

# The high density equation of state: constraints from accelerators and astrophysics

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The nuclear equation of state (EoS) at high densities and/or extreme isospin is one of the long-standing problems of nuclear physics. In the last years substantial progress has been made to constrain the EoS both, from the astrophysical side and from accelerator based experiments. Heavy ion experiments support a soft EoS at moderate densities while the possible existence of high mass neutron star observations favors a stiff EoS. Ab initio calculations for the nuclear many-body problem make predictions for the density and isospin dependence of the EoS far away from the saturation point. Both, the constraints from astrophysics and accelerator based experiments are shown to be in agreement with the predictions from many-body theory.

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<sup>†</sup>A footnote may follow.

## 1. Introduction

The isospin dependence of the nuclear forces which at present is only little constrained by data will be explored by the forthcoming radioactive beam facilities at FAIR/GSI, SPIRAL2/GANIL and RIA. Since the knowledge of the nuclear equation-of-state (EoS) at supra-normal densities and extreme isospin is essential for our understanding of the nuclear forces as well as for astrophysical purposes, the determination of the EoS was already one of the primary goals when first relativistic heavy ion beams started to operate. A major result of the SIS100 program at the GSI is the observation of a soft EoS for symmetric matter in the explored density range up to 2-3 times saturation density. These accelerator based experiments are complemented by astrophysical observations.

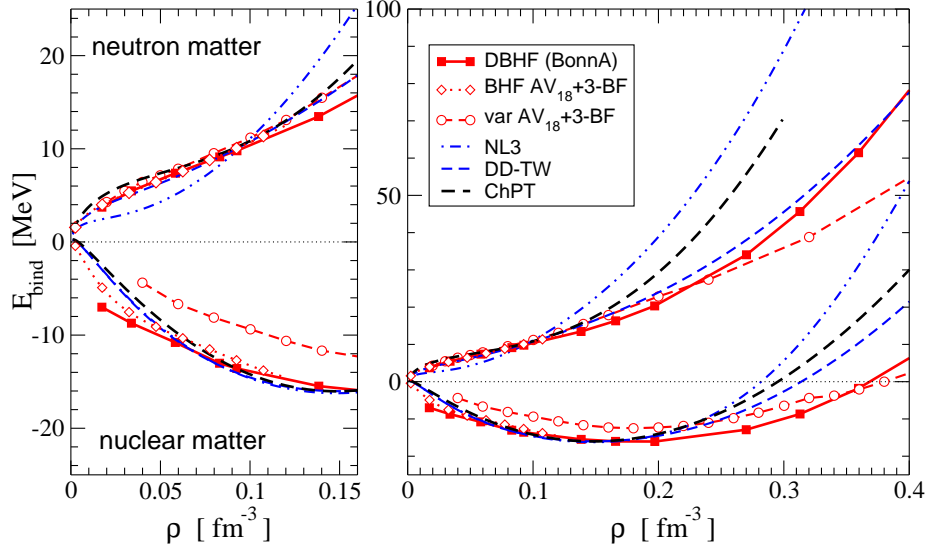
In particular the stabilization of high mass neutron stars requires a stiff EoS at high densities. There exist several observations pointing in this direction, e.g. the large radius of  $R > 12$  km for the isolated neutron star RX J1856.5-3754 (shorthand: RX J1856) [1]. Measurements of high masses are also reported for compact stars in low-mass X-ray binaries (LMXBs) as  $M = 2.0 \pm 0.1 M_{\odot}$  for the compact object in 4U 1636-536 [2]. For another LMXB, EXO 0748-676, constraints for the mass  $M \geq 2.10 \pm 0.28 M_{\odot}$  and the radius  $R \geq 13.8 \pm 0.18$  km have been reported [3]. Unfortunately, one of the most prominent high mass neutron star candidates, the J0751+1807 binary pulsar with an originally reported mass of  $M = 2.1 \pm 0.2 M_{\odot}$  [4] has been revisited and corrected down to  $M = 1.26 M_{\odot}$  [5]. However, very recently an extraordinary high value of  $M = 2.74 \pm 0.21 M_{\odot}$  ( $1\sigma$ ) has been reported for the millisecond pulsar PSR J1748-2021B [6].

Contrary to a naive expectation, high mass neutron stars do, however, not stand in contradiction with the observations from heavy ion reactions, see e.g. [7, 8]. Moreover, we are in the fortunate situation that *ab initio* calculations of the nuclear many-body problem predict a density and isospin behavior of the EoS which is in agreement with both observations.

Hence the present contribution starts with short survey on the predictions from many-body theory, turns then to heavy ion reactions and discusses finally the application to neutron stars.

## 2. The EoS from *ab initio* calculations

In *ab initio* calculations based on many-body techniques one derives the EoS from first principles, i.e. treating short-range and many-body correlations explicitly. This allows to make prediction for the high density behavior, at least in a range where hadrons are still the relevant degrees of freedom. A typical example for a successful many-body approach is Brueckner theory (for a recent review see [9]). In the following we consider non-relativistic Brueckner and variational calculations [10] as well as relativistic Brueckner calculations [11, 12, 13]. It is a well known fact that non-relativistic approaches require the inclusion of - in net repulsive - three-body forces in order to obtain reasonable saturation properties. In relativistic treatments part of such diagrams, e.g. virtual excitations of nucleon-antinucleon pairs are already effectively included. Fig. 1 compares now the predictions for nuclear and neutron matter from microscopic many-body calculations – DBHF [12] and the ‘best’ variational calculation with 3-BFs and boost corrections [10] – to phenomenological approaches (NL3 and DD-TW from [15]) and an approach based on chiral pion-nucleon dynamics [16] (ChPT+corr.). As expected the phenomenological functionals agree well at and below saturation density where they are constrained by finite nuclei, but start to deviate substantially at



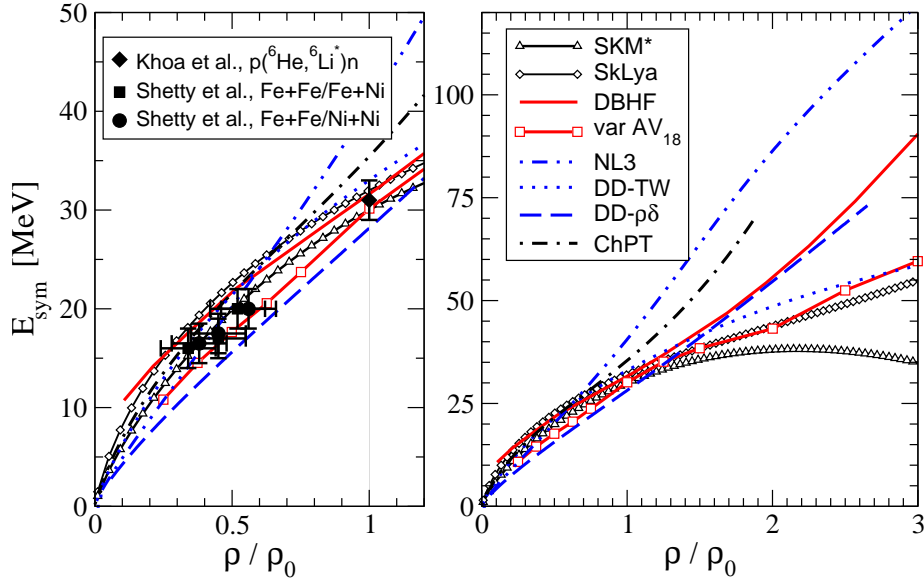
**Figure 1:** EoS in nuclear matter and neutron matter. BHF/DBHF and variational calculations are compared to phenomenological density functionals (NL3, DD-TW) and ChPT+corr.. The left panel zooms the low density range. The Figure is taken from Ref. [14].

supra-normal densities. In neutron matter the situation is even worse since the isospin dependence of the phenomenological functionals is less constrained. The predictive power of such density functionals at supra-normal densities is restricted. *Ab initio* calculations predict throughout a soft EoS in the density range relevant for heavy ion reactions at intermediate and low energies, i.e. up to about  $3 \rho_0$ . Since the  $nn$  scattering length is large, neutron matter at subnuclear densities is less model dependent. The microscopic calculations (BHF/DBHF, variational) agree well and results are consistent with 'exact' Quantum-Monte-Carlo calculations [17].

Fig. 2 compares the symmetry energy predicted from the DBHF and variational calculations to that of the empirical density functionals already shown in Fig. 1. In addition the relativistic DD- $\rho\delta$  RMF functional [20] is included. Two Skyrme functionals, SkM\* and the more recent Skyrme-Lyon force SkLya represent non-relativistic models. The left panel zooms the low density region while the right panel shows the high density behavior of  $E_{\text{sym}}$ .

The low density part of the symmetry energy is in the meantime relatively well constraint by data. Recent NSCL-MSU heavy ion data in combination with transport calculations are consistent with a value of  $E_{\text{sym}} \approx 31$  MeV at  $\rho_0$  and rule out extremely "stiff" and "soft" density dependences of the symmetry energy [21]. The same value has been extracted [18] from low energy elastic and (p,n) charge exchange reactions on isobaric analog states  $p(^6\text{He}, ^6\text{Li}^*)n$  measured at the HMI. At sub-normal densities recent data points have been extracted from the isoscaling behavior of fragment formation in low-energy heavy ion reactions with the corresponding experiments carried out at Texas A&M and NSCL-MSU [19].

However, theoretical extrapolations to supra-normal densities diverge dramatically. This is crucial since the high density behavior of  $E_{\text{sym}}$  is essential for the structure and the stability of neutron stars. The microscopic models show a density dependence which can still be considered as *asy-stiff*. DBHF [12] is thereby stiffer than the variational results of Ref. [10]. The density



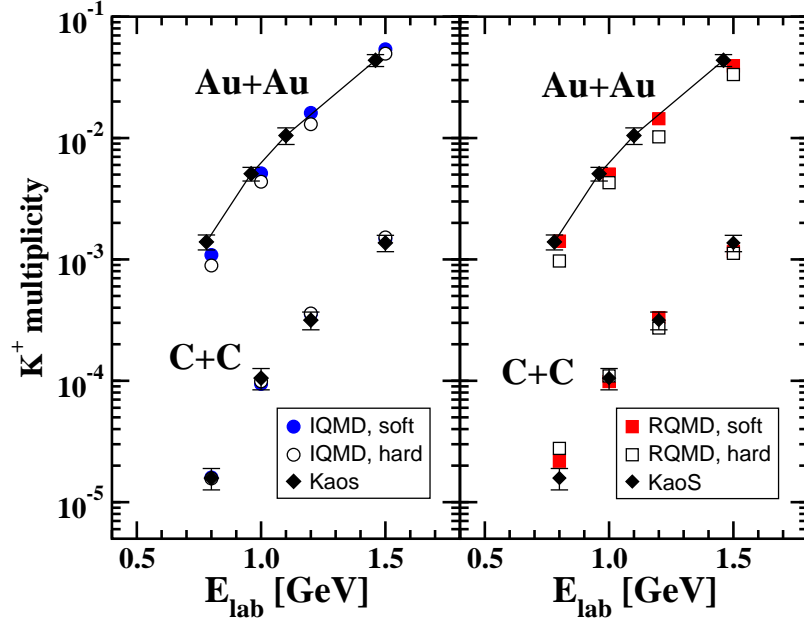
**Figure 2:** Symmetry energy as a function of density as predicted by different models. The left panel shows the low density region while the right panel displays the high density range. Data are taken from [18] and [19].

dependence is generally more complex than in RMF theory, in particular at high densities where  $E_{\text{sym}}$  shows a non-linear and more pronounced increase. Fig. 2 clearly demonstrates the necessity to better constrain the symmetry energy at supra-normal densities with the help of heavy ion reactions.

### 3. Constraints from heavy ion reactions

Experimental data which put constraints on the symmetry energy have already been shown in Fig. 2. The problem of multi-fragmentation data from low and intermediate energy reactions is that they are restricted to sub-normal densities up to maximally saturation density. However, from low energetic isospin diffusion measurements at least the slope of the symmetry around saturation density could be extracted [25]. This puts already an important constraint on the models when extrapolated to higher densities. It is important to notice that the slopes predicted by the ab initio approaches (variational, DBHF) shown in Fig. 2 are consistent with the empirical values. Further going attempts to derive the symmetry energy at supra-normal densities from particle production in relativistic heavy ion reactions [20, 26, 27] have so far not yet led to firm conclusions since the corresponding signals are too small, e.g. the isospin dependence of kaon production [28].

Firm conclusions could only be drawn on the symmetric part of the nuclear bulk properties. To explore supra-normal densities one has to increase the bombarding energy up to relativistic energies. This was one of the major motivation of the SIS100 project at the GSI where - according to transport calculation - densities between  $1 \div 3 \rho_0$  are reached at bombarding energies between  $0.1 \div 2$  AGeV. Sensitive observables are the collective nucleon flow and subthreshold  $K^+$  meson production. In contrast to the flow signal which can be biased by surface effects and the momentum dependence of the optical potential,  $K^+$  mesons turned out to an excellent probe for the high density phase of the reactions. At subthreshold energies the necessary energy has to be provided

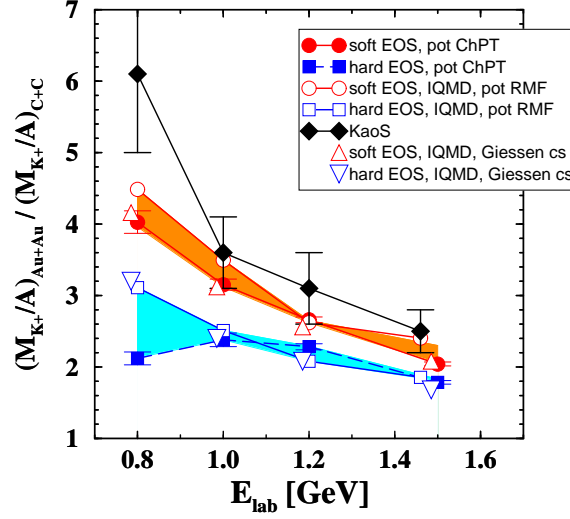


**Figure 3:** Excitation function of the  $K^+$  multiplicities in  $Au + Au$  and  $C + C$  reactions. RQMD [22] and IQMD [23] with in-medium kaon potential and using a hard/soft nuclear EoS are compared to data from the KaoS Collaboration [24]. The figure is taken from [14].

by multiple scattering processes which are highly collective effects. This ensures that the majority of the  $K^+$  mesons is indeed produced at supra-normal densities. In the following I will concentrate on the kaon observable.

Subthreshold particles are rare probes. However, within the last decade the KaoS Collaboration has performed systematic high statistics measurements of the  $K^+$  production far below threshold [24, 30]. Based on this data situation, in Ref. [22] the question if valuable information on the nuclear EoS can be extracted has been revisited and it has been shown that subthreshold  $K^+$  production provides indeed a suitable and reliable tool for this purpose. In subsequent investigations the stability of the EoS dependence has been proven [23, 29].

Excitation functions from KaoS [24, 31] are shown in Fig. 3 and compared to RQMD [22, 29] and IQMD [23] calculations. In both cases a soft ( $K=200$  MeV) and a hard ( $K=380$  MeV) EoS have been used within the transport approaches. Skyrme type forces supplemented by an empirical momentum dependence have been used. As expected the EoS dependence is more pronounced in the heavy  $Au+Au$  system while the light  $C+C$  system serves as a calibration. The effects become even more evident when the ratio  $R$  of the kaon multiplicities obtained in  $Au+Au$  over  $C+C$  reactions (normalized to the corresponding mass numbers) is built [22, 24]. Such a ratio has the advantage that possible uncertainties which might still exist in the theoretical calculations cancel out to large extent. Comparing the ratio shown in Fig. 4 to the experimental data from KaoS [24], where the increase of  $R$  is even more pronounced, strongly favors a soft equation of state. This result is in agreement with the conclusion drawn from the alternative flow observable [32, 33, 34, 35].



**Figure 4:** Excitation function of the ratio  $R$  of  $K^+$  multiplicities obtained in inclusive Au+Au over C+C reactions. RQMD [22] and IQMD [23] calculations are compared to KaoS data [24]. Figure is taken from [29].

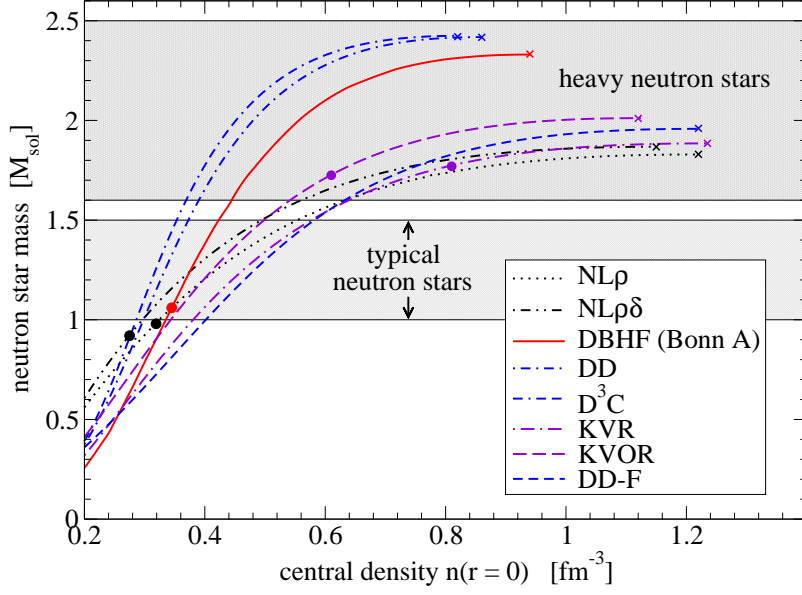
#### 4. Constraints from neutron stars

Measurements of “extreme” values, like large masses or radii, huge luminosities etc. as provided by compact stars offer good opportunities to gain deeper insight into the physics of matter under extreme conditions. There has been substantial progress in recent time from the astrophysical side.

The most spectacular observation was probably the recent measurement [4] on PSR J0751+1807, a millisecond pulsar in a binary system with a helium white dwarf secondary, which implied a pulsar mass of  $2.1 \pm 0.2 \left( {}^{+0.4}_{-0.5} \right) M_{\odot}$  with  $1\sigma$  ( $2\sigma$ ) confidence. This measurement has, however, been revisited and corrected down to  $M = 1.26 M_{\odot}$  [5].

There exist, however, several alternative observations pointing towards large masses, e.g. the large radius of  $R > 12$  km for the isolated neutron star RX J1856.5-3754 (shorthand: RX J1856) [1]. Measurements of high masses are also reported for compact stars in low-mass X-ray binaries (LMXBs) as  $M = 2.0 \pm 0.1 M_{\odot}$  for the compact object in 4U 1636-536 [2]. For another LMXB, EXO 0748-676, constraints for the mass  $M \geq 2.10 \pm 0.28 M_{\odot}$  and the radius  $R \geq 13.8 \pm 0.18$  km for the same object have been reported [3]. Very recently even an extremely high mass value of  $M = 2.74 \pm 0.21 M_{\odot}$  ( $1\sigma$ ) has been reported for the millisecond pulsar PSR J1748-2021B [6]. According to the authors of Ref. [6] there exists only a 1 % probability that the pulsar mass is below 2 solar masses, and a 0.10 % probability that it lies within the range of conventional neutron stars, i.e. between 1.20 and 1.44 solar masses. Such an anomalously large mass would of course strongly constrain the equation of state for dense matter, even excluding many-body approaches which reach maximum masses around  $M = 2.3 M_{\odot}$ .

In Ref. [37] we applied more conservative upper and lower limits for the maximum mass of  $1.6 - 2.5 M_{\odot}$  (initiated by the originally reported  $2\sigma$  range of PSR J0751+1807 [4]). However, even

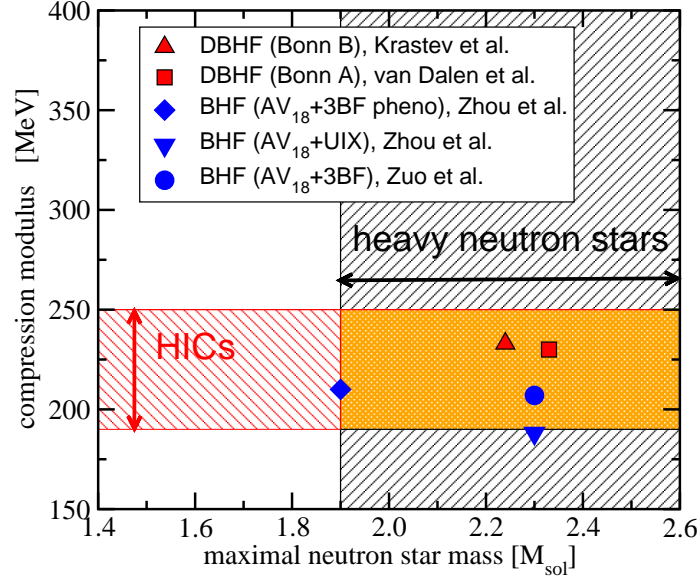


**Figure 5:** Mass versus central density for compact star configurations obtained for various relativistic hadronic EoS. Crosses denote the maximum mass configurations, filled dots mark the critical mass and central density values where the DU cooling process becomes possible. According to the DU constraint, it should not occur in “typical NSs” for which masses are expected from population synthesis [36] to lie in the lower grey horizontal band. The dark grey horizontal bands around indicate a conservative estimate for the possible range of maximal neutron star masses derived from recent observation.

such a weaker condition limits the softness of the EoS in neutron star (NS) matter considerably. The corresponding figure, Fig. 5, shows the mass versus central density for compact star configurations obtained by solving the TOV equations for a compilation of different hadronic EoS. These are relativistic mean field models and the microscopic DBHF model. For details see [37]. Crosses denote the maximum mass configurations, filled dots mark the critical mass and central density values where the DU cooling process becomes possible.

One might now be worried about an apparent contradiction between the constraints derived from neutron stars and those from heavy ion reactions. While heavy ion reactions favor a soft EoS, high neutron star masses require a stiff EoS. The corresponding constraints are, however, complementary rather than contradictory. Intermediate energy heavy-ion reactions, e.g. subthreshold kaon production, constrains the EoS at densities up to  $2 \div 3 \rho_0$  while the maximum NS mass is more sensitive to the high density behavior of the EoS. Combining the two constraints implies that the EoS should be *soft at moderate densities and stiff at high densities*. Such a behavior is predicted by microscopic many-body calculations (see Fig. 6). DBHF, BHF or variational calculations, typically, lead to maximum NS masses between  $2.1 \div 2.3 M_\odot$  and are therefore in accordance with most of the high mass neutron star measurement (except of masses around or above  $2.4 \div 2.5 M_\odot$ .)

This fact is illustrated in Fig. 6 which combines the results from heavy ion collisions and the maximal mass constraint. The figure shows the compression moduli for the EoS in symmetric nuclear matter as well as the maximum neutron star mass obtained with the corresponding models,



**Figure 6:** Combination of the constraints on the EoS derived from the maximal neutron star mass criterium and the heavy ion collisions constraining the compression modulus. Values of various microscopic BHF and DBHF many-body calculations are show.

i.e. non-relativistic Brueckner (BHF) as well as for relativistic Brueckner (DBHF) calculations [12, 38]. The BHF calculations differ essentially in the usage of different 3-body-forces (3-BFs). In particular the isospin dependence of 3-BFs is not yet well constrained by nuclear data which is reflected in the maximum masses obtained, not so much in the compression moduli. The DBHF calculations differ in the elementary NN interaction applied. However, here the results for both, compression moduli and maximum neutron star masses are rather stable.

Besides the maximum masses there exist several other constraints on the nuclear EoS which can be derived from observations of compact stars, see e.g. Refs. [37, 39]. Among these, the most promising one is the Direct Urca (DU) process which is essentially driven by the proton fraction inside the NS [40]. DU processes, e.g. the neutron  $\beta$ -decay  $n \rightarrow p + e^- + \bar{\nu}_e$ , are very efficient regarding their neutrino production, even in super-fluid NM and cool NSs too fast to be in accordance with data from thermally observable NSs. Therefore, one can suppose that no DU processes should occur below the upper mass limit for “typical” NSs, i.e.  $M_{DU} \geq 1.5 M_\odot$  ( $1.35 M_\odot$  in a weak interpretation). These limits come from a population synthesis of young, nearby NSs [36] and masses of NS binaries [4]. While the present DBHF EoS leads to too fast neutrino cooling this behavior can be avoided if a phase transition to quark matter is assumed [41]. Thus a quark phase is not ruled out by the maximum NS mass. However, corresponding quark EoS have to be almost as stiff as typical hadronic EoS [41].

## 5. Summary

Heavy ion reactions provide in the meantime reliable constraints on the isospin dependence of the nuclear EoS at sub-normal densities up to saturation density and for the symmetric part up to - as an conservative estimate - two times saturation density. These are complemented by astrophysical



constraints derived from the measurements of extreme values for neutron star masses. As long as the neutron star mass is below  $2.3 M_{\odot}$ , both, the heavy ion constraint as well as the astrophysical constraint is in fair agreement with the predictions from nuclear many-body theory. If, however, a maximum mass around or above  $2.5 M_{\odot}$  [6] will be established, this requires an extremely stiff EoS which demands for new physical pictures.

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