PROCEEDINGS OF SCIENCE

The effects of angular momentum conservation in relativistic heavy ion collisions

F. Becattini[*†]
Università di Firenze and INFN Sezione di Firenze, Firenze, Italy
E-mail: author@email

F. Piccinini
INFN Sezione di Pavia, Pavia, Italy
E-mail: piccinini@pv.infn.it

The effects of angular momentum conservation in peripheral heavy ion collisions at very high energy are investigated. If a sufficiently large fraction of the initial angular momentum of the interaction region is converted into vorticity, the azimuthal anisotropy (elliptic flow) gets enhanced and the transverse momentum spectra turn out to be further broadened. A distinctive signature of the existence of vorticity in the plasma is the generation of a net polarization of the emitted hadrons with peculiar kinematical features. These phenomena might be possibly observed at LHC, where the initial angular momentum of the colliding ions will be about a factor 30 larger than at RHIC.

Critical Point and Onset of Deconfinement 4th International Workshop
July 9-13 2007
GSI Darmstadt,Germany

[*] Speaker.
[†] The speaker would like to thank the organizers of the conference for their kind hospitality.

PoS(CPOD07)009

1. Introduction

Nuclei colliding at ultrarelativistic energies have a large initial orbital angular momentum L_0 if their impact parameter is of order of some fm; in fact, for symmetric nuclei, $L_0 \simeq A\sqrt{s_{NN}}b/2$ in natural units ($\hbar = 1$). For Au-Au collisions at RHIC energies $\sqrt{s_{NN}} = 200$ GeV and $L_0 \sim 5 \times 10^5$ at an impact parameter $b = 5$ fm. The angular momentum will be almost two orders of magnitude larger in the forthcoming Pb-Pb collisions at LHC, at $\sqrt{s_{NN}} = 5.5$ TeV, with $L_0 \sim 1.4 \times 10^7$. Such large values of the initial angular momentum may give rise, as we will show, to significant observables effects provided that at least some fraction of it gets converted into intrinsic angular momentum of a subregion of the quark-gluon plasma (QGP) fireball.

According to the to-date generally accepted description of the collision process, a locally equilibrated plasma is formed after a relatively short time (of the order of 1 fm/c) followed by a quasi-ideal hydrodynamical expansion. This kind of approach proved to be able to reproduce the large observed values of the elliptic flow in peripheral collisions, at a finite impact parameter, and the transverse momentum spectra of particles in the low p_T region. In the ideal fluid description essentially no room is left for rotational collective effects, i.e. vortex formation, because the vorticity field $\boldsymbol{\omega} = (1/2)\nabla \times \mathbf{v}$ vanishes in the usual Bjorken initial conditions. However, in presence of dissipative effects, vorticity may indeed develop and, if viscosity is large enough, or if the system is kept together for a sufficiently long time, then part of the angular momentum could get transformed into a significant amount of intrinsic spin. Furthermore, if the Bjorken scaling hypothesis is released, a dependence of the initial longitudinal velocity on the transverse coordinates is very likely and this involves a non-vanishing vorticity from the very beginning of the hydrodynamical evolution which drives an enhancement of the expansion rate and, consequently, of the elliptic flow; we refer the reader to the ref. [1] for a more detailed discussion.

In general, the entropy increase implied by dissipative effects will drive the system towards the global equilibrium configuration, which is a rigidly rotating fireball at rest [2, 3]. Of course, the system expands and will never be able to reach full equilibrium (as it is clear from the observed hadron spectra); nevertheless, it may happen that vorticity, i.e. intrinsic rotation, becomes a non-negligible effect if viscosity is sufficiently large or if it is not vanishing from the very beginning of the hydrodynamical regime. Estimating the vorticity from *ab initio* dissipative hydrodynamical calculations is a formidable task. In this paper, we do not cope with this problem, rather we point out some of the possible observable effects which can probe the formation of spinning subsystems within the plasma by using simple full-equilibrium calculations.

2. Angular momentum conservation in heavy ion collisions

In the usual picture of a peripheral heavy ion collision at ultrarelativistic energy the overlapping region of the two incoming nuclei gives rise to QGP whereas the non-overlapping nucleons keep flying almost unaffected. Thereby, only a fraction of the initial angular momentum L_0 is left to the interaction region, while the largest part is carried away by the fragments (see fig. 1). The angular momentum of the interaction region takes its origin from the inhomogeneity of the density profile in the transverse plane. This is much clearly seen in a longitudinal projection: the colliding strips of nucleons have, in peripheral collisions, different number of nucleons. While the central

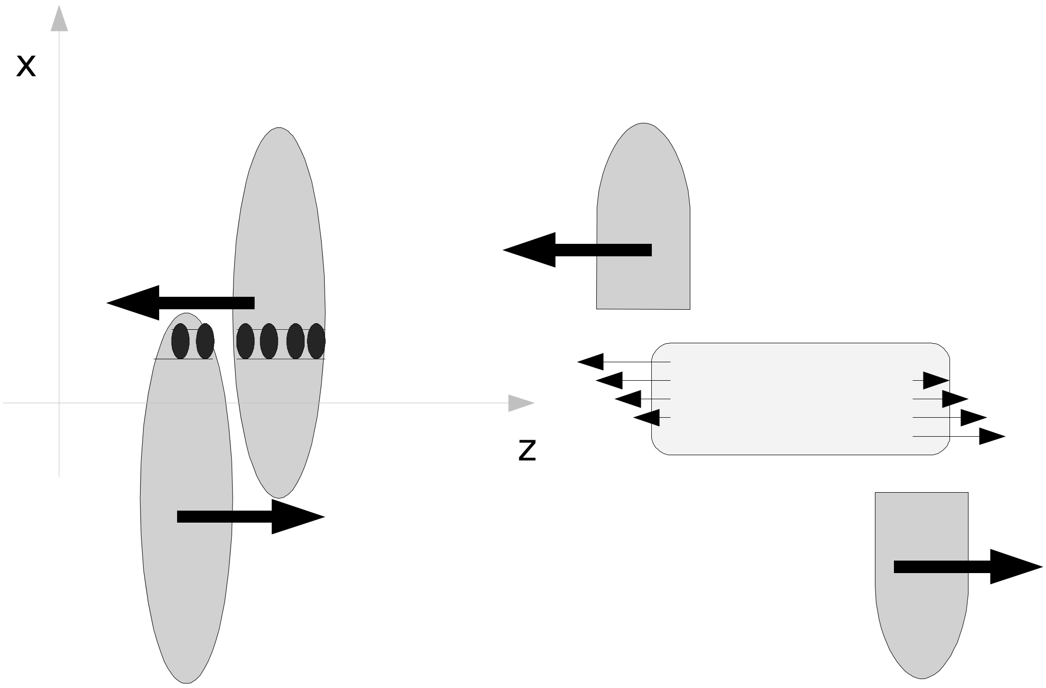

Figure 1: Sketch of a peripheral heavy ion collision at very high energy in the longitudinal projection. The initial momentum distribution of the interaction region (right) should have a gradient along the axis x orthogonal to the collision axis z stemming from the different transverse densities of the colliding strips (left).

strips have the same weight, the strips above it will have a net momentum directed along the negative z axis and conversely the ones below it (see fig. 1). Only if the two colliding objects were homogeneous in the transverse plane, the angular momentum of the interaction region would be vanishing. Yet, the nuclei *are not* homogenous in the transverse plane; for instance, if they are assumed to be homogenous spheres in their rest frame, their transverse density (or thickness function) would be proportional to $\sqrt{R^2 - r^2}$, r being the distance from the centre of the nucleus and R its radius.

In fig. 2 we show the angular momentum of the interaction region J for two colliding Gold nuclei at $\sqrt{s_{NN}} = 200$ GeV, assuming they are spheres with radius 7.0 fm. It is seen that the angular momentum attains a maximal value at an impact parameter of 2.5 fm and quickly drops thereafter. The maximal value of J is about 7.2×10^4, i.e. 29% of the initial orbital angular momentum L_0 of the colliding nuclei at that impact parameter.

The angular momentum J pertaining to the interaction region is then very large and strongly dependent on the impact parameter b. This angular momentum should be then non-vanishing also at the onset of the hydrodynamical expansion. For it to be non-vanishing, either the initial four-velocity distribution of the fluid or the proper energy density and pressure should be dependent on the azimuthal angle in the transverse plane. The first assumption, breaking the Bjorken scaling hypothesis, is the most natural in our view and it is the only possible if thermalization is instantaneous [1]. The second assumption is adopted in current hydrodynamical calculations [4, 5] but it implies different hydro evolution equations [1], lack of vorticity and it is therefore distinguishable from the

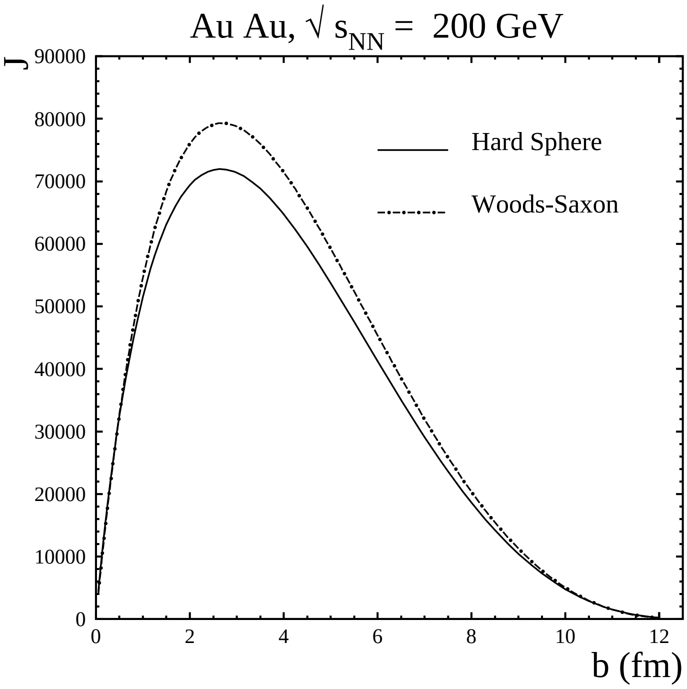

Figure 2: Angular momentum J of the interaction region as a function of the impact parameter for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

first hypothesis.

Once the correct initial conditions have been set such that $J \neq 0$, the effectiveness of the angular momentum in producing observable effects crucially depends on what fraction of it gets converted into intrinsic angular momentum; in hydrodynamical language, on how much rotation or vorticity enters in the angular momentum balance, as has been mentioned in the Introduction. The angular momentum associated to vorticity will be denoted as J_ω, such that $0 < J_\omega < J$. The value of J_ω will depend, as has been mentioned, on the initial value of the vorticity and the strength of dissipative coefficients like shear viscosity. Other recent studies [6] argue that he amount of vorticity in the fluid is sensitive to initial conditions and especially if Bjorken scaling is released.

3. Elliptic flow, p_T broadening and polarization

We carried out our calculations for a simple fully equilibrated source at hadronization, i.e. a rigidly spinning subsystem of the plasma with angular momentum $\mathbf{J}_\omega$ and fixed angular velocity $\boldsymbol{\omega} = 1/2 \mathrm{rot}\, \mathbf{v}$ parallel to it and linked to $\mathbf{J}_\omega$ by a thermodynamic relation which is linear for small ω/T values [3].

Among the most remarkable effects that could be observed if J_ω, hence ω, is sufficiently large, is an enhancement of elliptic flow. This is owing to the fact that in a rotating system particles have a larger momentum if directed along the reaction plane, i.e. orthogonally to $\mathbf{J}_\omega$ (see fig. 3); in classical terms, this is a consequence of centrifugal potential. The corresponding coefficient $v_2^{(J)}$

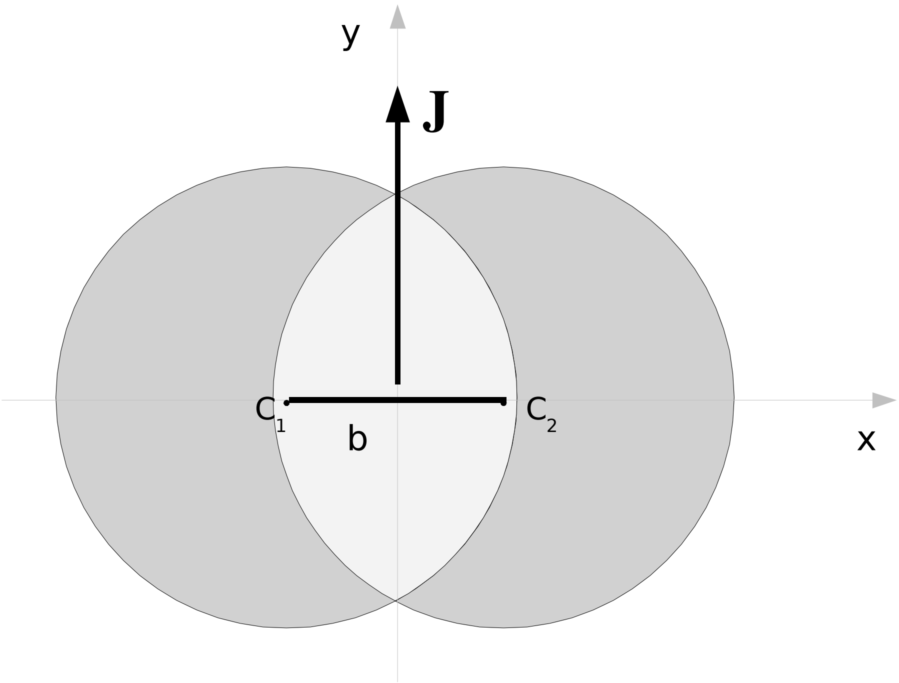

Figure 3: Sketch of a peripheral heavy ion collision at very high energy in the transverse projection. The overlap almond-shaped region is marked in light grey and has an overall angular momentum directed along the symmetry axis *y*.

reads [3]:

$$v_2^{(J)} = \frac{\int \mathrm{d}^3x \frac{\mathrm{K}_1(m_T\sqrt{1-|\boldsymbol{\omega}\times\mathbf{x}|_\parallel^2}/T)}{\sqrt{1-|\boldsymbol{\omega}\times\mathbf{x}|_\parallel^2}} \mathrm{I}_2\left(\frac{p_T z\omega}{T}\right)}{\int \mathrm{d}^3x \frac{\mathrm{K}_1(m_T\sqrt{1-|\boldsymbol{\omega}\times\mathbf{x}|_\parallel^2}/T)}{\sqrt{1-|\boldsymbol{\omega}\times\mathbf{x}|_\parallel^2}} \mathrm{I}_0\left(\frac{p_T z\omega}{T}\right)} \tag{3.1}$$

where I_n are modified Bessel functions and T is the global temperature. It should be stressed that the global temperature T in a spinning relativistic gas is related to the *local* proper temperature T_0 by the relation [3]:

$$T_0(r) = \frac{T}{\sqrt{1-\omega^2 r^2}} \tag{3.2}$$

where r is the distance from the rotation axis $\boldsymbol{\omega}$. Since it is the local, and not the global, temperature which determines the phase of the system, the decoupling should occur when the lowest local temperature reaches the critical value T_c for the quark-hadron transition, that is when:

$$\frac{T}{\sqrt{1-\omega^2 R^2}} = T_c \tag{3.3}$$

being R the maximal distance from the rotation axis.

That also a finite angular momentum of the interaction region may induce an anisotropy in the particle azimuthal spectra has been argued long ago by Hagedorn [7] and recently rediscussed in ref. [8]. It should be emphasized that $v_2^{(J)}$ is a possible extra contribution to v_2, supplementing the traditional one stemming from initial geometrical eccentricity of the nuclear overlap region. Yet,

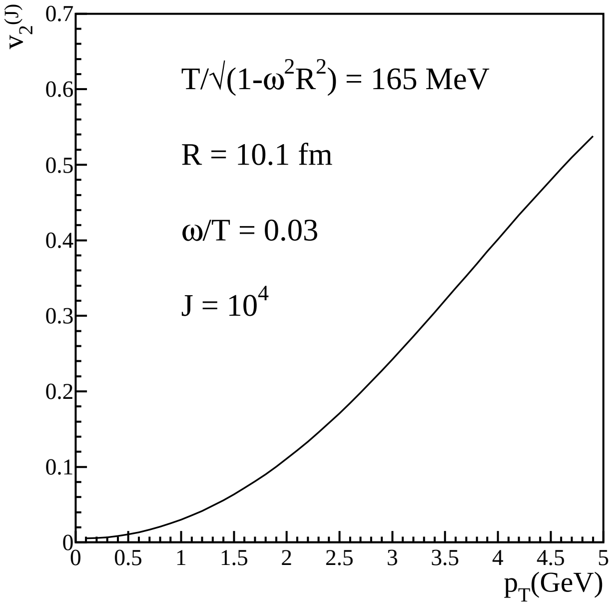

Figure 4: Elliptic flow coefficient $v_2^{(J)}$ as a function of p_T for hadrons originated from a spherical spinning plasma at a chemical freeze-out T=165 MeV and radius 10.1 fm for $\omega/T = 0.03$.

the behaviour of the "rotational" $v_2^{(J)}$ as a function of p_T is very similar to the traditional one, as shown in fig. 4 for $\omega/T = 0.03$ at the local chemical freeze-out temperature of $T_c = 165$ MeV and a source radius $R = 10.1$ fm. Therefore, it is almost impossible to disentangle the two contributions from a fit to the data and one should rather make a full hydrodynamical calculation. It should be pointed out that the formula (3.1) refers to primary particles only in the Boltzmann statistics and turns out to be almost independent of the particle mass. We also stress that $v_2^{(J)}$ shown in fig. 4 only refers to the particles emitted from a spinning source. Of course, particles emitted from non-spinning (i.e. vorticity-free) regions of the source will dilute this rotational contribution to the anisotropy.

Besides generating an azimuthal anisotropy, the presence of a spinning source will also enhance the broadening of the p_T spectra with respect to a static source. This is clearly seen in the formula [3]:

$$\frac{\mathrm{d}n_j}{\mathrm{d}p_T} \propto \int \mathrm{d}^3x \frac{p_T m_T \mathrm{K}_1(m_T \sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathbf{x}|_\parallel^2}} \mathrm{I}_0\left(\frac{p_T|\omega\times\mathbf{x}|_\perp}{T}\right) \tag{3.4}$$

which tells us that the presence of the angular momentum nearly amounts to the a local radial flow velocity $\boldsymbol{\omega}\times\mathbf{x}$. Once again, the effect is very similar to that driven by pressure gradients in a perfect fluid, although, in case of spinning regions, it would be more enhanced in semi-peripheral collisions, where J, hence J_ω, turns out to be larger.

Altogether, both elliptic flow and p_T broadening are not unique consequences of an intrinsic rotation. There is, however, a distinctive signature thereof: a polarization of the emitted hadrons along the angular momentum direction (in the lab frame). A possible relation between the large angular momentum in peripheral heavy ion collisions and polarization of the final hadrons has been pointed out first in ref. [9], where it has been studied within a perturbative QCD framework. We have used a completely different approach and determined it for an equilibrated system, by invoking the statistical hadronization dogma which turned out to be succesfull in describing hadronic multiplicities over a wide range of energies: every multihadronic state compatible with conservation laws is equally likely. Since the total angular momentum is not vanishing, particles will not fill available spin states uniformly and a net polarization will show up. A calculation of the proper polarization vector $\mathbf{\Pi}_0$ for a relativistic rotating ideal gas has been carried out by the authors [3]:

$$\mathbf{\Pi}_0 = \frac{1}{2}\tanh\frac{\omega}{2T}\left[\frac{\varepsilon}{m}\hat{\boldsymbol{\omega}} - \frac{\hat{\boldsymbol{\omega}}\cdot\mathbf{pp}}{m(\varepsilon+m)}\right] \tag{3.5}$$

for spin 1/2 particles and:

$$\mathbf{\Pi}_0 = \frac{2\sinh(\omega/T)}{2\cosh(\omega/T)+1}\left[\frac{\varepsilon}{m}\hat{\boldsymbol{\omega}} - \frac{\hat{\boldsymbol{\omega}}\cdot\mathbf{pp}}{m(\varepsilon+m)}\right] \tag{3.6}$$

for spin 1 particles; ε is the energy and $\mathbf{p}$ the momentum of the particle. The polarization turns out to be maximal for particles emitted orthogonally to $\mathbf{J}_\omega$, i.e. along the reaction plane, and increasing for increasing p_T [3].

It is interesting to note that the polarization (more generally the spin density matrix) depends on the ratio between angular velocity and global temperature, that is, using (3.2) on $\gamma\omega/T_0$, being T_0 the local temperature. Therefore, it can be conjectured, by invoking locality principle, that a polarization should appear in a generic accelerated hydrodynamical cell at local equilibrium fully determined by local quantities. Hence,$\boldsymbol{\omega}$ is to be plausibly replaced by the vector:

$$\boldsymbol{\omega} \to \frac{1}{v^2}\mathbf{v}\times\mathbf{a} \tag{3.7}$$

which is the local angular velocity for a general trajectory, according to the Frenet formulae. If this conjecture is true, every hadronizing hydrodynamical cell will produce hadrons with polarization vector (3.5),(|refinteger) with $\boldsymbol{\omega}$ equal to the right hand side of (3.7).

4. Conclusions

We have pointed out that angular momentum conservation may give rise to observable effects in peripheral heavy ion collisions at very high energy, where the initial angular momentum is very large. Besides an enhancement of the elliptic flow parameter v_2 and a further broadening of transverse momentum spectra, the most characteristic signature of the formation of spinning subregions within the plasma would be a polarization of the emitted particles, orthogonal to the reaction plane and maximal for particles with momentum parallel to the reaction plane. A quantitative assessment of these effects is very difficult as the fraction of angular momentum associated with intrinsic rotation J_ω is an unknown quantity. This depends on how much vorticity exists at the beginning and gets created during the hydrodynamical expansion of the system, which in turn requires a more accurate knowledge of initial conditions and dissipative constants. Because of the larger initial angular momentum, such effects could be better observed at LHC.

References

[1] F. Becattini, F. Piccinini and J. Rizzo, “Angular momentum conservation in heavy ion collisions at very high energy,” arXiv:0711.1253.

[2] L. D. Landau and E. M. Lifshitz, “Statistical Physics", Pergamon Press, 1980.

[3] F. Becattini and F. Piccinini, “The ideal relativistic spinning gas: polarization and spectra,” arXiv:0710.5694.

[4] U. W. Heinz and P. F. Kolb, J. Phys. G **30**, S1229 (2004).

[5] T. Hirano and K. Tsuda, Phys. Rev. C **66**, 054905 (2002).

[6] P. Romatschke, U. Romatschke, “How perfect is the RHIC fluid?,” arXiv:0706.1522.

[7] R. Hagedorn and U. Wambach, Nucl. Phys. B **123**, 382 (1977).

[8] P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C **52** 187 (2007).

[9] Z. T. Liang and X. N. Wang, Phys. Rev. Lett. **94**, 102301 (2005).