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Gauss-Bonnet interactions influence on the radion stabilisation in inflating backgrounds

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5d models with branes in presence of Gauss-Bonnet interactions are analyzed. The Goldberger-Wise mechanism is introduced and considered for static and inflating backgrounds. The necessary and sufficient conditions for radion stabilization are given for the static case. The influence of the Gauss-Bonnet terms on the radion mass and the inter-brane distance is analyzed.

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1. Introduction

The idea that has been intensively studied recently is that the Standard Model fields are confined on a brane in a higher-dimensional space-time. This concept has become quite popular since Hořava & Witten presented [1] their 11d model with 10d branes, motivated by M-Theory. Many 5d models with 4d branes, eg. effective models derived from the Hořava-Witten model, have been discussed since then. However, for any such model to be physically sensible, the distance between the branes should be stabilized. In other words the squared mass of the radion - a scalar field related to that distance - has to be positive.

Thus the first problem arises, namely what definition of the radion should be adopted. Various definitions for the radion have been proposed in the literature. As pointed out in [2], a sensible minimal requirement for the radion is to satisfy linearized Einstein equations.

One of the possible extensions of the standard gravity is to add to the action the interactions of higher order in the curvature tensor. According to the α' expansion in the string theory [3] the lowest order correction is given by the Gauss-Bonnet (GB) term

$$R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,. \tag{1.1}$$

Following the idea presented by Goldberger & Wise [4], stabilization of the branes can be achieved by introducing an additional bulk scalar field Φ. Thus, the considered 5d model is described on the $M^4 \times S^1/\mathbb{Z}_2$ orbifold by the action

$$S = \int \mathrm{d}^4x\,\mathrm{d}y\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R + \alpha R^2_{GB}\right] - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi) - \sum_{i=1}^{2}\delta(y-y_i)U_i(\Phi)\right\}, \tag{1.2}$$

where the branes are localized at y_1 and y_2, $V(\Phi)$ and $U_i(\Phi)$ are the scalar field potentials in the bulk and on the branes, respectively.

2. Background equations of motion and boundary conditions

Describing 5d space-time with 4d inflating de Sitter sections and two branes, the following ansatz is assumed for the metric and the scalar field

$$\mathrm{d}s^2 = a(y)^2\left\{-\mathrm{d}t^2 + e^{2Ht}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \mathrm{d}y^2\right\}, \qquad \Phi = \phi(y)\,, \tag{2.1}$$

where $a(y)$ is the warp factor and H is the 4d Hubble constant. For the given ansatz the scalar and tensor background equations of motion yield (in units $\kappa = 1$)

$$\phi'' + 3\frac{a'}{a}\phi' - a^2V' = 0\,, \qquad \left\{\frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2 + H^2\right\}\frac{\xi}{a^2} + \frac{1}{3}\phi'^2 = 0\,, \tag{2.2}$$

$$3\left\{\left(\frac{a'}{a}\right)^2 - H^2\right\}\left[1 + \frac{\xi}{a^2}\right] - \frac{1}{2}\phi'^2 + a^2V = 0\,, \tag{2.3}$$

where a useful notation has been introduced, namely $\xi = a^2 - 4\alpha\left\{(a'/a)^2 - H^2\right\}$, and primes denote differentiation with respect to the appropriate arguments (either y for a and ϕ, or Φ for V).

Integrating the equations of motion over the infinitesimal intervals around the branes yields the boundary conditions

$$\lim_{y\to y_1^+(y_2^-)} \frac{\phi'}{a} = \pm\frac{1}{2}U_i', \qquad \lim_{y\to y_1^+(y_2^-)} \left\{\frac{a'}{a^4}\left[a^2 - 4\alpha\left(\frac{1}{3}\left(\frac{a'}{a}\right)^2 - H^2\right)\right]\right\} = \mp\frac{1}{6}U_i, \quad (2.4)$$

which describe the jumps of the $\mathbb{Z}_2$ odd functions $a'(y)$ and $\phi'(y)$ at the $\mathbb{Z}_2$ fixed points y_1 and y_2.

3. Scalar perturbations

The next step is to derive the equations of motion for scalar perturbations in an inflating background in the presence of the Gauss-Bonnet interactions. Scalar perturbations around the background metric (2.1) can be introduced in the generalized longitudinal gauge as

$$\mathrm{d}s^2 = a^2\left\{(1+2F_1)\left[-\mathrm{d}t^2 + e^{2Ht}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right] + (1+2F_2)\mathrm{d}y^2\right\}, \qquad \Phi = \phi + F_3, \quad (3.1)$$

where the small perturbations F_i depend on all of the coordinates.

Consequently, the linearized Einstein equations yield

$$\frac{\xi'}{\xi}F_1 + \frac{a'}{a}F_2 = 0, \qquad (\xi F_1)' + \frac{1}{3}a^2\phi' F_3 = 0, \quad (3.2)$$

$$\frac{\xi}{a^2}\left\{(\Box + 4H^2)F_1 + 4\frac{a'}{a}F_1' - 4\left(\frac{a'}{a}\right)^2 F_2\right\} + \frac{1}{3}\phi'^2 F_2 + \left\{\frac{1}{3}\phi'' + \frac{a'}{a}\phi'\right\}F_3 - \frac{1}{3}\phi' F_3' = 0, \quad (3.3)$$

and the boundary conditions read

$$\lim_{y\to y_1^+(y_2^-)} \left\{F_3' - F_2\phi'\right\} = \pm\frac{1}{2}aF_3U_i''. \quad (3.4)$$

According to eq. (3.2), the three scalar perturbations are not independent. Thus, employing these equations, F_2 and F_3 can be eliminated. Separating the variables as

$$F_1(t,\vec{x},y) = \sum_{m^2} F_{m^2}(y)\left\{\int \mathrm{d}^3k\, f_{(m^2,k)}(t)e^{i\vec{k}\vec{x}}\right\}, \quad (3.5)$$

a dynamical equation of motion can be derived, namely

$$F_{m^2}'' + \left\{2\frac{\xi'}{\xi} - \frac{a'}{a} - 2\frac{\phi''}{\phi'}\right\}F_{m^2}' + \left\{\frac{\xi''}{\xi} - \frac{\xi' a'}{\xi a} - 2\frac{\xi'\phi''}{\xi\phi'} - \frac{a^3\xi'}{3a'\xi^2}(\phi')^2 + m^2 + 4H^2\right\}F_{m^2} = 0, \quad (3.6)$$

where the separation constant m^2 turns out to be the scalars mass squared in the effective 4d description. After the elimination of F_2 and F_3 (and F_1''), the boundary conditions become

$$\pm b_{1(2)} \lim_{y\to y_1^+(y_2^-)} \left\{F_{m^2}' + \frac{\xi'}{\xi}F_{m^2}\right\} + \left[m^2 + 4H^2\right] \lim_{y\to y_1^+(y_2^-)} F_{m^2} = 0, \quad (3.7)$$

$$\text{where} \qquad b_{1(2)} = \lim_{y\to y_1^+(y_2^-)} \left\{\frac{1}{2}aU_{1(2)}'' \pm \frac{a'}{a} \mp \frac{\phi''}{\phi'}\right\}. \quad (3.8)$$

4. Radion mass and stability conditions

Defining a new variable $Q_{m^2} = \xi F_{m^2}$ (we henceforth omit the subscript m^2), the dynamical equation of motion simplifies considerably, as

$$-(pQ')' + qQ = \lambda\, pQ\,, \tag{4.1}$$

where $p = 3/(2a\phi'^2)$, $q = (a^2\xi')/(2a'\xi^2)$, $\lambda = m^2 + 4H^2$. Although this equation is exactly of the form of a Sturm-Liouville differential equation, its boundary conditions are non-standard - eigenvalue λ dependent:

$$\frac{\partial Q}{\partial n}(y_i) - \frac{\lambda}{b_i} Q(y_i) = 0\,, \tag{4.2}$$

so the differential equation's usual analysis [5] requires certain modifications (details in [6]).

The lowest eigenvalue of the equivalent variational problem is

$$\lambda_0 = \min_{Q} \left\{ \frac{\int_{y_1}^{y_2} \left[pQ'^2 + qQ^2\right]}{\int_{y_1}^{y_2} \left[pQ^2\right] + b_1^{-1}(pQ^2)|_{y_1} + b_2^{-1}(pQ^2)|_{y_2}} \right\}, \tag{4.3}$$

where Q denotes all smooth functions defined on the extra dimension interval $[y_1, y_2]$.

Hence, using $Q = \text{const}$, the expression for the mass of the lightest perturbation, which shall be identified as the radion, yields

$$m_0^2 \leq -4H^2 + \frac{\int \mathrm{d}y (a^2\xi')/(a'\xi^2)}{3\left\{\int \mathrm{d}y (a\phi'^2)^{-1} + \sum [b_i a(y_i)\phi'^2(y_i)]^{-1}\right\}}\,, \tag{4.4}$$

and can be considered as the radion mass bound in an inflating ($H^2 > 0$) background. Thus the stability of the inter-brane distance is attained if $\lambda_0 > 4H^2$.

For the non-inflating branes (i.e. when $H = 0$) $\lambda = m^2$ and the brane system is stable if $\lambda_0 > 0$. The sufficient and necessary stability conditions (to be fulfilled for all y) are found to be (c.f. [6] for details)

$$\phi'(y) \neq 0\,, \qquad \frac{\xi'(y)}{a'(y)} > 0\,, \qquad b_i > 0\,. \tag{4.5}$$

5. Role of Gauss-Bonnet interactions

Two main issues concerning the introduction of the Gauss-Bonnet interactions should be discussed. First of all two classes of solutions can be distinguished[1]. The difference between them is fundamental and obvious when considering their GB coefficient going to zero limit. More explicitly, solutions from one class converge to the solutions in the Einstein-Hilbert theory, whereas solutions from the other class diverge. However, it can be shown via the analysis of the inter-brane distance stability that these "new" solutions are disfavored.

The final question to answer is whether the assumed existence of the Gauss-Bonnet interactions has any influence on the inter-brane distance stability. According to the quantitative analysis

[1] c.f. eq. (2.3), which is linear in the combination $[(a'/a)^2 - H^2]$ for $\alpha = 0$, but quadratic in this combination for any $\alpha \neq 0$.

that was performed in [6], the presence of the Gauss-Bonnet interactions with a negative coefficient ($\alpha < 0$) have a model dependent influence and, in general, yields worse stability. However, the Gauss-Bonnet interactions introduced with a positive coefficient ($\alpha > 0$, as predicted by the string theory) improve the stability of the brane positions, as the radion mass squared increases. Furthermore, the inter-brane distance decreases (fig. 1). Detailed results of further quantitative (numerical) analysis for the specific models (parameter ranges) can be found in [6].

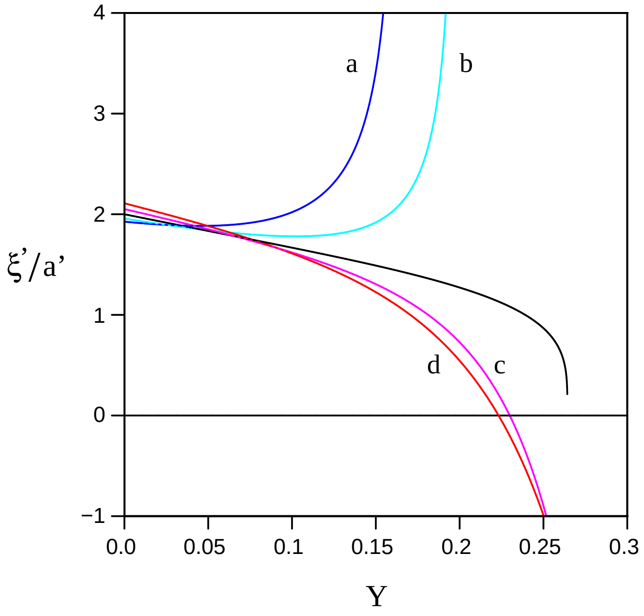

Figure 1: Ratio ξ'/a' (where $\xi'/a' > 0$ is one of the stability conditions given in (4.5)), as a function of the physical distance Y for different values of α: 0.01 (a); 0.005 (b); -0.005 (c); -0.01 (d). The curve without a label corresponds to a model without the GB term ($\alpha = 0$).

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