measurable; ii) the matrix Elements at simulated masses were well determined and (iii) non-perturbative renormalization was implemented successfully. On the other hand the quark masses were too high to demonstrate explicitly the validity of the chiral extrapolation and the Lellouch-Lüscher factor was not implemented. We are now in a position to overcome these shortcomings.

I have stated above that in order to compute the amplitudes at NLO in the chiral expansion it is not sufficient in general to compute only $K \to \pi$ and $K \to$ vacuum matrix elements. It is necessary to calculate $K \to \pi\pi$ matrix elements. An exception is the evaluation of the matrix elements of the electroweak penguin operators [17] for which preliminary results were presented in 2005 [18].

4. Miscellany

In this section I briefly discuss two approaches which are being developed to improve our understanding of $K \to \pi\pi$ decays; the application of twisted boundary conditions to improve the momentum resolution and an investigation into the rôle of the charm quark in the $\Delta I = 1/2$ rule.

4.1 (Partially) Twisted Boundary Conditions and the Lellouch-Lüscher Factor.

The Lellouch-Lüscher factor relating the $K \to \pi\pi$ matrix elements in finite-volume to the physical decay amplitudes contains the derivative of the phase-shift. As we have seen, the phase-shift can be determined from the two-particle spectrum in finite-volume, but only at discrete momenta. Indeed the momentum resolution is very poor, e.g. for a lattice with $L = 24a$ and $a \simeq 0.1$ fm the components of momentum are separated by about 1/2 GeV.

Using twisted boundary conditions, $q(x_i + L) = e^{i\theta_i} q(x_i)$, the momentum spectrum is modified (relative to periodic boundary conditions) to

$$p_i = n_i \frac{2\pi}{L} + \frac{\theta_i}{L}\,. \tag{4.1}$$

For quantities which do not involve final state interactions (e.g. masses, decay constants, form-factors) the finite-volume corrections are exponentially small also with twisted boundary conditions [19]. Moreover they are also exponentially small for *partially twisted boundary conditions* in which the sea quarks satisfy periodic boundary conditions but the valence quarks satisfy twisted boundary conditions [19, 20], so that we do not need to perform new simulations for every choice of θ_i.

By using (partially) twisted boundary conditions, the derivative of the phase shift ($\delta'(q^{*2})$) can be evaluated for $\Delta I = 3/2$ decays [21]. To see this consider the propagation of two π^+ mesons with momenta θ/L and $(\theta - 2\pi)/L$, where we can vary θ. This can be arranged for example by choosing twisted boundary conditions for the valence u quark. From the two-pion spectrum, as explained above, we can determine $\delta(q^{*2})$, where q^* is the corresponding centre-of-mass relative momentum. Since θ can be varied with small intervals, we can also evaluate the derivative $\delta'(q^2)$. To illustrate this consider the following preliminary results, obtained using $N_f = 2+1$ Domain Wall Fermion configurations from the RBC & UKQCD collaborations, whose properties are described in ref. [22]. We start by measuring the lowest energy in the finite volume (E_{FV}) which is different from the sum of the energies of two free pions with momenta θ/L and $(\theta - 2\pi)/L$. From the total energy, E_{FV}, and momentum, $(2\theta - 2\pi)/L$, we determine q^* and in the left-hand plot of fig. 1 we show the energy shift as a function of q^*. From the energy shift we can determine the phase-shift $\delta(q^*)$ as described above (see centre plot) and from the behaviour of δ as a function of q^* we can obtain its derivative and hence the Lellouch-Lüscher factor relating the finite-volume matrix element and the physical decay amplitude (right-hand plot). At each stage we compute the quantities directly for the masses used in the simulation (the plots are shown for one particular choice of the masses). We are therefore able to control the finite volume effects in the lattice computations.

In addition to enabling one to calculate the finite-volume corrections in lattice calculations of $\Delta I = 3/2$ decays with periodic boundary conditions, the use of twisted boundary conditions extends the kinematic range for which the matrix elements can be computed. Since in the procedure which we have used the relative momentum in the moving frame is always $2\pi/L$, the range of accessible values of q^* is limited, but nevertheless, represents an extension over periodic boundary conditions alone.

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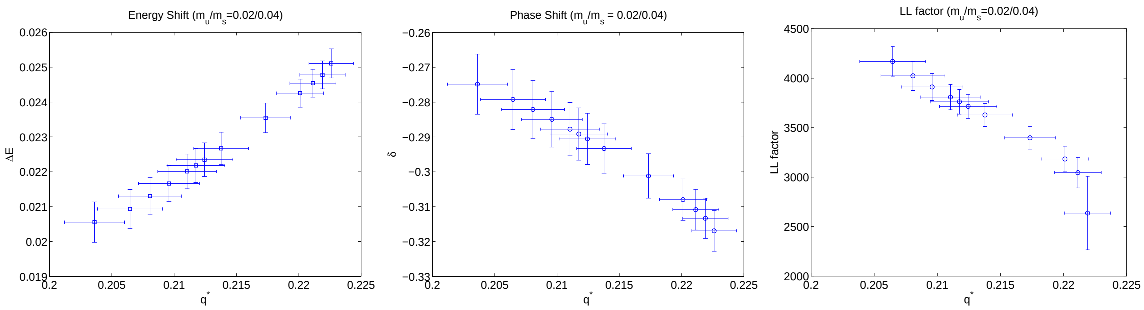

Figure 1: The energy shift (left), the phase-shift (centre) and the Lellouch-Lüscher factor (right) as a function of q^* for an illustrative pair of light quark ($ma = 0.02$) and strange quark ($m_s a = 0.04$) masses.

4.2 Rôle of the Charm Quark in the $\Delta I = 1/2$ Rule

There has been a recent proposal to study the rôle of the charm quark in the $\Delta I = 1/2$ rule [23, 24]. The programme, which has only been partially implemented to date, consists of 3 steps:

1. In the $SU(4)$ limit with all four masses very light, ($m_c = m_s = m_u = m_d \ll \Lambda_{\chi\mathrm{PT}}$) there are two LECs, g^+ and g^-, which have been evaluated by matching a quenched QCD simulation onto χPT (in the ε regime):

$$g^+ = 0.51 \pm 0.09 \quad \text{and} \quad g^- = 2.6 \pm 0.5 \quad \Rightarrow \quad \frac{A_0}{A_2} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3g^-}{2g^+}\right) \simeq 6\,. \qquad (4.2)$$

   $\Lambda_{\chi\mathrm{PT}}$ is the scale of chiral symmetry breaking. The authors conclude that: *Even though the enhancement is not large enough to match the experiment, it already indicates that penguin operator/contractions cannot be the whole story.*

2. The mass of the charm quark is now increased, while still remaining within the chiral regime ($\Lambda_{\chi\mathrm{PT}} \gg m_c \gg m_s = m_u = m_d$). The matching from step 1 to this effective theory can be done analytically. This has been done at lowest order but *unfortunately NLO couplings are needed to have predictability.*

3. Finally the charm quark mass is increased to its physical value ($m_c \geq \Lambda_{\chi\mathrm{PT}} \gg m_s = m_u = m_d$) and this remains to be done in the future.

5. Summary and Conclusions

In this talk, I have tried to demonstrate that there has been a considerable amount of theoretical progress in formulating $K \to \pi\pi$ decays in a form suitable for lattice simulations. There is now the opportunity of achieving significant numerical results for $K \to \pi\pi$ decay amplitudes.

(i) For $I = 2$ final states, there is now no barrier to calculating the matrix elements precisely.

(ii) For $I = 0$, $\pi\pi$ states, as explained in section 2.3, we still need to learn how to calculate the disconnected diagrams with sufficient precision.

I started the talk with a reminder about the results from the RBC and CP-PACS collaborations obtained in 2001 at lowest order in χPT in quenched simulations. We will hear from Bob Mawhinney, that interesting new calculations are underway to repeat the calculations in unquenched simulations and at lower masses.

References

[1] H. Burkhardt *et al.* [NA31 Collaboration], Phys. Lett. B **206** (1988) 169;
V. Fanti *et al.* [NA48 Collaboration], Phys. Lett. B **465** (1999) 335 [arXiv:hep-ex/9909022];
A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **83** (1999) 22 [arXiv:hep-ex/9905060].

[2] L. Maiani and M. Testa, Phys. Lett. B **245** (1990) 585.

[3] M. Luscher, *Commun. Math. Phys.* **104** 177 (1986); *Commun. Math. Phys.* **105** 153 (1986); *Nucl. Phys. B* **354** 531 (1991); *Nucl. Phys. B* **364** 237 (1991).

[4] T. Blum *et al.* [RBC Collaboration], *Phys. Rev. D* **68** 114506 (2003) [arXiv:hep-lat/0110075].

[5] J. I. Noaki *et al.* [CP-PACS Collaboration], *Phys. Rev. D* **68** 014501 (2003) [arXiv:hep-lat/0108013].

[6] R.Mawhinney, these proceedings

[7] T.Kaneko, these proceedings

[8] A.Jüttner, these proceedings

[9] K. Rummukainen and S. A. Gottlieb, *Nucl. Phys. B* **450** 397 (1995) [arXiv:hep-lat/9503028].

[10] L. Lellouch and M. Luscher, *Commun. Math. Phys.* **219** 31 (2001) [arXiv:hep-lat/0003023].

[11] C. J. D. Lin, G. Martinelli, C. T. Sachrajda and M. Testa, *Nucl. Phys. B* **619** 467 (2001) [arXiv:hep-lat/0104006].

[12] C. h. Kim, C. T. Sachrajda and S. R. Sharpe, *Nucl. Phys. B* **727** (2005) 218 [arXiv:hep-lat/0507006]; PoS **LAT2005** (2005) 359 [arXiv:hep-lat/0510022].

[13] N. H. Christ, C. Kim and T. Yamazaki, *Phys. Rev. D* **72** 114506 (2005) [arXiv:hep-lat/0507009].

[14] D. B. Kaplan, AIP Conf. Proc. **892** (2007) 50 [arXiv:nucl-th/0611025].

[15] T. Yamazaki [RIKEN-BNL-Columbia Collaboration], POS LAT2005 (2005) 351 [arXiV:hep-lat/0509135]; arXiv:hep-lat/0610051.

[16] P. Boucaud, V. Gimenez, C. J. Lin, V. Lubicz, G. Martinelli, M. Papinutto and C. T. Sachrajda, Nucl. Phys. B **721** (2005) 175 [arXiv:hep-lat/0412029].

[17] J. Laiho and A. Soni, Phys. Rev. D **71** (2005) 014021 [arXiv:hep-lat/0306035].

[18] J. Noaki, PoS **LAT2005** (2006) 350 [arXiv:hep-lat/0510019].

[19] C. T. Sachrajda and G. Villadoro, *Phys. Lett. B* **609** 73 (2005) [arXiv:hep-lat/0411033].

[20] P. F. Bedaque and J. W. Chen, *Phys. Lett. B* **616** 208 (2005) [arXiv:hep-lat/0412023].

[21] C. Kim and C. T. Sachrajda, preliminary.

[22] D. J. Antonio *et al.* [RBC Collaboration], Phys. Rev. D **75** (2007) 114501 [arXiv:hep-lat/0612005].

[23] L. Giusti, P. Hernandez, M. Laine, P. Weisz and H. Wittig, JHEP **0411** (2004) 016 [arXiv:hep-lat/0407007].

[24] L. Giusti, P. Hernandez, M. Laine, C. Pena, J. Wennekers and H. Wittig, Phys. Rev. Lett. **98** (2007) 082003 [arXiv:hep-ph/0607220].