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NNLO QCD corrections to the $\mathbf{m_c}$ dependent matrix elements in $\overline{B} \to X_s\gamma$

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Recent developements in the calculation of the NNLO QCD corrections to the charm quark mass dependent matrix elements in $\overline{B} \to X_s\gamma$ are reported. Special emphasis is put on the new results of the virtual $\mathcal{O}\left(\alpha_s^2\right)$ fermionic contribution to these matrix elements [1].

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1. Introduction

The inclusive rare $\overline{B}\to X_s\gamma$ decay is a natural framework for high precision studies of FCNC, thanks to its low sensitivity to non-perturbative effects. As a loop induced process in the Standard Model (SM), it is highly sensitive to new physics [2]. In order to obtain stringent constraints on extensions of the SM from this decay, accurate measurments and precise theoretical predictions with a good control of perturbative and non-perturbative corrections have to be provided.
On the experimental side, the latest measurements by CLEO, Belle and BaBar [3] have been combined by the Heavy Flavor Averaging Group into the current world average (WA) that reads for a photon energy cut of $E_\gamma > 1.6$ GeV [4] in the $\overline{B}$-meson rest-frame

$$\mathscr{B}(\overline{B}\to X_s\gamma)^{\rm exp}_{E_\gamma>1.6\,\rm GeV} = \left(3.55\pm 0.24\,^{+0.09}_{-0.10}\pm 0.03\right)\times 10^{-4}, \tag{1.1}$$

where the first error is given by the statistic and systematic uncertainty, the second one is due to the theory input on the shape function, and the third one is caused by the $b\to d\gamma$ contamination . This average is in good agreement with the recent theoretical estimate including known next-to-next-to-leading-order (NNLO) effects [5]

$$\mathscr{B}(\overline{B}\to X_s\gamma)^{\rm theo}_{E_\gamma>1.6\,\rm GeV} = (3.15\pm 0.23)\times 10^{-4}, \tag{1.2}$$

where the error consists of four types of uncertainties added in quadrature: non-perturbative (5%), parametric (3%), higher-order (3%) and m_c-interpolation ambiguity (3%). The total error of the present experimental WA of about 7% in Eq. (1.1) is expected to be reduced at future B factories to approximately 5%. In view of this accuracy, SM calculations need to be improved with the same precision level by completing the NNLO QCD program.
QCD corrections to the partonic decay rate $\Gamma(b\to s\gamma)$ contain large logarithms of the form $\alpha_s^n(m_b)\ln^m(m_b/M_W)$, with $m\le n$, which should be resummed with the help of renormalization-group techniques. A convenient framework is an effective low-energy theory obtained from the SM by decoupling the heavy electroweak bosons and the top quark . The resulting effective Lagrangian is a product of the Wilson coefficients $C_i(\mu)$ with local flavor-changing operators $Q_i(\mu)$ up to dimension six.
A consistent calculation of $b\to s\gamma$ at the NNLO level requires three steps: *i*) evaluation of $C_i(\mu_0)$ at the matching scale $\mu_0\sim M_W$ by requiring equality of Green's functions in the full and the effective theory up to leading order in (external momenta)/M_W to $\mathscr{O}\left(\alpha_s^2\right)$. All the relevant Wilson coefficients have already been calculated [6, 7] to this precision, by matching the four-quark operators $Q_1,\ldots,Q_6$ and the dipole operators Q_7 and Q_8 at the 2- and 3-loop level respectively. *ii*) calculation of the operator mixing under renormalization, by deriving the effective theory Renormalization Group Equations (RGE) and evolving $C_i(\mu)$ from μ_0 down to the low-energy scale $\mu_b\sim m_b$, using the anomalous-dimension matrix (ADM) to $\mathscr{O}\left(\alpha_s^3\right)$. Here, the 3-loop renormalization in the $\{Q_1,\ldots,Q_6\}$ and $\{Q_7,Q_8\}$ sectors was found in [8, 9], and results for the 4-loop mixing of $Q_1,\ldots,Q_6$ into Q_7 and Q_8 were recently provided in [10], thus completing the anomalous-dimension matrix. *iii*) determination of the on-shell matrix elements of the various operators at $\mu_b\sim m_b$ to $\mathscr{O}\left(\alpha_s^2\right)$. This task is not complete yet, although a number of contributions is known. The 2-loop matrix element of the photonic dipole operator Q_7, together with the corresponding

bremsstrahlung, was found in [11, 12], confirmed in [13] and subsequently extended to include the full charm quark mass dependence in [14]. In [15], the $\mathscr{O}\left(\alpha_s^2 n_f\right)$ contributions were found to the 2-loop matrix elements of Q_7 and Q_8, as well as to the 3-loop matrix elements of Q_1 and Q_2, using an expansion in the quark mass ratio m_c^2/m_b^2. Diagrammatically, these parts are generated by inserting a 1-loop quark bubble into the gluon propagator of the 2-loop Feynman diagrams. Naive non-abelianization (NNA) is then used to get an estimate of the complete corrections of $\mathscr{O}\left(\alpha_s^2\right)$ by replacing n_f with $-\frac{3}{2}\beta_0$. Moreover, the contributions of the dominant operators at $\mathscr{O}\left(\alpha_s^2\beta_0\right)$ to the photon energy spectrum have been computed in [16].

A rather important and difficult piece that is still missing to date is the complete $\mathscr{O}\left(\alpha_s^2\right)$ calculation of the matrix elements of the four-quark operators Q_1 and Q_2. These operators contain the charm quark, and the main source of uncertainty at the NLO level is related to the ambiguity associated to the choice of scale and scheme for m_c [18]. As these matrix elements start contributing for the first time at $\mathscr{O}(\alpha_s)$, the choice of scale and scheme for m_c is a NNLO effect in the branching ratio. Therefore a calculation of $\langle s\gamma|Q_{1,2}|b\rangle$ at $\mathscr{O}\left(\alpha_s^2\right)$ is crucial to reduce the overall theoretical uncertainty in $\mathscr{B}(\overline{B}\to X_s\gamma)$. In [17], the full matrix elements of Q_1 and Q_2 have been computed in the large m_c limit, $m_c \gg m_b/2$. Subsequently, an interpolation in the charm quark mass has been done down to the physical region, under the assumption that the β_0-part is a good approximation at $m_c = 0$. This is the source of the interpolation uncertainty mentioned below Eq. (1.2). Reducing this uncertainty requires the evaluation of the 3-loop $\langle s\gamma|Q_{1,2}|b\rangle$ at $m_c = 0$, whereas removing it involves their calculation at the physical value of m_c, namely dealing with hundreds of 3-loop on-shell vertex diagrams with two scales m_b and m_c which is a formidable task. Both of these calculations are being pursued in [19], and we will comment on the current status in the next section. An important subset of diagrams contributing to the virtual 3-loop on-shell calculation of $\langle s\gamma|Q_{1,2}|b\rangle$ for $m_c \neq 0$ is the fermionic part which constitutes a major input both for the NNA and for the interpolation of the non-NNA terms between $m_c \gg m_b/2$ and $m_c < m_b/2$, and are thus crucial for the accuracy of Eq. (1.2). A result for these diagrams was presented in [15] assuming that $n_f = 5$ massless fermions are present in the quark loop inserted into the gluon propagator. An independent check of this calculation as well as the validity of the massless approximation, and new results for the missing diagrams with heavy b and c quark loops have been recently given in [1].

2. Calculation of the matrix elements $\langle s\gamma|Q_{1,2}|b\rangle$

The $\mathscr{O}\left(\alpha_s^2\right)$ calculation of the matrix elements $\langle s\gamma|Q_{1,2}|b\rangle$ is done within the framework of an effective theory with the Lagrangian

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i(\mu). \tag{2.1}$$

Adopting the operator definitions of [20], the physical operators that are relevant for our calculation

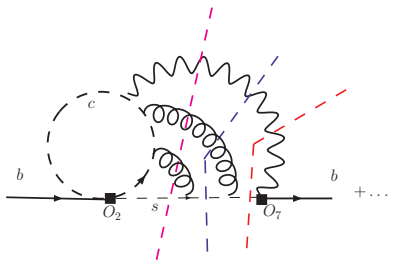

Figure 1: An example graph of a cut 4-loop b-quark self-energy.

together with the size of their Wilson coefficients read

$$\begin{aligned} Q_{1,2} &= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)\,, & C_{1,2}(m_b) &\sim 1\,,\\ Q_{3\text{–}6} &= (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma_i' q)\,, & |C_{3\text{–}6}(m_b)| &< 0.07\,,\\ Q_7 &= e/g_s^2\,\overline{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu}b_R)\,F_{\mu\nu}\,, & C_7(m_b) &\sim -0.3\,,\\ Q_8 &= 1/g_s\,\overline{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu}T^a b_R)\,G^a_{\mu\nu}\,, & C_8(m_b) &\sim -0.15\,, \end{aligned} \tag{2.2}$$

where Γ and Γ' stand for various products of Dirac and color matrices. A possible way of getting the complete matrix elements at $m_c = 0$ is by interfering the operators Q_1 and Q_2 with the magnetic dipole operator Q_7, then cutting the resulting 4-loop propagator diagrams in all possible ways that contain a photon and an s-quark in the final state. In total, 506 diagrams are generated this way each of which involves up to 5-particle cuts if final states with $c\bar{c}$ production are considered. A sample graph is shown in FIG. 1. Since charmed hadrons in the final state are excluded experimentally, the $\mathcal{B}(\overline{B}\to X_s\gamma)$ does not contain contributions from $c\bar{c}$ production. Thus, a perturbative calculation of $b\to X_s^{parton}\gamma$ should be done accordingly. In order to avoid logarithmic divergences resulting at $m_c = 0$ from $\ln m_c$ terms, cuts through the c-quark loop inserted into the gluon propagator have been kept in our calculation. Their contribution will be subtracted at the measured value of m_c after performing the interpolation. Moreover, since only the real part of the interference between the matrix elements of Q_2 and Q_7 contributes to the decay of $b\to X_s^{parton}\gamma$, we do not distinguish between masters that differ only in their imaginary part. This reduces the number of masters to less than 200. Details related to their calculation will be given elsewhere [19].
As far as the $\mathcal{O}\left(\alpha_s^2\right)$ 3-loop virtual correction to $\langle s\gamma|Q_{1,2}|b\rangle$ at $m_c\neq 0$ is concerned, the generated 420 vertex diagrams have been expressed through 21231 scalar integrals that depend on the scales m_b and m_c. With the help of Laporta algorithm [21], they have been subsequently reduced to 476 masters. The latter are being evaluated using a combined approach, namely Mellin-Barnes technique together with differential equations solved numerically. The same techniques have been applied in the calculation of the fermionic contribution, therefore we refer the reader to [1, 22] for all related details.

3. Results for the fermionic diagrams

As was mentioned in the introduction, we have calculated the fermionic diagrams with three different quark loop insertions into the gluon propagator, namely a massless as well as heavy b and

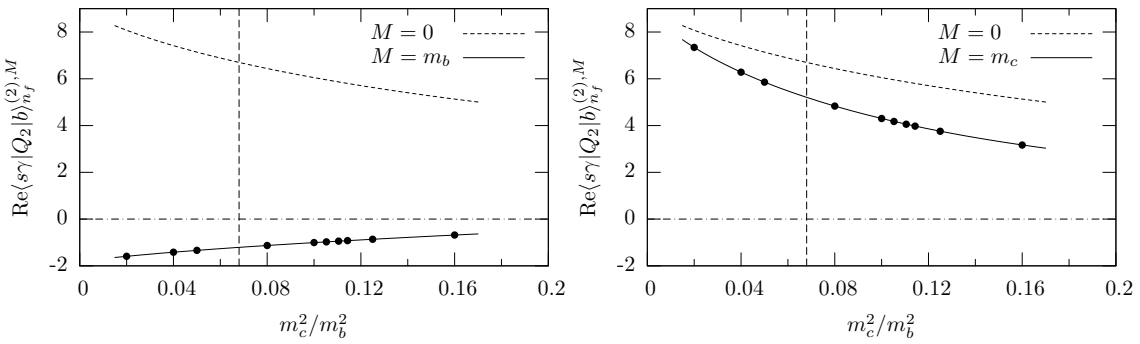

Figure 2: Plots of $\mathrm{Re}\langle s\gamma|Q_2|b\rangle_{n_f}^{(2),M}$ as function of m_c^2/m_b^2 with $M=m_b$ (a) and $M=m_c$ (b) and ($\mu_b=m_b$, $n_f=1$). For comparison, we also show the $M=0$ case.

c quark loops. Since the massless case was discussed in detail in [15], we constrain ourself here to the new results related to the missing contributions from heavy loops and compare them with the massless approximation results. As, at $\mathscr{O}(\alpha_s^2 n_f)$, the matrix elements of Q_1 and Q_2 are related to each other by $\langle s\gamma|Q_1|b\rangle=-1/(2N_c)\langle s\gamma|Q_2|b\rangle$, we just give results for the matrix elements of Q_2. The normalization of our amplitude is defined as follows

$$\langle s\gamma|Q_2|b\rangle_{\mathscr{O}(\alpha_s^2 n_f)}=\left(\frac{\alpha_s}{4\pi}\right)^2\frac{e}{8\pi^2}\,m_b\,n_f\,\langle s\gamma|Q_2|b\rangle_{n_f}^{(2),M}\,\overline{u}_s R\not{\varepsilon}\not{q}u_b \tag{3.1}$$

where m_b denotes the b-quark pole mass, ε and q are the photon polarization and momentum, $R=(1+\gamma_5)/2$ is the right handed projection operator, and n_f is the number of active flavors of a given mass. The superscript (2) counts the powers of α_s and $M=(0,m_b\,\mathrm{or}\,m_c)$ denotes the mass of the quark running in the loop inserted into the gluon propagator. The plots in FIG.2 summarize the outcome of our calculation. It turned out that the massless approximation overestimates the massive *b* result by a large factor, and moreover, has the opposite sign. On the other hand, less pronounced but non-negligible effects were observed for the massive *c*-quark case.

4. Conclusions

A complete $\mathscr{O}\left(\alpha_s^2\right)$ calculation of the matrix elements $\langle s\gamma|Q_{1,2}|b\rangle$ is crucial to reduce the overall uncertainty in the current NNLO estimate of the $\mathscr{B}(\overline{B}\to X_s\gamma)$. This calculation is being pursued in [19]. Taking new results for the complete NNLO fermionic contribution into account, an enhancement of 1.1% for $\mu_b=2.5$ GeV is observed in the current estimate of the branching ratio.

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