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Bose-Einstein Condensation of Pions

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Particle number fluctuations are studied in the ideal pion gas approaching Bose-Einstein condensation. Two different cases are considered: Bose condensation of pions at large charge densities ρ_Q and Bose condensation at large total densities of pions ρ_π. Calculations are done in grand canonical, canonical and microcanonical ensembles. At high collision energy, in the samples of events with a fixed number of all pions, N_π, one may observe a prominent signal. When N_π increases the scaled variances for particle number fluctuations of both neutral and charged pions increase dramatically in the vicinity of the Bose-Einstein condensation line. As an example, the estimates are presented for $p+p$ collisions at the beam energy of 70 GeV.

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Long time ago in 1924 the Bose statistics was discovered [1], and one year later the phenomenon of Bose-Einstein condensation (BEC) [2] was predicted. Tremendous efforts were required however to confirm BEC experimentally. The atomic gases are transformed into a liquid or solid before reaching the BEC point. The only way to avoid this is to consider extremely low densities. At these conditions the thermal equilibrium in the atomic gas is reached much faster than the chemical equilibrium. The life time of the metastable gas phase is stretched to seconds or minutes. This is enough to observe the BEC signatures. Small density leads, however, to small temperature of BEC. Only in 1995 two experimental groups succeeded to create the 'genuine' BE condensate by using new developments in cooling and trapping techniques [3]. Leaders of these two groups, Cornell, Wieman, and Ketterle, won the 2001 Nobel Prize for this achievement.

Pions are spin-zero mesons. They are the lightest hadrons copiously produced in high energy collisions. In the present letter we argue that the pion number fluctuations may give a prominent signal of approaching the BEC point. In fact, there is the BEC line in a plane of pion density and temperature. The pion system should be in a state of thermal, but not chemical, equilibrium to reach the BEC line. This can be achieved by selecting the samples of events with high pion multiplicities. Multipion states are formed in high energy nucleus-nucleus collisions, as well as in the elementary particle ones. There were several suggestions to search for BEC of π-mesons (see, e.g., Ref. [4]). However, complete statistical mechanics calculations of pion number fluctuations have never been presented. There is a qualitative difference in properties of the mean multiplicity and of the scaled variance of multiplicity fluctuations in different statistical ensembles. The results obtained with grand canonical ensemble (GCE), canonical ensemble (CE), and microcanonical ensemble (MCE) for the mean multiplicity approach to each other in the large volume limit. This reflects the thermodynamic equivalence of the statistical ensembles. Recently it has been found [5–9] that corresponding results for the scaled variance are different in different ensembles, and this difference is preserved in the thermodynamic limit. To extract the matter properties from analysis of event-by-event fluctuations, one needs to fix the samples of high energy events, and choose the corresponding statistical ensemble for their analysis. This is discussed below (see also [7]).

Let us start with a well known example of non-relativistic ideal Bose gas. The occupation numbers, $n_{\mathbf{p}}$, of single quantum states, labelled by 3-momenta $\mathbf{p}$, are equal to $n_{\mathbf{p}} = 0, 1, \ldots, \infty$. In the GCE their average values, fluctuations, and correlations are the following [10]:

$$\langle n_{\mathbf{p}} \rangle = \frac{1}{\exp\left[\left(\frac{\mathbf{p}^2}{2m} - \mu\right)/T\right] - 1}, \quad \langle (\Delta n_{\mathbf{p}})^2 \rangle = \langle n_{\mathbf{p}} \rangle (1 + \langle n_{\mathbf{p}} \rangle) \equiv \upsilon_{\mathbf{p}}^2, \quad \langle \Delta n_{\mathbf{p}} \Delta n_{\mathbf{k}} \rangle = \upsilon_{\mathbf{p}}^2 \, \delta_{\mathbf{pk}}, \quad (1)$$

where $\Delta n_{\mathbf{p}} \equiv n_{\mathbf{p}} - \langle n_{\mathbf{p}} \rangle$, m denotes the particle mass, T and μ are the system temperature and chemical potential, respectively (throughout the paper we use the units with $\hbar = c = k = 1$). The average number of particles in the GCE reads [10]:

$$\langle N \rangle \; \equiv \; \overline{N}(V, T, \mu) \; = \; \sum_{\mathbf{p}} \langle n_{\mathbf{p}} \rangle \; = \; \frac{V}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(\frac{p^2}{2m} \; - \; \mu\right)/T\right] \; - \; 1} \; , \quad (2)$$

where V is the system volume. We consider particles with spin equal to zero, thus the degeneracy factor equals 1. In the thermodynamic limit, $V \to \infty$, the sum over momentum states is transformed into the momentum integral, $\sum_{\mathbf{p}} \ldots = (V/2\pi^2) \int_0^\infty \ldots p^2 dp$. This substitution, assumed in all formulae below, is valid if the chemical potential in the non-relativistic Bose gas is restricted to $\mu < 0$ (or $\mu < m$ in relativistic formulation). When the temperature T decreases at fixed particle number density $\rho \equiv \overline{N}/V$, the chemical potential μ increases and becomes equal to zero at $T = T_C$, known as the BEC temperature. At this point from Eq. (2) one finds, $\overline{N}(V, T = T_C, \mu = 0) = V[mT_C/(2\pi)]^{3/2}\zeta(3/2)$, where $\zeta(3/2) \cong 2.612$ is the Riemann zeta-function. This gives,

$$T_C \;=\; 2\pi[\zeta(3/2)]^{-2/3}\,\frac{\rho^{2/3}}{m} \;\cong\; 3.31\,\frac{\rho^{2/3}}{m}\,. \qquad (3)$$

At $\mu = 0$ and $T < T_C$, a macroscopic part, N_C (called the BE condensate), of the total particle number occupies the lowest energy level $\mathbf{p} = 0$. At $\mu = 0$ and $T < T_C$ the GCE average number of particles in the BE condensate is equal to $N_C = \overline{N}[1-(T/T_C)^{3/2}]$.

Introducing $\Delta N \equiv N - \langle N\rangle$ one finds the particle number fluctuations in the GCE,

$$\langle(\Delta N)^2\,\rangle \;=\; \sum_{\mathbf{p},\mathbf{k}}\langle\Delta n_{\mathbf{p}}\,\Delta n_{\mathbf{k}}\rangle \;=\; \sum_{\mathbf{p}} v_{\mathbf{p}}^2\,, \qquad \omega \equiv \frac{\langle(\Delta N)^2\,\rangle}{\langle N\,\rangle} \;=\; \frac{\sum_{\mathbf{p}} \upsilon_{\mathbf{p}}^2}{\sum_{\mathbf{p}}\langle n_{\mathbf{p}}\rangle} \;=\; 1 \;+\; \frac{\sum_{\mathbf{p}}\langle n_{\mathbf{p}}\rangle^2}{\sum_{\mathbf{p}}\langle n_{\mathbf{p}}\rangle}\,. \qquad (4)$$

The limit $-\mu/T \gg 1$ gives $\langle n_{\mathbf{p}}\rangle \ll 1$. This corresponds to the Boltzmann approximation, and then from Eqs. (2,4) it follows: $\overline{N}(V,T,\mu) \cong V\exp(\mu/T)(mT/2\pi)^{3/2}$ and $\omega \cong 1$. When μ increases the scaled variance ω becomes larger, $\omega > 1$. This is the well known Bose enhancement effect for the particle number fluctuations. From Eq. (4) at $\mu \to 0$ one finds $\omega \to \infty$. Thus, the anomalous particle number fluctuations appear in the GCE formulation when the system approaches the BEC point. Two comments are appropriate here. First, for finite systems ω remains finite, and $\omega = \infty$ emerges from Eq. (4) at $\mu = 0$ in the thermodynamic limit $V \to \infty$, when the sums over $\mathbf{p}$ are transformed into the momentum integrals. Second, the anomalous fluctuations of the particle number at the BEC point correspond to the GCE description. In the CE and MCE, the number of particles N in a non-relativistic system is fixed by definition, thus, $\omega_{c.e.} = \omega_{m.c.e.} = 0$.

The average values of the occupation numbers in the relativistic ideal gas of pions equal to:

$$\langle n_{\mathbf{p},j}\rangle \;=\; \frac{1}{\exp[(\sqrt{\mathbf{p^2}+m_\pi^2}\;-\;\mu_j)/T]\;-\;1}\,, \qquad (5)$$

where index j enumerates 3 isospin pion states, π^+, π^-, and π^0, the energy of one-particle states is taken as, $\varepsilon_{\mathbf{p}} = (\mathbf{p^2}+m_\pi^2)^{1/2}$ with $m_\pi \cong 140$ MeV being the pion mass (we neglect a small difference between the masses of charged and neutral pions). The inequality $\mu_j \le m_\pi$ is a general restriction in the relativistic Bose gas, and $\mu_j = m_\pi$ corresponds to the BEC. In Ref. [6] we discussed in details the Bose gas with one conserved charge in the CE $(V, T, Q = const)$, i.e. the $\pi^+\pi^-$-gas with fixed electric charge. This corresponds to the GCE (V, T, μ_Q), thus, in Eq. (5) $\mu_+ = \mu_Q$ and $\mu_- = -\mu_Q$ for π^+ and π^-, respectively. Approaching the BEC of π^+ at $\mu_Q \to m_\pi$, one finds the relation between T_C and $\rho_Q \equiv \rho_+ - \rho_-$ (see Fig. 1, Left). The picture of BEC of π^- at $Q < 0$ and $\mu_Q \to -m_\pi$ is obtained by a mirror reflection. BEC starts at $T = T_C$ when $\mu_Q = \mu_Q^{max} = m_\pi$. It gives:

$$\rho_Q(T = T_C, \mu_Q = m_\pi) \;=\; \frac{T_C\,m_\pi^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_2\,(n\,m_\pi/T_C)\,\sinh(n\,m_\pi/T_C), \qquad (6)$$

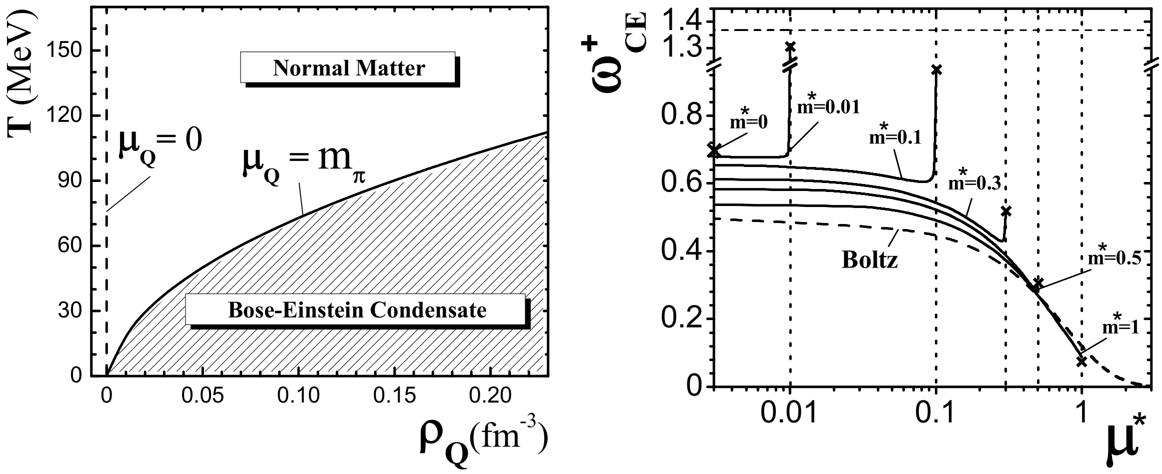

Figure 1: Left: The phase diagram of the relativistic ideal Bose gas. The solid line shows Bose condensation temperature as a function of the conserved charge density ρ_Q given by Eq. (6) at $\mu_Q = m_\pi$. The dashed line shows $\rho_Q = \rho_+ - \rho_- = 0$ at $\mu_Q = 0$. **Right:** The scaled variances $\omega^+_{c.e.}$ in pion Bose gas, are shown as functions of $\mu^* \equiv \mu_Q/T$. The solid lines present $\omega^+_{c.e.}$ at $m^* \equiv m_\pi/T = 0.01,\ 0.1,\ 0.3,\ 0.5,\ 1$. The vertical dotted lines $\mu^* = m^*$ demonstrate the restriction $\mu^* \leq m^*$ in the Bose gas. The dashed horizontal line presents a value of $\zeta(2)/\zeta(3) \simeq 1.368$ which is an upper limit for $\omega^\pm_{c.e.}$ reached at $\mu^* = m^* \to 0$. The crosses at $\mu^* = m^*$ correspond to the points of Bose condensation. The crosses at $\mu^* = 0$ correspond to $\omega^\pm_{c.e.}(\mu^* = 0, m^* \to 0$. The dashed line corresponds to $\omega^+_{c.e.}$ in classical (Boltzmann) pion gas (see Ref. [6] for details).

where K_2 is the modified Hankel function. At $T_C/m_\pi \ll 1$, Eq. (6) gives, $T_C = \frac{2\pi}{[\zeta(3/2)]^{2/3}}\,\rho_Q^{2/3} m_\pi^{-1} \cong 3.31\rho_Q^{2/3} m_\pi^{-1}$, which coincides with the non-relativistic formula (3). In the ultrarelativistic limit, $T_C/m_\pi \gg 1$, Eq. (6) gives

$$T_C \;=\; \sqrt{3}\,\rho_Q^{1/2}\,m_\pi^{-1/2}\;. \tag{7}$$

The results presented in Eqs. (6-7) are well known (see, e.g., Ref. [11]). For this system, the particle number fluctuations near the BEC line within GCE and CE were recently studied in Ref. [6]. The scaled variance $\omega^+ \equiv \langle(\Delta N_+)^2\rangle/\langle N_+\rangle$ in the GCE goes to infinity. This is similar to the non-relativistic case. On the other hand, the scaled variance for negative particles, $\omega^- \equiv \langle(\Delta N_-)^2\rangle/\langle N_-\rangle$, remains finite and even decreases with μ_Q. The pion numbers N_+ and N_- fluctuate in the both GCE and CE. However, the exact conservation imposed in the CE on the system charge, $Q = N_+ - N_-$, suppresses anomalous fluctuations at the BEC point: $\omega^+_{c.e.}$ (see Fig. 1, Right) is finite with the upper limit, $\zeta(2)/\zeta(3) \cong 1.368$ (see details in Ref. [6]).

A formation of the pion system with large electric charge density ρ_Q in high energy collisions does not look realistic. In what follows we discuss a rather different pion system which may be created in high multiplicity events [7]. We use the MCE $(V, E, Q = 0, N_\pi = const)$ formulation, where the total system energy E, electric charge $Q \equiv N_+ - N_- = 0$, and total number of pions,

$$N_\pi \;=\; N_0 \;+\; N_+ \;+\; N_-\;, \tag{8}$$

will be fixed. Such a system can be also described in the GCE $(V,T,\mu_Q{=}0,\mu_\pi)$ formulation, with $\mu_+ = \mu_\pi + \mu_Q$, $\mu_- = \mu_\pi - \mu_Q$, and $\mu_0 = \mu_\pi$ in Eq. (5). We restrict $\mu_Q{=}0$ and consider BEC when $\mu_\pi \to m_\pi$. The $\mu_Q{=}0$ corresponds to zero electric charge, $Q{=}0$ or $N_+ = N_-$, in the pion system.

The pion density is equal to $\rho_\pi(T,\mu_\pi) = \sum_{\mathbf{p},j}\langle n_{\mathbf{p},j}\rangle/V$. The phase diagram of the ideal pion gas in $\rho_\pi - T$ plane is presented in Fig. 2 (Left). BEC starts at $T = T_C$ when $\mu_\pi = \mu_\pi^{max} = m_\pi$. It gives [7]:

$$\rho_\pi(T = T_C, \mu_\pi = m_\pi) \ = \ \frac{3\,T_C\,m_\pi^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_2\,(n\,m_\pi/T_C)\,\exp(n\,m_\pi/T_C). \qquad (9)$$

Note an essential difference between Eq. (9) and Eq. (6): a presence of $\exp(nm_\pi/T_C)$ in Eq. (9), instead of $\sinh(nm_\pi/T_C)$ in Eq. (6). The Eq. (9) gives the BEC line shown by the solid line in Fig. 2 (Left). If $T_C/m_\pi \ll 1$, from Eq. (9) one finds, $T_C = 2\pi[3\zeta(3/2)]^{-2/3}\rho_\pi^{2/3}m_\pi^{-1} \cong 1.59\rho_\pi^{2/3}m_\pi^{-1}$. This again corresponds to the non-relativistic limit (3) discussed above, but with a degeneracy factor $g_\pi{=}3$. In the ultrarelativistic limit, $T_C/m_\pi \gg 1$, Eq. (9) leads to a new relation [7]:

$$T_C = [\pi^2/3\zeta(3)]^{1/3}\rho_\pi^{1/3} \ \cong 1.4\ \rho_\pi^{1/3}\ , \qquad (10)$$

which differs from Eq. (7) and does not include the dependence on m_π.

Let us consider the region in $\rho_\pi - T$ plane between the $\mu_\pi{=}0$ and $\mu_\pi{=}m_\pi$ lines. The lines of fixed energy density, $\varepsilon(T,\mu_\pi) = \sum_{\mathbf{p},j}\varepsilon_{\mathbf{p}}\,\langle n_{\mathbf{p},j}\rangle/V$, are shown as dotted lines in Fig. 2 (Left) inside this region for three fixed values of ε. An increase of ρ_π at constant ε leads to the increase of μ_π and decrease of T. In this letter we discuss how the system approaches the BEC line $(\mu_\pi{=}m_\pi, T{=}T_C)$, and do not touch the region $(\mu_\pi{=}m_\pi, T < T_C)$ below this line where the non-zero BE condensate is formed. The GCE (V,T,μ_Q,μ_π), MCE (V,E,Q,N_π), and CE (V,T,Q,N_π) are equivalent for average quantities, including average particle multiplicities, in the thermodynamic limit. Thus, Eq. (9) and phase diagram in Fig. 1 remain the same in all statistical ensembles. However, the pion number fluctuations are very different in different ensembles. Before starting to calculate the pion number fluctuations let us make several comments.

As an example we consider the high multiplicity events in $p+p$ collisions at IHEP (Protvino) accelerator with the beam energy of 70 GeV (see Ref. [12] on the experimental project “Thermalization”, team leader V.A. Nikitin). In the reaction $p+p \to p+p+N_\pi$ with small final proton momenta in the c.m.s., the total c.m. energy of created pions is $E \cong \sqrt{s} - 2m_p \cong 9.7$ GeV. The trigger system designed at JINR (Dubna) selects the events with $N_\pi > 20$ in this reaction. This makes it possible to accumulate the samples of events with fixed $N_\pi = 30 \div 50$ and the full pion identification during the next 2 years [13]. Note that for this reaction the kinematic limit is $N_\pi^{max} = E/m_\pi \cong 70$. We stress that the IHEP experiment will measure the both charged and neutral pions (see Ref. [14]). A reliable measurement of N_0 number in each event is a crucial point for the identification of the BEC line suggested in this letter. The BEC signatures discussed below become useless if the π^0 number cannot be measured reliably.

The pion system in the thermal equilibrium is expected to be formed for high multiplicities. The volume of the pion gas system is estimated as, $V = E/\varepsilon(T,\mu_\pi)$, and the number of pions equals to $N_\pi = V\rho_\pi(T,\mu_\pi)$. The values of N_π at $\mu_\pi = 0$ and $\mu_\pi = m_\pi$ for 3 different values of energy density ε are shown in Fig. 2 (Left) for the fixed total pion energy of $E = 9.7$ GeV. Note that the

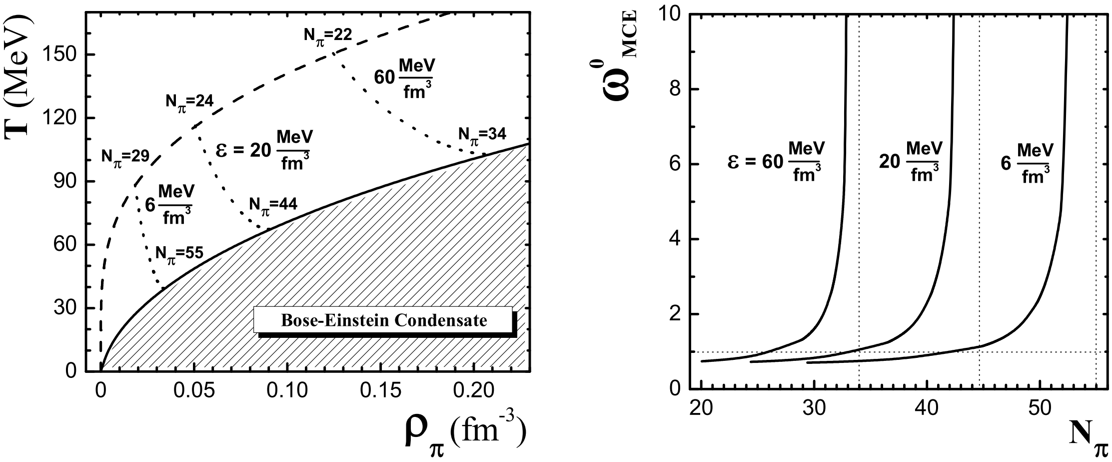

Figure 2: Left: The phase diagram of the pion gas with $\mu_Q = 0$ [7]. The dashed line corresponds to $\rho_\pi(T, \mu_\pi = 0)$, and the solid line to BEC. The dotted lines show the states with fixed energy densities: $\varepsilon = 6, 20, 60$ MeV/fm^3. The N_π numbers in the figure correspond to $\mu_\pi = 0$ and $\mu_\pi = m_\pi$ at these energy densities for the total pion energy, $E = 9.7$ GeV. **Right:** The scaled variance of neutral pions in the MCE is presented as the function of the total number of pions [7]. Three solid lines correspond to different energy densities: $\varepsilon = 6, 20, 60$ MeV/fm^3. The total energy of the pion system is assumed to be fixed, $E = 9.7$ GeV. The vertical dotted lines correspond to the points on the BEC line at the specific values of the energy density.

statistical approach to hadron production in p+p collisions has been used successfully to calculate the particle number ratios within the CE [15] and MCE [16]. Such an approach is usually applied to a sample of the minimum bias events. Our suggestion has two new points. First, it is a selection of the sample of events with high pion multiplicity N_π. Most part of the available energy is then spent to the pion production. Thus, a strong longitudinal motion seen in the inclusive data is suppressed in high multiplicity events because of the energy conservation. The pion system may approach the state of global thermal equilibrium with the thermodynamical parameters close to the BEC line. To search the BEC effects we suggest to study the specific event-by-event fluctuations of the number of pions. This is a second new point of our suggestion. The typical expected temperature of the pion gas approaching the BEC line is about of $T = 60 - 90$ MeV (see Fig. 1). One can then calculate the average thermal energy per particle, a rough estimate gives: $3T/2 = 90 - 135$ MeV. On the other hand, a pion from the ρ-meson decay has a much larger 'kinetic energy' of about 245 MeV in the ρ-meson rest frame. The pion energy becomes even larger due to a presence of non-zero rho-meson momenta in the c.m.s. of the p+p collision. Thus, we conclude that for the high multiplicity events discussed in this letter a presence of large number of resonances decaying into pions is strongly suppressed because of the energy conservation.

For $Q = 0$, the average pion multiplicities, $\langle N_0 \rangle = \langle N_\pm \rangle = N_\pi/3$, are the same in all statistical ensembles for large systems. This thermodynamic equivalence is not, however, valid for the scaled variances of pion fluctuations. The system with the fixed electric charge, $Q = 0$, the total pion number, N_π, and total energy of the pion system, E, should be treated in the MCE. The volume V is one more (and unknown) MCE parameter. The calculations below are carried out in a large volume limit, thus, parameter V does not enter explicitly in the formulae for the scaled variances.

The microscopic correlators in the MCE $(V,E,Q=0,N_\pi)$ equal to (see also Refs. [7–9]):

$$\langle \Delta n_{\mathbf{p}}^j \Delta n_{\mathbf{k}}^j \rangle_{m.c.e.} = \upsilon_{\mathbf{p},j}^2\ \delta_{\mathbf{pk}}\delta_{ji} - \frac{\upsilon_{\mathbf{p},j}^2\,\upsilon_{\mathbf{k},i}^2}{|A|}\Big[q_j q_i M_{qq} + M_{\pi\pi} + \varepsilon_{\mathbf{p}i}\varepsilon_{\mathbf{k}j} M_{\varepsilon\varepsilon} - (\varepsilon_{\mathbf{p}i} + \varepsilon_{\mathbf{k}j}) M_{\pi\varepsilon}\Big]\,, \quad (11)$$

where $q_+=1$, $q_-=-1$, $q_0=0$,

$$\upsilon_{\mathbf{p},j}^2 \ =\ \upsilon_{\mathbf{p}}^2 \ =\ \langle n_{\mathbf{p}}\rangle\ (1\ +\ \langle n_{\mathbf{p}}\rangle)\ , \qquad \langle n_{\mathbf{p}}\rangle \ =\ \{\exp[(\sqrt{\mathbf{p^2}+m_\pi^2}-\mu_\pi)/T]\ -\ 1\}^{-1}\ , \quad (12)$$

$|A|$ is the determinant and M_{ij} are the minors,

$$M_{qq} = \Delta(\pi^2)\,\Delta(\varepsilon^2) - (\Delta(\pi\varepsilon))^2, \quad M_{\pi\pi} = \Delta(q^2)\,\Delta(\varepsilon^2), \quad M_{\varepsilon\varepsilon} = \Delta(q^2)\,\Delta(\pi^2), \quad M_{\pi\varepsilon} = \Delta(q^2)\,\Delta(\pi\varepsilon), \quad (13)$$

of the correlation matrix A,

$$A \ =\ \begin{pmatrix} \Delta(q^2) & 0 & 0 \\ 0 & \Delta(\pi^2) & \Delta(\pi\varepsilon) \\ 0 & \Delta(\pi\varepsilon) & \Delta(\varepsilon^2) \end{pmatrix}\ . \quad (14)$$

The matrix A (14) has the following elements:

$$\begin{aligned} \Delta(q^2)\ &=\ \sum_{\mathbf{p},j} q_j^2 \upsilon_{\mathbf{p},j}^2\ =\ 2\sum_{\mathbf{p}} \upsilon_{\mathbf{p}}^2\ , \qquad & \Delta(\pi^2)\ &=\ \sum_{\mathbf{p},j} \upsilon_{\mathbf{p},j}^2\ =\ 3\sum_{\mathbf{p}} \upsilon_{\mathbf{p}}^2\ , \\ \Delta(\varepsilon^2)\ &=\ \sum_{\mathbf{p},j} \varepsilon_{\mathbf{p}}^2 \upsilon_{\mathbf{p},j}^2\ =\ 3\sum_{\mathbf{p}} \varepsilon_{\mathbf{p}}^2 \upsilon_{\mathbf{p}}^2\ , \qquad & \Delta(\pi\varepsilon)\ &=\ \sum_{\mathbf{p},j} \varepsilon_{\mathbf{p}} \upsilon_{\mathbf{p},j}^2\ =\ 3\sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} \upsilon_{\mathbf{p}}^2\ . \end{aligned} \quad (15)$$

Note that the first term in the r.h.s. of Eq. (11) corresponds to the GCE. Correlations between differently charged pions, $j\neq i$, and between different single modes, $\mathbf{p}\neq\mathbf{k}$, are absent in the GCE:

$$\omega^+\ =\ \omega^-\ = \omega^0\ \equiv \omega\ =\ 1\ +\ \frac{\sum_{\mathbf{p}}\langle n_{\mathbf{p}}\rangle^2}{\sum_{\mathbf{p}}\langle n_{\mathbf{p}}\rangle}\ , \quad (16)$$

similar to non-relativistic result (4), but with $\langle n_{\mathbf{p}}\rangle$ given by the relativistic relation (12). In the GCE the numbers N_+, N_-, and N_0 fluctuate independently of each other. The Bose effects in the pion system are small if $\mu_\pi = 0$. For $\mu_\pi = 0$, one finds $\omega = 1.01 \div 1.12$ in the temperature interval $T = 40 - 160$ MeV (note that $\omega = 1$ in the Boltzmann approximation). The Bose effects increase with μ_π, and $\omega \to \infty$ at $\mu_\pi \to m_\pi$, i.e. approaching the BEC line the GCE calculations give anomalous fluctuations for N_+, N_-, and N_0.

The MCE $(V,E,Q=0,N_\pi=const)$ formulation means the restrictions of the exactly fixed total system energy E, electric charge $Q=N_+-N_-=0$, and total number of pions N_π (8) for the each microscopic state of the system. This changes the pion number fluctuations. From Eq. (11) one notices that the MCE fluctuations of each mode $\mathbf{p}$ are reduced, and the (anti)correlations between different modes $\mathbf{p}\neq\mathbf{k}$ and between different charge states appear. This results in a suppression of all scaled variances $\omega^j_{m.c.e.}$ in comparison with the corresponding ones ω in the GCE. A nice feature of the MCE microscopic correlators (11) is that although being different from that in the GCE, they are expressed with the quantities calculated in the GCE. The MCE scaled variances depend on two GCE parameters: T and μ_π.

The substitution of Eqs. (12-15) in Eq. (11) and straightforward calculations lead to the following MCE scaled variance for neutral pions [7]:

$$\omega^0_{m.c.e.} ~=~ \frac{\sum_{\mathbf{p},\mathbf{k}}\langle\Delta n^0_{\mathbf{p}}\,\Delta n^0_{\mathbf{k}}\rangle_{m.c.e.}}{\sum_{\mathbf{p}}\langle n^0_{\mathbf{p}}\rangle} ~=~ \frac{2}{3}\,\omega~, \tag{17}$$

where ω is given by Eq. (16) and corresponds to pion fluctuations in the GCE. Due to the conditions, $N_+ = N_-$ and $N_+ + N_- + N_0 = N_\pi$, and equal average multiplicities, $\langle N_0\rangle = \langle N_+\rangle = \langle N_-\rangle = N_\pi/3$, it follows [7]:

$$\omega^{\pm}_{m.c.e.} ~=~ \frac{1}{4}\,\omega^0_{m.c.e.} ~=~ \frac{1}{6}\,\omega~, \qquad \text{and} \qquad \omega^{ch}_{m.c.e.} ~=~ \frac{1}{2}\,\omega^0_{m.c.e.} ~=~ \frac{1}{3}\,\omega~, \tag{18}$$

where $N_{ch} \equiv N_+ + N_-$. The behavior of $\omega^0_{m.c.e.}$ (17) is shown in Fig. 2 (Right). To make a correspondence with N_π values, we consider again the $p+p \rightarrow p+p+N_\pi$ collisions at the beam energy of 70 GeV and take the pion system energy to be equal to $E = 9.7$ GeV. Despite of the MCE suppression the scaled variances for the number fluctuations of π^0 and $\pi^\pm$ increase dramatically and abruptly when the system approaches the BEC line.

The following inequalities are always hold for particle number fluctuations in different ensembles: $\omega^j_{m.c.e.} < \omega^j_{c.e.} < \omega^j_{g.c.e.}$. Therefore, if the anomalous BEC fluctuations are present in the MCE, they are also exist (and even larger) in the CE and GCE. The reverse statement is not true. The anomalous BEC fluctuations of the GCE may disappear in the CE or MCE. We found that for the system with $N_\pi = const$ and $Q = 0$ the anomalous BEC fluctuations are not washed out by exact conservation laws of the CE and MCE. This is an advantage of the system with $N_\pi = const$ and $Q = 0$. Let us repeat again (see discussion just after Eq. (7) and Ref. [6]) that the anomalous BEC fluctuations at high charge density ρ_Q disappear in the CE and/or MCE. As another instructive example let us consider the MCE $(V,E,Q{=}0,N_{ch} = const)$, i.e. fixed $N_{ch} = N_+ + N_-$, instead of fixed N_π (8). The corresponding GCE formulation gives the following pion chemical potential: $\mu_+ = \mu_\pi$, $\mu_0 = 0$, $\mu_- = \mu_\pi$ in Eq. (5) ($\mu_Q = 0$, as before, because of $Q = 0$ condition). When $\mu_\pi \rightarrow m_\pi$ the system approaches the BEC line for π^+ and π^-. The thermodynamic behavior and position of this BEC line can be easily found. Approaching the BEC line one can also find $\omega^\pm \rightarrow \infty$ in the GCE. The pion number fluctuations are, however, very different in the both CE $(V,T,Q = 0,N_{ch})$ and MCE $(V,E,Q = 0,N_{ch})$. In the statistical ensembles with fixed N_{ch} and Q no anomalous BEC fluctuations are possible. The numbers of N_+ and N_- are completely fixed by the conditions $Q = N_+ - N_- = 0$ and $N_{ch} = N_+ + N_- = const$, thus, $\omega^\pm_{c.e.} = \omega^\pm_{m.c.e.} = 0$. The number N_0 fluctuates, but $\mu_0 = 0$, thus, neutral pions are far away from the BEC line and their fluctuations are small, $\omega^0 \approx 1$, in all statistical ensemble formulations.

The broad distributions over N_0 and N_{ch} close to the BEC line also implies large fluctuations of the $f \equiv N_0/N_{ch}$ ratio. These large fluctuations were suggested (see, e.g., Ref. [17]) as a possible signal for the disoriented chiral condensate (DCC). The DCC leads to the distribution of f in the form, $dW(f)/df = 1/(2\sqrt{f})$. The thermal Bose gas corresponds to the f-distribution centered at $f = 1/2$. Therefore, f-distributions from BEC and DCC are very different, and this gives a possibility to distinguish between these two phenomena.

The calculations presented in this letter should be improved by taking into account the finite size effects, pion-pion interactions, and some other effects. Examples discussed in Refs. [5, 8]

demonstrate that the thermodynamical limit for the average multiplicities and scaled variances is reached rather quickly. Normally, if the average pion multiplicity is about of $N_\pi = 10$, the deviations from thermodynamic limit in the ideal pion gas are only a few percents. The Bose-Einstein condensation is a phase transition phenomenon. Thus, the infinite volume limit is of a principal importance. A strict mathematical meaning of the phase transition (and its order) has only sense in the infinite volume limit. Of course, the real systems are finite. For our applications this means that the scaled variance for neutral pions shown in Fig. 2 does not increase up to 'infinity', but it is restricted from above in the finite system. A detailed study of the finite size effects for the BEC is now under investigation and will be published elsewhere. The BEC temperature of about $T = 60-90$ MeV corresponds to the pion number density of $\rho_\pi = 0.1-0.15$ fm^{-3} (see Fig. 1). This particle density is not too large (it is smaller than the normal nuclear density). This probably may justify the ideal pion gas approximation considered in the present letter as a first step in the modelling of multi-pion states. The effects of pion interactions will be discussed in the future studies. Preliminary estimates suggest that the BEC signatures suggested in the present letter may survive the complications. A crucial point is the analysis of the samples of high N_π events. The required N_π values for the BEC are much larger than the average pion multiplicity per collision, thus, these high N_π events are rather rare and give negligible contributions to inclusive observables in high energy collisions. With increasing of N_π in the sample with fixed total energy, the temperature of the pion system has to decrease and it approaches the BEC line. This can happen in different ways: at constant energy density ε, at constant pion density ρ_π, or with decreasing of both ε and ρ_π. The pion system should move to the BEC line one way or another. In the vicinity of the BEC line (no BE condensate is yet formed) one observes an abrupt and anomalous increase of the scaled variances of neutral and charged pion number fluctuations. This could (may be even should) be checked experimentally.

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