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Event shapes in e^+e^- annhilation at NNLO

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We report first results on the calculation of NNLO corrections to event shape distributions in electron-positron annhilation. The corrections are sizeable for all variables, however their magnitude is substantially different for different observables. We observe that inclusion of the NNLO corrections yields a considerably better agreement between theory and experimental data both in shape and normalisation of the event shape distributions.

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1. Introduction

For more than a decade experiments at LEP (CERN) and SLC (SLAC) gathered a wealth of high precision high energy hadronic data from electron-positron annihilation at a range of centre-of-mass energies [1, 2, 3]. This data provides one of the cleanest ways of probing our quantitative understanding of QCD. This is particularly so because the strong interactions occur only in the final state and are not entangled with the parton density functions associated with beams of hadrons. As the understanding of the strong interaction, and the capability of making more precise theoretical predictions, develops, more and more stringent comparisons of theory and experiment are possible, leading to improved measurements of fundamental quantities such as the strong coupling constant [4].

In addition to measuring multi-jet production rates, more specific information about the topology of the events can be extracted. To this end, many variables have been introduced which characterise the hadronic structure of an event. With the precision data from LEP and SLC, experimental distributions for such event shape variables have been extensively studied and have been compared with theoretical calculations based on next-to-leading order (NLO) parton-level event generator programs [5, 6, 7], improved by resumming kinematically-dominant leading and next-to-leading logarithms (NLO+NLL) [8] and by the inclusion of non-perturbative models of power-suppressed hadronisation effects [9].

The precision of the strong coupling constant determined from event shape data has been limited up to now largely by the scale uncertainty of the perturbative NLO calculation. We report here on the first calculation of NNLO corrections to event shape variables, and discuss their phenomenological impact.

2. Event shape variables

In order to characterise hadronic final states in electron-positron annihilation, a variety of event shape variables have been proposed in the literature, for a review see e.g. [10]. These variables can be categorised into different classes, according to the minimal number of final-state particles required for them to be non-vanishing: In the following we shall only consider three particle final states which are thus closely related to three-jet final states.

Among those shape variables, six variables [11] were studied in great detail: the thrust T, the normalised heavy jet mass ρ, the wide and total jet broadenings B_W and B_T, the C-parameter and the transition from three-jet to two-jet final states in the Durham jet algorithm Y_3.

The perturbative expansion for the distribution of a generic observable y up to NNLO at e^+e^- centre-of-mass energy $\sqrt{s}$, for a renormalisation scale μ^2 is given by

$$\begin{aligned}\frac{1}{\sigma_{\rm had}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(s,\mu^2,y) = &\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)\frac{\mathrm{d}\bar{A}}{\mathrm{d}y}+\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left(\frac{\mathrm{d}\bar{B}}{\mathrm{d}y}+\frac{\mathrm{d}\bar{A}}{\mathrm{d}y}\beta_0\log\frac{\mu^2}{s}\right)\\ &+\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3\left(\frac{\mathrm{d}\bar{C}}{\mathrm{d}y}+2\frac{\mathrm{d}\bar{B}}{\mathrm{d}y}\beta_0\log\frac{\mu^2}{s}\right.\\ &\left.+\frac{\mathrm{d}\bar{A}}{\mathrm{d}y}\left(\beta_0^2\log^2\frac{\mu^2}{s}+\beta_1\log\frac{\mu^2}{s}\right)\right)+\mathcal{O}(\alpha_s 4)\,. \qquad (2.1)\end{aligned}$$

The dimensionless perturbative coefficients $\bar{A}$, $\bar{B}$ and $\bar{C}$ depend only on the event shape variable y. They are computed by a fixed-order parton-level calculation, which includes final states with three partons at LO, up to four partons at NLO and up to five partons at NNLO. LO and NLO corrections to event shapes have been available already for a long time [5, 6, 7].

The calculation of the NNLO corrections is carried out using a newly developed parton-level event generator programme EERAD3 which contains the relevant matrix elements with up to five external partons [12, 13, 14, 15]. Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft. In order to extract these infrared divergences and combine them with the virtual corrections, the antenna subtraction method [16] was extended to NNLO level [17] and implemented for $e^+e^- \to 3$jets and related event-shape variables [18]. The analytical cancellation of all infrared divergences serves as a very strong check on the implementation. EERAD3 yields the perturbative A, B and C coefficients as histograms for all infrared-safe event-shape variables related to three-particle final states at leading order. From these, $\bar{A}$, $\bar{B}$ and $\bar{C}$ are computed by normalising to the total hadronic cross section. As a cross check, the A and B coefficients have also been obtained from an independent integration [7] of the NLO matrix elements [5], showing excellent agreement.

For small values of the event shape variable y, the fixed-order expansion, eq. (2.1), fails to converge, because the fixed-order coefficients are enhanced by powers of $\ln(1/y)$. In order to obtain reliable predictions in the region of $y \ll 1$ it is necessary to resum entire sets of logarithmic terms at all orders in α_s. A detailed description of the predictions at next-to-leading-logarithmic approximation (NLLA) can be found in Ref. [19].

3. NNLO results

The precise size and shape of the NNLO corrections depend on the observable in question. Common to all observables is the divergent behaviour of the fixed-order prediction in the two-jet limit, where soft-gluon effects at all orders become important, and where resummation is needed. For several event shape variables (especially T and C) the full kinematical range is not yet realised for three partons, but attained only in the multi-jet limit. In this case, the fixed-order description is also insufficient since it is limited to a fixed multiplicity (five partons at NNLO). Consequently, the fixed-order description is expected to be reliable in a restricted interval bounded by the two-jet limit on one side and the multi-jet limit on the other side.

In this intermediate region, we observe that inclusion of NNLO corrections (evaluated at the Z-boson mass, and for fixed value of the strong coupling constant) typically increases the previously available NLO prediction. The magnitude of this increase differs considerably between different observables[20], it is substantial for T (18%), B_T (17%) and C (15%), moderate for ρ and B_W (both 10%) and small for Y_3 (6%). For all shape variables, we observe that the renormalisation scale uncertainty of the NNLO prediction is reduced by a factor 2 or more compared to the NLO prediction. Inclusion of the NNLO corrections modifies the shape of the event shape distributions. We observe that the NNLO prediction describes the shape of the measured event shape distributions over a wider kinematical range than the NLO prediction, both towards the two-jet and the multi-jet limit. To illustrate the impact of the NNLO corrections, we compare the fixed-order predictions for

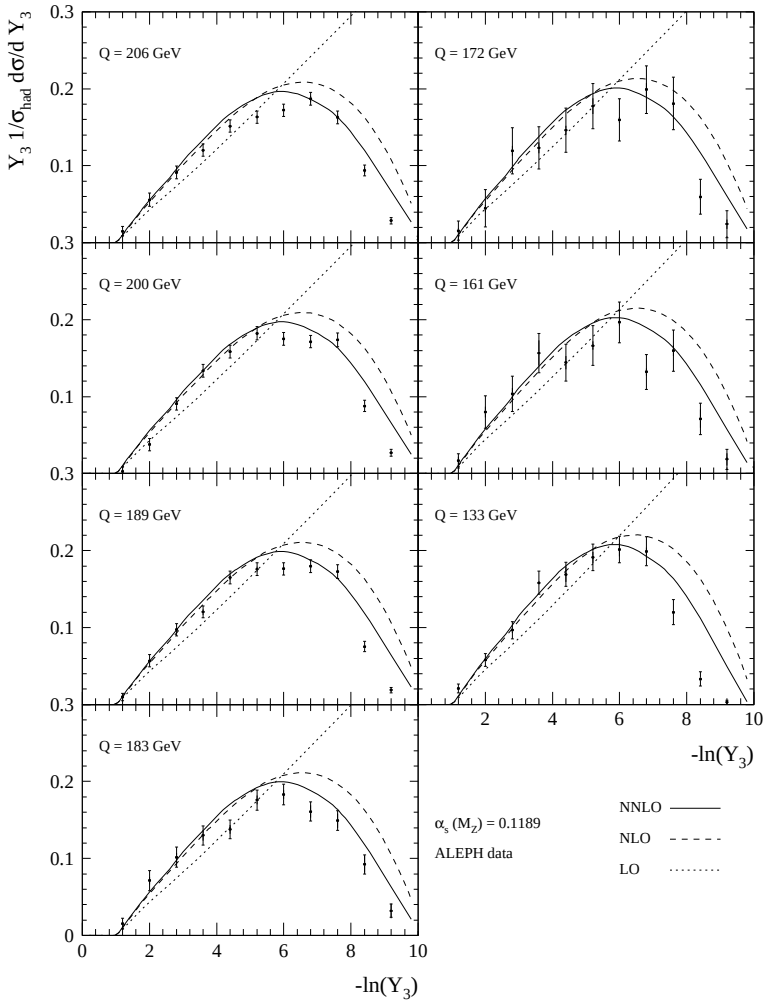

Figure 1: Perturbative fixed-order predictions for the Y_3-distribution, compared to LEP2 data from ALEPH.

Y_3 to LEP2-data obtained by the ALPEH experiment in Figure 1, which illustrates especially the improvement in the approach to the two-jet region (large $-\ln(Y_3)$).

The information contained in the event shape distributions can be restructured by computing individual moments. Moments of event shape distributions have been studied theoretically and experimentally in particular in view of understanding non-perturbative power corrections [9]. Consequently, perturbative NNLO corrections will improve the discrimination between higher perturbative orders and genuine non-perturbative effects. For the first moment $\langle 1-T \rangle$ of the thrust distribution, we find the integrated coefficients

$$\mathscr{A} = 2.101 \qquad \mathscr{B} = 44.98 \qquad \mathscr{C} = 1095 \pm 130\,,$$

which yields for $\sqrt{s} = \mu = M_Z$:

$$\langle 1-T \rangle (\alpha_s(M_Z) = 0.1189) = 0.0398\,(\mathrm{LO}) \;+\; 0.0146\,(\mathrm{NLO}) \;+\; 0.0068\,(\mathrm{NNLO})\,.$$

Work on moments of the event shapes is ongoing.

4. Determination of the strong coupling constant

Using the newly computed NNLO corrections to event shape variables, we performed[21] a new extraction of α_s from data on the standard set of six event shape variables, measured by the ALEPH collaboration [1] at centre-of-mass energies of 91.2, 133, 161, 172, 183, 189, 200 and 206 GeV. The combination of all NNLO determinations from all shape variables yields

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo}).$$

We observe a clear improvement in the fit quality when going to NNLO accuracy. Compared to NLO the value of α_s is lowered by about 10%, but still higher than for NLO+NLLA [1], which shows the obvious need for a matching of NNLO+NLLA for a fully reliable result. The scatter among the α_s-values extracted from different shape variables is lowered considerably, and the theoretical uncertainty is decreased by a factor 2 (1.3) compared to NLO (NLO+NNLA).

These observations visibly illustrate the improvements gained from the inclusion of the NNLO corrections, and highlight the need for further studies on the matching of NNLO+NLLA, and on the derivation of NNLLA resummation terms.

5. Outlook

Our results for the NNLO corrections open up a whole new range of possible comparisons with the LEP data. The potential of these studies is illustrated by the new determination of α_s reported here, which can be further improved by the matching NLLA+NNLO, currently in progress. Similarly, our results will also allow a renewed study of power corrections, now matched to NNLO.

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