A QCD axion from a warped extra dimension

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In this talk I am reporting on a number of realizations of axions existing in a multi-'throat' generalization of the warped throat geometry of a Randall-Sundrum slice of AdS_5. A Peccei-Quinn scale in the phenomenologically allowed axion window is realized by warping. In compactifications with two or more throats it is shown to be possible to simultaneously solve the Standard Model hierarchy problem by the Randall-Sundrum mechanism while implementing interesting warped axion models. The constructions discussed are related to holographic duals of previously studied models of composite axions arising from strongly coupled four-dimensional dynamics.

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1. Introduction

The strong CP problem comprises one of the outstanding puzzles of the Standard Model (SM) of elementary particle physics. Due to the non-trivial QCD vacuum caused by instanton solutions,

$$S_\theta = \frac{\overline{\theta}}{64\pi^2}\varepsilon_{\mu\nu\lambda\sigma}G^{a\mu\nu}G^{a\lambda\sigma} \tag{1.1}$$

is a *physical* CP violating contribution to the QCD action. In Eq. (1.1), $\overline{\theta} = \theta - \arg\det M_q$ describes CP violation by the bare QCD vacuum θ as well as by the phase of the CKM-matrix which must be $\mathcal{O}(1)$ in order to explain the observed CP violation in the *K*- and *B*-meson systems. However, the non-observation of a neutron electric dipole moment implies $|\overline{\theta}| \lesssim 10^{-9}$. Unless explained otherwise, this implies that the QCD vacuum contribution has to cancel the CKM-phase almost perfectly, which means severe fine-tuning.

One solution to the strong CP problem, proposed by Peccei and Quinn [1] (see also Weinberg and Wilczek [2]) is to endow the SM with additional matter content and a global $U(1)_{PQ}$ symmetry, spontaneously broken at a scale f_{PQ}. The associated Goldstone boson - the axion *a* - couples to QCD via

$$\Delta S = \frac{a}{32\pi^2 f_{PQ}}G\tilde{G}. \tag{1.2}$$

If the Peccei-Quinn (PQ) charges of the matter fields of the theory are assigned such that the theory has a global $U(1)_{PQ}SU(3)_c^2$ anomaly, then strong QCD dynamics generates an effective potential for *a* such that $\overline{\theta}$ is dynamically set to zero by the vacuum expectation value of *a*. Laboratory experiments as well as astrophysical and cosmological considerations constrain the PQ scale to the "axion window" $10^9\,\text{GeV} \lesssim f_{PQ} \lesssim 10^{12}\,\text{GeV}$. Thus, solving the strong CP problem via the axion mechanism comes at a price: the hierarchy $m_{\text{weak}} \ll f_{PQ} \ll M_{\text{pl}}$ must be explained by the underlying theory.

As shown by Randall and Sundrum [3], large hierarchies in energy scales can be explained naturally in models with a warped extra dimension, which commonly occurs in "throats" in string constructions with flux. Recently, several authors argued that geometries with multiple throats and a range of warp factors are not unnatural in string compactifications [4, 5, 6]. This motivates us to address the problem of the $m_{\text{weak}} \ll f_{PQ} \ll M_{\text{pl}}$ hierarchy in a multi-throat setup.

2. An axion model on a multi-throat background

Our model is based on a warped 5d geometry with two throats – one containing the SM, in particular the Higgs and the SM fermions, while the other throat contains the axion. The geometry of both throats will be approximated by a standard Randall-Sundrum slice of AdS_5 with the metric

$$ds^2 = (kz)^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu - d^2z), \tag{2.1}$$

where for simplicity we choose a common curvature radius $1/k$ in both throats. The z coordinates of the two throats lie in the ranges $z_i \in [1/k, L_i]$, where $kL_{SM} = M_{\text{pl}}/m_{\text{weak}}$ in order to establish the $M_{\text{pl}} - m_{\text{weak}}$ hierarchy, and we denote $L_{PQ} \equiv L$. The positions of the UV branes are identified, *i.e.* the throats are glued together "back to back" (*c.f.* Fig. 1). In this setup, the 4D effective
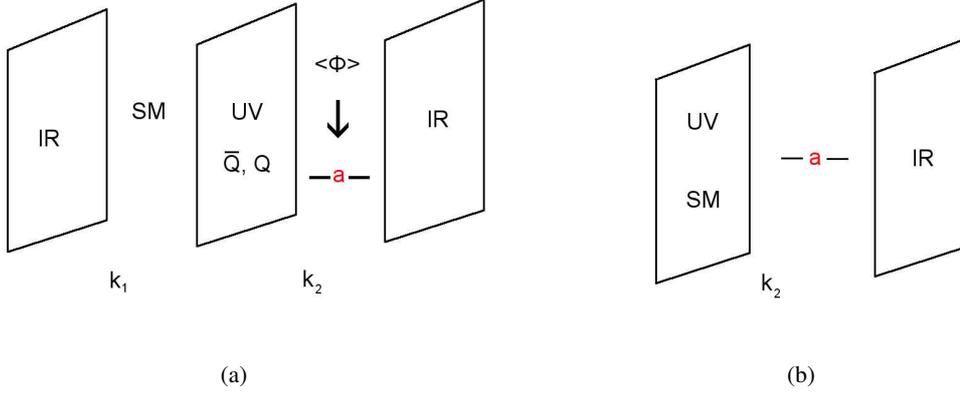
Figure 1: Illustration of a two throat axion toy model. a) The standard model lives in left AdS_5 throat while the field Φ (either scalar or gauge field) realizing the axion lives in the right throat. b) Once the PQ symmetry is broken, the axion effectively sees the Standard Model as UV confined matter.

Planck mass is given in terms of the underlying 5d Planck mass M_5 by $M_{\rm pl}^2 = 2M_5^3/k$, which only differs from the one-throat result by the factor of two.

In Ref. [7] we developed implementations of an axion from a 5D complex scalar field as well as from a 5D gauge field, which both successfully establish the desired energy scale. In both scenarios, the SM matter fields and the axion are localized in different throats, communicating only via UV boundary localized operators. Hence, we can be agnostic about the implementation in the SM throat and model all matter fields as UV brane localized operators (*c.f.* Fig. 1 for an illustration). We allow the gauge fields to propagate in both throats in order to be able to vary the localization of the axion-gluon coupling in the PQ throat.

In this talk I focus on the gauge field realization. When compactifying a 5D gauge field A_M, A_5 gives rise to a scalar degree of freedom in the 4D effective theory. If A_5 inherits a shift symmetry and couples appropriately to QCD, it satisfies all requirements to serve as an axion. As a minimal model, consider an abelian gauge field in the bulk of the PQ throat with the action

$$S = \int d^5x \sqrt{-g} \left(-\frac{1}{4g_5^2} g^{MN} g^{PQ} F_{MP} F_{NQ} - \frac{1}{2\xi g_5^2} \left[g^{\mu\nu} \partial_\mu A_\nu + z\xi g^{55} \partial_5 \left(\frac{A_5}{z} \right) \right]^2 \right), \qquad (2.2)$$

where the gauge-fixing is chosen such that no mixing between A_μ and A_5 occurs. With boundary conditions

$$A_\mu \bigg|_{\rm bdy} = 0, \qquad \partial_5 \left(\frac{A_5}{z} \right) \bigg|_{\rm bdy} = 0, \qquad (2.3)$$

the A_μ zero-mode is projected out, while the A_5 zero-mode is IR localized

$$A_5(x,z) = Nza(x), \qquad (2.4)$$

where $N \approx \sqrt{2kg_5^2/L}$ is determined from the canonical normalization of the kinetic term for $a(x)$. The gauge transformations which leave the boundary conditions Eq. (2.3) invariant, correspond to a global shift symmetry on $a(x)$ [7]. The axion-gluon coupling can be achieved via a mixed Chern-Simons term. In the simplest setup with all SM fields (including the gluon) in the SM throat, the

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coupling must be localized on the UV brane where - due to the imposed boundary conditions - only A_5 and the 4D gluon zero-mode are non-vanishing. The mixed Chern-Simons term is

$$S_{int} = \int d^4x \frac{\alpha}{64\pi^2 M_5} A_5 \Big|_{\text{UV}} \varepsilon^{\mu\nu\lambda\sigma} G^a_{\mu\nu} G^a_{\lambda\sigma}, \tag{2.5}$$

where G denotes the QCD field strength and α is an $\mathcal{O}(1)$ constant. This interaction precisely corresponds to the axion-gluon coupling in Eq. (1.2). However in this 5D model, the Peccei-Quinn scale is *calculable*. Canonical normalization of the kinetic term for $a(x)$ yields $f_{PQ} = M_5 kL/\sqrt{2kg_5^2}$. For natural parameters (with the standard assumptions $M_{\text{pl}} \gtrsim M_5 \gtrsim k$ and $g_5^2 \sim 1/M_5$) this implies $f_{PQ} \sim M_5^2 L \gg M_5 \sim M_{\text{pl}}$, *i.e.* f_{PQ} is warped *up* due to the volume suppression of the coupling, rather than warped down as desired.

The situation improves if QCD is assumed to propagate in both throats. Now the 5D Chern-Simons term appears in the whole PQ throat:

$$S_{int} = \int d^5x \frac{\alpha}{64\pi^2 M_5} \varepsilon^{SMNPQ} A_S G^a_{MN} G^a_{PQ}. \tag{2.6}$$

As the A_μ zero-mode is projected out, Eq. (2.6) yields the interaction

$$S_{int} = \int d^5x \frac{\alpha}{64\pi^2 M_5} A_5 \varepsilon^{\mu\nu\lambda\sigma} G^a_{\mu\nu} G^a_{\lambda\sigma} \tag{2.7}$$

in terms of the light fields of the effective theory. Here, the gluon kinetic term must be normalized as well, and the axion coupling is not volume suppressed. This scenario leads to

$$f_{PQ} = \sqrt{\frac{2}{kg_5^2}} \frac{1}{L_1} \simeq \frac{M_{\text{pl}}}{M_5 L_1}. \tag{2.8}$$

f_{PQ} is warped down to the IR brane scale $1/L_1$. To obtain a phenomenologically viable PQ scale we require a PQ throat with warping $kL = 10^6...10^9$, which is milder than the standard warping required for the SM throat ($kL_{SM} = M_{\text{pl}}/m_{\text{weak}} \sim 10^{15}$). The mass of the first Kaluza-Klein excitation of A_μ is of order $1/L_1 \sim f_{PQ}$, not imposing any further phenomenological constraint. We conclude that the setup provides a phenomenologically viable, natural realization of the $m_{\text{weak}} \ll f_{PQ} \ll M_{\text{pl}}$ hierarchy.

Apart from successfully creating the hierarchies, the realization of the axion via a 5D gauge field on AdS_5 provides two further advantages:

- *Protection of the axion potential by a gauge symmetry:*
  A common problem for axion models arises because the axion potential is very sensitive to corrections which can shift the axion vev. The potential is protected by the global U(1) symmetry, but global symmetries are generically expected to be broken by quantum gravity effects such that higher dimensional Planck suppressed operators must be forbidden by other means [8].

  In the model presented here, higher dimensional operators are forbidden by a U(1) *gauge* symmetry which - due to the no-hair theorem - even holds when gravity becomes strong.

- *Dual interpretation as a composite axion model:*
  Using the AdS/CFT correspondence,[1] the axion realization presented here is dual to composite axion model as summarized in Table 1 and discussed in detail in Ref. [7]. Composite axion models, in which f_{PQ} is explained by dimensional transmutation, have been proposed a long time ago, but calculational power has been limited because of strong coupling. We presented a dual AdS_5 model which is weakly coupled and perfectly calculable.

AdS_5	$\leftrightarrow$	4D CFT
5D $U(1)$ gauge symmetry	$\leftrightarrow$	global $U(1)$ symmetry
IR Dirichlet b.c. on A_μ	$\leftrightarrow$	global $U(1)$ is spont. broken
$\rightarrow$ IR Neumann b.c. on A_5	$\leftrightarrow$	Goldstone boson (a)
A_5 is IR localized	$\leftrightarrow$	a is composite
UV Dirichlet b.c. on A_μ	$\leftrightarrow$	$U(1)$ is explicitly broken
$\rightarrow$ UV Neumann b.c. on A_5	$\leftrightarrow$	a acquires a (small) mass

Table 1: *AdS/CFT* correspondence of the 5D axion from $A_5^{(0)}$ with a composite 4D axion.

Acknowledgments

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[1]See Ref. [9] for the dual interpretation of gauge fields on AdS_5.