PROCEEDINGS OF SCIENCE

A Matrix Formulation for Small-x RG Improved Evolution

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After recalling the small-x resummation methods which generalize DGLAP and BFKL approaches to QCD evolution equations, I present a recent k-factorized matrix formulation in which quarks and gluons are treated on the same ground and exact NLO and NLx calculations are incorporated. I then produce results for the resummed eigenvalue functions and the splitting function matrix which show an overall gentle matching of resummation effects to fixed order quantities. The shallow dip occurring in previous treatments of P_{gg} is confirmed, and found in P_{gq} also.

8th International Symposium on Radiative Corrections (RADCOR)
October 1-5 2007
Florence, Italy

[∗]Speaker.
[†]Work supported in part by a PRIN grant, MIUR, Italy.

1. Generalizing BFKL and DGLAP equations in matrix form

The physical question underlying this talk is, at large, to provide a reliable description of rising "hard" cross sections and structure functions at high energies, and a precise determination of parton splitting functions at small-x, while keeping their well known behaviour at larger-x. More precisely, I will deal with the problem of providing a small-x resummation of parton evolution in *matrix* [1] form, so as to treat by k-factorization [2] quarks and gluons on the same ground and in a collinear factorization scheme as close as possible to $\overline{\mathrm{MS}}$.

The issue of a small-x generalization of DGLAP [3] and BFKL [4] evolutions has a long story [5, 6, 7, 8, 9], whose outcome is, at present, a certain consensus on the criteria and the mechanism of the evolution-kernel construction. Here I will summarize their application to the matrix case and the ensuing resummed results for the partonic splitting function matrix.

The BFKL equation (1976) *predicts* rising cross-sections but leading log predictions overestimate the hard Pomeron exponent, while NLL corrections are large and negative [5] , and may make it ill-defined. On the other hand, low order DGLAP evolution is consistent with the rise of HERA SF, with marginal problems (hints of a negative gluon density). Therefore, we need to *reconcile* BFKL and DGLAP approaches: in the last decade, various (doubly) resummed approaches have been devised [6, **?**, 8, 9] whose main idea is to incorporate *RG constraints* in the BFKL kernel, by calculating some effective (resummed) BFKL eigenvalue $\chi_{eff}(\gamma)$ or the "dual" DGLAP anomalous dimension $\Gamma_{eff}(\omega)$. So far, only the gluon channel has been treated self-consistently, while the quark channel is added by k-factorization of the $q-\bar{q}$ dipole.

The purpose of our matrix approach is to generalize DGLAP self-consistent evolution for quarks and gluons in *k-factorized matrix* form, so as to be consistent, at small x, with BFKL gluon evolution. One of the outcomes is to define, by construction, some *unintegrated* partonic densities at any x, even if the general issue of their factorization [10] is not actually treated. The main construction criteria for our matrix kernel are to incorporate exactly NLO DGLAP matrix evolution and the NLx BFKL kernel and to satisfy RG constraints in both ordered and antiordered collinear regions, and thus the $\gamma \leftrightarrow 1-\gamma+\omega$ symmetry [6]. An important role is played also by what I will call the *minimal-pole assumption* in the γ- and ω- expansions, as explained below.

Let me recall that the DGLAP evolution equations for the PDFs $f_a(Q^2)$ in the hard scale Q^2 define the anomalous dimension matrix $\Gamma(\omega)$, with the moment index $\omega = \partial/\partial Y$ conjugated to $Y = \log 1/x$:

$$\frac{\partial}{\partial t} f_a = \frac{\partial}{\partial \log Q^2} f_a = [\Gamma(\omega)]_{ab} f_b \tag{1.1}$$

On the other hand, the BFKL evolution equation in Y for the unintegrated gluon PDF $\mathscr{F}(Y, \mathbf{k}^2)$ defines the kernel $\mathscr{K}(\gamma)$, with $\gamma = \partial/\partial t$ conjugated to $t = \log \mathbf{k}^2$:

$$\omega \mathscr{F} = \frac{\partial}{\partial Y} \mathscr{F} = \mathscr{K}(\gamma) \mathscr{F} \tag{1.2}$$

If $\mathbf{k}$-factorization is used, DGLAP evolution of the Green's function G corresponds to either the *ordered* $k \gg k' \gg ... k_0$ or the *antiordered* momenta, while BFKL incorporates all possible orderings.

At frozen α_s, our RG-improved matrix kernel, generalizing the above evolutions, is expanded in the form $\mathscr{K}(\bar{\alpha}_s, \gamma, \omega) = \bar{\alpha}_s \mathscr{K}_0(\gamma, \omega) + \bar{\alpha}_s^2 \mathscr{K}_1(\gamma, \omega)$ and satisfies the minimal-pole assumption in the γ- and ω- expansions ($\gamma = 0 \leftrightarrow$ ordered $\mathbf{k}$'s)

$$\begin{aligned} \mathscr{K}(\bar{\alpha}_s,\gamma,\omega) &= (1/\gamma)\, \mathscr{K}^{(0)}(\bar{\alpha}_s,\omega) + \mathscr{K}^{(1)}(\bar{\alpha}_s,\omega) + O(\gamma) \\ &= (1/\omega)\, {}_0\mathscr{K}(\bar{\alpha}_s,\gamma) + {}_1\mathscr{K}(\bar{\alpha}_s,\gamma) + O(\omega) \end{aligned} \tag{1.3}$$

from which DGLAP anomalous dimension matrix Γ and BFKL kernel χ are derived

$$\Gamma_0 = \mathscr{K}_0^{(0)}(\omega); \quad \Gamma_1 = \mathscr{K}_1^{(0)}(\omega) + \mathscr{K}_0^{(1)}(\omega)\Gamma_0(\omega); \ldots \tag{1.4}$$

$$\chi_0 = [{}_0\mathscr{K}_0(\gamma)]_{gg};\ \chi_1 = [{}_0\mathscr{K}_1(\gamma) + {}_0\mathscr{K}_0(\gamma)\ {}_1\mathscr{K}_0(\gamma)]_{gg}; \ldots \tag{1.5}$$

Such expressions are used to constrain $\mathscr{K}_0$ and $\mathscr{K}_1$ iteratively to yield the known NLO and NLx evolutions, and approximate momentum conservation. Furthermore, the RG constraints in both ordered and antiordered collinear regions are met by the $\gamma \leftrightarrow 1+\omega-\gamma$ symmetry of the kernel which corresponds, in $(\mathbf{k},x)$ space, to the $\mathbf{k} \leftrightarrow \mathbf{k}'$ and $x \leftrightarrow xk^2/k'^2$ symmetry of the matrix elements and thus relates the ordered and antiordered regions mentioned before. Finally, the running coupling is introduced by setting

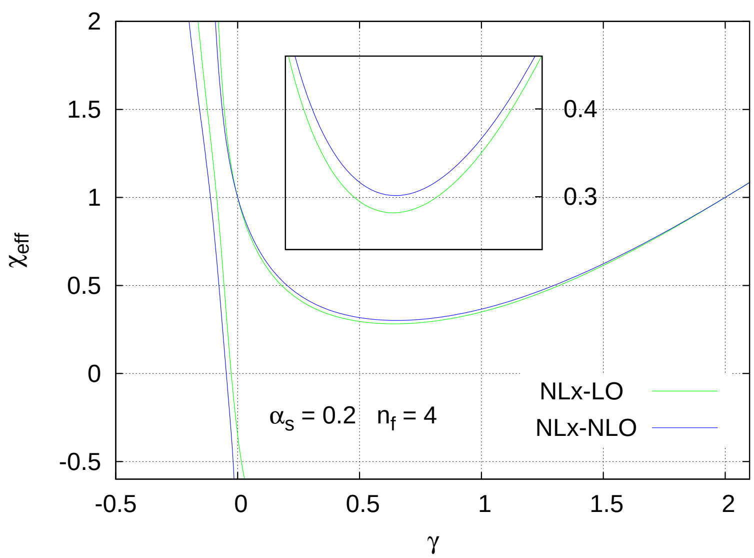
$$\mathscr{K}(\mathbf{k},\mathbf{k}';x) = \bar{\alpha}_s(\mathbf{k}_>^2)\mathscr{K}_0(\mathbf{k},\mathbf{k}';x) + \bar{\alpha}_s^2(\mathbf{k}_>^2)\mathscr{K}_1(\mathbf{k},\mathbf{k}';x) \tag{1.6}$$

where we understand that the scale $\mathbf{k}_>^2 \equiv \max(\mathbf{k}^2,\mathbf{k}'^2)$ is replaced by $(\mathbf{k}-\mathbf{k}')^2$ in front of the BFKL kernel χ_0^ω.

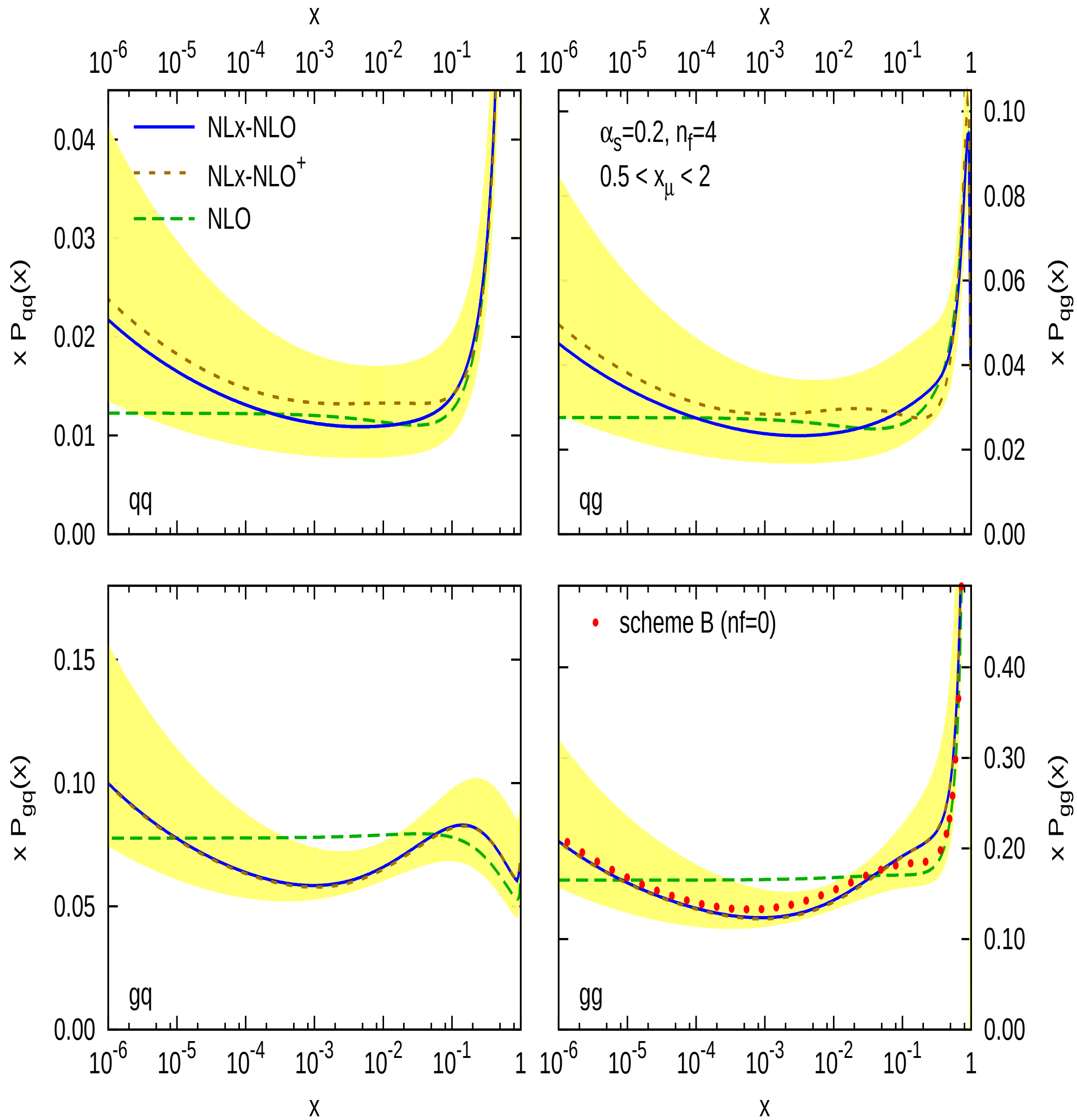
We remark that reproducing both low order DGLAP and BFKL evolutions provides novel *consistency relations* between the matrix $\mathbf{k}$-factorization scheme and the $\overline{\mathrm{MS}}$ scheme. They turn out to be satisfied at NLO/NLx accuracy, while a small violation would appear at NNLO. In fact, the simple- pole assumption in ω-space implies [1] that $[\Gamma_2]_{gq} = (C_F/C_A)[\Gamma_2]_{gg}$ at order α_s^3/ω^2, violated by (n_f/N_c^2)-suppressed terms (≤ 0.5 % for $n_f \leq 6$) in $\overline{\mathrm{MS}}$ [11]. For this reason we do not attempt full inclusion of the NNLO in our scheme.

2. Results for the resummed eigenvalues and the splitting matrix

There are two, frozen α_s, resummed eigenvalue functions: $\omega = \chi_\pm(\alpha_s,\gamma)$, corresponding to the leading and subleading anomalous dimensions $\gamma_\pm(\omega,\alpha_s)$, as depicted in Fig. 1.



The leading eigenvalue function shows fixed points at $\gamma = 0, 2$ and $\omega = 1$, corresponding to momentum conservation in both collinear and anti-collinear limits. Due to the matrix structure for $n_f \neq 0$, a new subleading eigenvalue χ_- appears. The n_f-dependence of $\chi_+(\alpha_s, \gamma)$ is modest, and the NL*x*-LO scheme recovers the known gluon-channel result (in agreement with [8]) at $n_f = 0$. Finally, a level crossing of χ_- and χ_+, present in the $n_f = 0$ limit, disappears at $n_f = 4$.



The results for the splitting function matrix $P_{ab}(x)$, including running coupling effects, are shown in Fig. 2 for $\alpha_s = 0.2$, and compared to *NLO* entries. The *NLO*$^+$ scheme includes, besides NLO, also NNLO terms $\sim \alpha_s^3/\omega^2$, while scheme B refers to previous results [7] for the gluon channel only. We have numerically checked that the infrared cutoff independence insures (matrix) collinear factorization We note that, at intermediate $x \simeq 10^{-3}$, the resummed P_{gg} and P_{gq} entries show a *shallow dip*, similarly to the one-channel case. Furthermore, the small-x rise of the novel P_{qg} and P_{qq} entries is delayed down to $x \simeq 10^{-4}$. Finally, the scale uncertainty band (for a rescaling parameter $0.25<x_\mu^2<4$) is larger for the (small) P_{qa} entries, as perhaps expected from the fact that, in this case, the constant small-x behaviour starts at NLO.

To sum up, we have proposed a small-x evolution scheme in matrix form in which quarks and gluons are treated on the same ground and the splitting functions are already (closely) in the $\overline{\mathrm{MS}}$ scheme. We fix the NLO/NLx matrix factorization scheme by further requiring "ordering-antiordering symmetry" and "minimal poles". We find that the Hard Pomeron and the leading eigenvalue function are stable, with modest n_f-dependence, while a new subleading eigenvalue is obtained. The resummed splitting functions P_{ga} show a shallow dip, and the small$x$ increase of P_{qa} is delayed to $x \simeq 10^{-4}$. Overall, we find a *gentle matching* of low order with resummation. In order to complete this program, we still need coefficients with comparable accuracy; but we could take first the LO impact factors with "exact kinematics" [12]. On the whole, it looks quite nice!

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PoS(RAD COR 2007)029