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An inference method of luminosity spectrum in a future e^+e^- linear collider

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We propose a inference method to determine the luminosity spectrum precisely in a future high luminosity electron positron linear collider. We introduce a statistical method recently developed in the information technology. In the ordinal electron positron collides, colliding beam energy is tuned with monocromatic energy, and luminosity can be measured by just counting the number of Bhabha events. However, the high luminosity e^+e^- linear collider no more produces a monochromatic energy spectrum, but a continuous and rather broad energy spectrum due to the beamstrahlung. A precise knowledge of this energy spectrum alias the luminosity spectrum, the precision experiment in the linear collider should be confronted with a crucial problem. The proposed model is formulated as a mostly exact luminosity distribution function, and parameter fitting is carried out for the data generated by simulation. This shows that the beam parameters of colliding electrons and positrons can be determined with an uncertainty of several percents by using 10k Bhabha events in an ideal detector condition, by which the luminosity spectrum is described.

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1. Introduction

The high luminosity e^+e^- collider has been studied as an energy frontier future project in high energy physics, and is expected to be a good place for the precision experiments. It is well-known, however, that the colliding beam energies of electrons and positrons have no more monochromatic energy spectra at interaction point, but have broad energy spectra even if each beam was originally tuned to have monochromatic energy spectrum. This is due to the beamstrahlung, where electrons and positrons feel the field produced by the opposite beam with the large density, and loose their energies by radiating photons before colliding. The collision of electron and positron beams with such broad energy spectra makes a broad luminosity spectrum in the center of mass system, which is called the luminosity spectrum. This broad luminosity spectrum gives a large defect to the precision experiments[2]. A precise determination of the luminosity spectrum is indispensable for experimental studies. In the ordinal e^+e^- collider experiments, the luminosity could be measured just by counting Bhabha events, since the beamstrahlung was negligible and it was reasonable to assume that the event shape of Bhabha process is back-to-back. In a high luminosity linear collider, however, Bhabha events are not always in the back-to-back event shape and must be identified by measuring the final e^+ and e^- 4-momenta, by which the measured CM energy $\sqrt{s}$ of the event is calculated.

In this talk, which is in part based on Ref.[1], we propose a new method to determine the luminosity spectrum. We introduce the statistical methods, especially based on the Bayesian statistics. We derive an almost exact formula of the Bhabha event distribution fiction, which is decomposed into the differential cross-section of Bhabha process, the error distribution function of the 4-momentum measurement, and the luminosity spectrum function represented by the Yokoya-Chen function. The luminosity spectrum is determined by inference of the beam parameters in the Yokoya-Chen function. The numerical simulations are performed to generate Bhabha events at the ILC, and the potentiality of the method is investigated.

2. Determination of luminosity spectrum by Bhabha event

2.1 Previous works

In the e^+e^- collider experiments the luminosity has been determined by using Bhabha events, since it is a well known elementary process whose cross section can be precisely calculated by using the perturbation theory. In the ordinal experiments, electrons and positrons are accelerated to have monochromatic beam energy, and the colliding beams generates the luminosity with the monochromatic energy spectrum. Therefore the luminosity can be measured just counting the number of Bhabha events. The luminosity is calculated by

$$\#(\text{Bhabha events}) = (\text{cross section}) \times (\text{luminosity}), \tag{2.1}$$

where it is assumed the cross section of Bhabha process can be calculated analytically with required accuracy. However, in high luminosity linear collider, due to beamstrahlung the beam energy has no more monochromatic spectrum but continuous board one. Fig. 1, for example, shows beam energy spectrum of electrons and positrons at the interaction point. Therefore, the Bhabha events

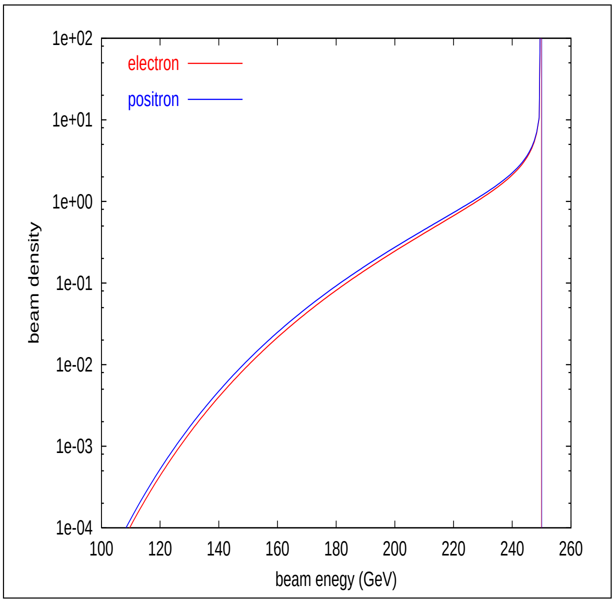

Figure 1: A logalithmic plot of beam energy distributions of electrons and positron at the interaction point, which is used in the simulation.

have energy dependency, and the luminosity must be determined as the function of $\sqrt{s}$ parameter by measuring the pair of 4-momenta of the electron and positron in Bhabha events.

Several pioneer works have been reported on the luminosity spectrum measurement for a high luminosity linear collider[4][5][6][16]. Most of them proposed to use the Bhabha events and measure the accolinearity angle of the final e^+ and e^- momenta to calculate $\sqrt{s}$ of the event. Here it is assumed that the nominal beam energy is known from the beam parameter of the collider or from the independent beam energy measurement. The accolinearity angle is used instead of measuring their 4-momenta directly, since the energy measurement of a high energy electron or positron could be suffered from a low precision while the accolinearity angle could be measured much more precise than the 4-momenta. (See Fig.2.) In the work of Ref.[5], the determination of the luminosity spectrum by measuring the accolinearity angle of Bhabha events was studied. In a special case, where the beam parameters of electrons and positrons are identical, they could extract the beam parameters, namely, beam-energy spread, the topological beam size and the number of particles in a bunch, from the measured CM energy spectrum by a likelihood fitting. Wherein the CM energy $\sqrt{s}$ is approximately calculated by $\sqrt{s} = \sqrt{s_0} - |\Delta p|$, where $\sqrt{s_0}$ is the nominal CM energy obtained from an independent energy measurement and Δp is the momentum difference between the colliding electron and positron which is calculated with accolinearity angle.

However, there are several limitations in this method. For example, $|\Delta p|$ is positive definite, $\sqrt{s}$ is always less than or equal to the nominal CM energy. Because of the beam-energy spread those events each of which CM energy is above the nominal value are produced, while their calculated $\sqrt{s}$ values are always less than the nominal value. Moreover, when the initial electron and positron have lost their energies nearly by the same amount of energy, Δp becomes almost zero and the $\sqrt{s}$ value is nearly equal to $\sqrt{s_0}$ regardless of the lost energy being large. The only source of

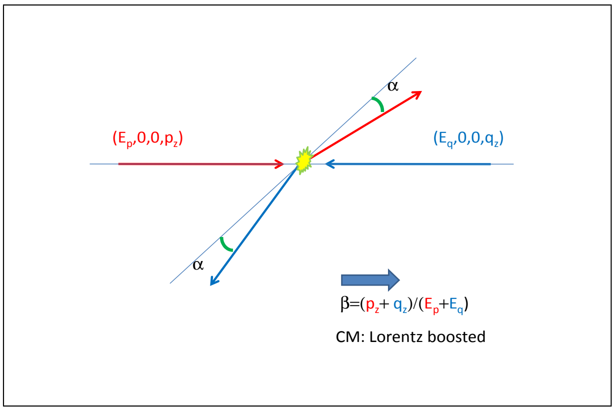

Figure 2: The Bhabha process. Due to beamstraghlung, electrons and positrons colide no more in CM system. Colliding electrons and positrons are boosted wth velocity β.

these problems is to use the accolinearity angle for calculating the CM energy, but the most crucial problem in this method is that when the e^+ and e^- beam parameters are different, though this is the usual case, the beam parameters can not be estimated just from the measured CM energy spectrum because of lack of information.

3. The formalism

3.1 Bhabha events as the distribution function

We derive the statistical model to measure the luminosity spectrum in terms of Bayesian statistics. In this paper, the luminosity spectrum is represented by the beam parameter by using the empirical formula of luminosity spectrum. The measurement of the luminosity spectrum can be represented by the conditional probability of model parameters under data: $\Pr\left(\alpha_{+,}\alpha_{-}|\{\left(q'_{+},q'_{-}\right)\}\right)$,where (α_{+},α_{-}) represents the parameters to describe the luminosity, and $D=\{\left(q'_{+},q'_{-}\right)\}$ a set of 4-momenta of Bhabha events. By the Bayes' theorem, using a model distribution (or a likelihood) $l\,(D;\alpha_{+,}\alpha_{-})=\Pr(D|\alpha_{+},\alpha_{-})$, the posterior distribution is written by

$$\Pr\left(\alpha_{+,}\alpha_{-}|D\right)=\frac{l\,(D;\alpha_{+,}\alpha_{-})\Pr(\alpha_{+,}\alpha_{-})}{\Pr(D)}, \tag{3.1}$$

where (α_{+},α_{-}) denotes a hypothesis, $\Pr(\alpha_{+,}\alpha_{-})$ the prior of the hypothesis. The denominator $\Pr(D)$ represents a normalization factor of the posterior $\Pr(D)=\int d\alpha_{+}d\alpha_{-}\,l\,(D;\alpha_{+,}\alpha_{-})\Pr(\alpha_{+,}\alpha_{-})$. The model distribution function can be described by Bhabha event distribution, $N(q'_{+},q'_{-})\,d^4q'_{+}d^4q'_{-}$, which is decomposed into the luminosity spectrum function $L(\cdot)$, the cross-section of Bhabha process $d\sigma(\cdot)$, and detection error function $G(\cdot)$:

$$\begin{aligned}\mathcal{N}(q'_{+},q'_{-}) =&C^{-1}\int d^4p_{+}d^4p_{-}d^4q_{-}d^4q_{+}G(q'_{+},q'_{-};\omega_q,q_{+},q_{-})\\ &\times\sigma(q_{+},q_{-};p_{+},p_{-})L(p_{+},p_{-};\alpha_{+},\alpha_{-}),\end{aligned} \tag{3.2}$$

where event distribution is the marginal distribution of three kinds of 4-momenta; those of colliding beam $p_{\pm}$, those of scattered e^+e^- from Bhabha process (theoretical and not observable) $q_{\pm}$, and those of detected Bhabha event $q'_{\pm}$, respectively. C denotes the normalization factor to satisfy $C^{-1}\int \mathcal{N}(q_+,q_-)d^4q_+d^4q_- = 1$.

Before we derive the detailed distribution function, we rewrite the Bhabha events in terms of the distribution functions. Using the total luminosity, $L(\cdot;\alpha_+,\alpha_-) := \int dp_+dp_-L(p_+,p_-;\alpha_+,\alpha_-)$, the normalized luminosity is defined as the probability function:

$$\tilde{L}(p_+,p_-;\alpha_+,\alpha_-) := \frac{L(p_+,p_-;\alpha_+,\alpha_-)}{L(\cdot;\alpha_+,\alpha_-)}. \tag{3.3}$$

For each pair of (p_+,p_-), we also obtain the normalized cross section:

$$\tilde{\sigma}(q_+,q_-;p_+,p_-) := \frac{\sigma(q_+,q_-;p_+,p_-)}{\sigma(\cdot;p_+,p_-)},$$

where $\sigma(\cdot;p_+,p_-) := \int dq_+dq_-\sigma(q_+,q_-;p_+,p_-)$ is total cross section for (p_+,p_-).

The noise distribution function can be reduced to the product of error functions

$$G(q'_+,q'_-;\omega_q,q_+,q_-) = G(q'_+;q_+,\omega_{q_+})G(q'_-;q_-,\omega_{q_-}), \tag{3.4}$$

since measurement of 4-momenta of electrons and positrons are independent. The detection errors depend on position of the detectors, and 4-momenta of incident particle, and its error function can be parametrize by $G(q';q,\omega_q)$ with condition $\int dq'G(q';q,\omega_q) = 1$. Thus we rewrite (3.2) to

$$\mathcal{N}(q'_+,q'_-) = \int dq_-dq_+dp_+dp_-\mathcal{N}(q'_+,q'_-,q_+,q_-,p_+,p_-;\omega,\alpha_+,\alpha_-) \tag{3.5}$$

$$\begin{aligned}&\mathcal{N}(q'_+,q'_-,q_+,q_-,p_+,p_-;\omega,\alpha_+,\alpha_-)\\ &= G(q'_+,q'_-;\omega_q,q_+,q_-)\tilde{\sigma}(q_+,q_-;p_+,p_-)\frac{\sigma(\cdot;p_+,p_-)L(p_+,p_-;\alpha_+,\alpha_-)}{N}\end{aligned} \tag{3.6}$$

where N is total number of events,

$$\begin{aligned}N &= \int dq'_-dq'_+dq_-dq_+dp_+dp_-G(q'_+,q'_-;\omega_q,q_+,q_-)\sigma(q_+,q_-;p_+,p_-)L(p_+,p_-;\alpha_+,\alpha_-)\\ &= \int dp_+dp_-L(p_+,p_-;\alpha_+,\alpha_-)\sigma(\cdot;p_+,p_-).\end{aligned} \tag{3.7}$$

Therefore $\mathcal{N}(q'_+,q'_-,q_+,q_-,p_+,p_-;\omega,\alpha_+,\alpha_-)$ can be considered as the universal distribution, that is decomposed into conditional distribution $G(q'_+,q'_-;\omega_q,q_+,q_-)$, $\tilde{\sigma}(q_+,q_-;p_+,p_-)$ and $\sigma(\cdot;p_+,p_-)$ $\times L(p_+,p_-;\alpha_+,\alpha_-)/N$. When we assume the prior $\Pr(\omega,\alpha_+,\alpha_-) = \Pr(\omega)\Pr(\alpha_+)\Pr(\alpha_-)$, we can construct the Bayesian network for continuum variable. It should be notice that The Bhabha events distribution function can be represented as the graphical model, and Figure 3 shows the graphical representation of the Bhabha events distribution (3.6).

3.2 The details of the statistical model

We have derived the statistical model for the luminosity spectrum using Bhabha events. Next, we discuss the details of distribution functions. Here, we consider the simplified model, since we want to investigate the potential of our method. In general, detector error function can be

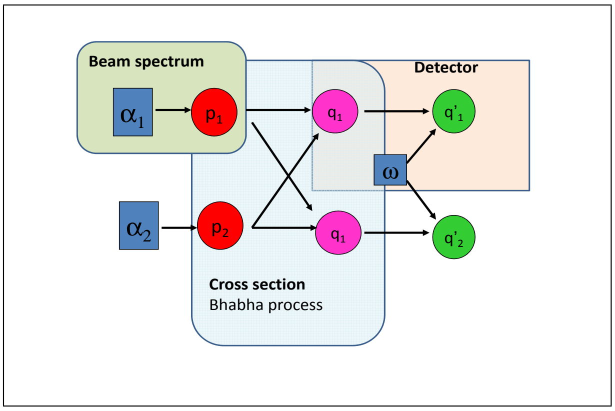

Figure 3: The graphical representation of the beam energy spsctrum function.

represented by the multi-valuated gauss function of 4-momentumm, however, we consider the case of no detection errors, i.e., zero variance case:

$$G_\pm(q_\pm; q'_\pm, \omega_{q'}) = \delta^{(4)}(q'_\pm - q_\pm). \tag{3.8}$$

The cross section of Bhabha process is given by

$$d\sigma(q'_+, q'_-; p_+, p_-) = \left\|\mathscr{M}(q'_+, q'_-; p_+, p_-)\right\|^2 d\Phi(q'_+, q'_-; p_+, p_-), \tag{3.9}$$

$$d\Phi(q'_+, q'_-; p_+, p_-) = \delta^{(4)}(q'_+ + q'_- - p_+ - p_-)\frac{d^3q_+}{2E_+}\frac{d^3q_-}{2E_-}, \tag{3.10}$$

and the invariant amplitude is calculated using Feynman graphs of Bhabha process. In this paper, the lowest order contribution from perturbation theory, i.e., one photon exchange process, is considered for the simplicity of analysis [17]. It should be noticed that higher order of corrections can be included to our model straightforwardly. The integral in eq(3.2) can be calculated analytically and is given by a simple form;

$$N(q_+, q_-) = C^{-1} d\sigma(q_+, q_-; p_+, p_-) L(p_+, p_-; \alpha_+, \alpha_-), \tag{3.11}$$

where $q_\pm = (E_\pm, q^x_\pm, q^y_\pm, q^z_\pm)$, $E_\pm = \sqrt{\vec{q}^{\,2}_\pm + m_e^2}$, $p_\pm = (\varepsilon_\pm, 0, 0, \pm\varepsilon_\pm)$, $\varepsilon_\pm = \frac{1}{2}(E_+ + E_- \pm (q^z_+ + q^z_-))$, $q^x_+ = -q^x_-$, $q^y_+ = -q^y_-$, and sign ($\pm$) in $p_\pm$ should be read for the same order.

The luminosity spectrum function $L(p_+, p_-; \alpha_+, \alpha_-)$ can be represented by colliding beams, and it is assumed to approximately obtained by product of the beam distribution function $B(p;\alpha)$ of the pair of the electron and positron,

$$L(p_+, p_-; \alpha_+, \alpha_-) = B(p_+; \alpha_+) B(p_-; \alpha_-), \tag{3.12}$$

where the colliding electrons and positrons travel along the z-axis with zero crossing angle, i.e., $p = (E, 0, 0, \pm E)$ $(E \gg m_e)$:

$$B(p;\alpha) = \delta(p^x)\delta(p^y)\delta(p^E - |p^z_\pm|)Y(p^z;\alpha). \tag{3.13}$$

As for the beam spectrum function $B(p;\alpha)$, the Yokoya-Chen function is used, which gives the empirical formula of the luminosity spectrum at interaction point afterbeamstrahlung [7][8]. In the approximated luminosity spectrum function, the Yokoya-Chen function $Y(p^z;\alpha)$ is modified by the τ–averaged function in momentum space:

$$Y(E;\alpha) = \langle \psi(E,\tau;\alpha)\rangle_\tau := \int_0^1 d\tau \psi(E,\tau;\alpha), \tag{3.14}$$

$$\psi(E,\tau;\alpha) \simeq e^{-N_\gamma \tau}\left[\delta(E-E_0) + \frac{e^{-y}}{E_0-E} h(\tau N_1 y^{1/3})\right], \tag{3.15}$$

$$h(x) \simeq \sqrt{\frac{3}{8\pi}}\left[\frac{\sqrt{x/3}}{1+0.53x^{-5/6}}\right]^{3/4} \exp\left(4\left(\frac{x}{3}\right)^{3/4}\right), \tag{3.16}$$

where new variables y and N_1 are introduced for the simplicity of notation; $y = (E/E_0-1)/\xi_1$, $N_1 = N_{cl}/(1+\xi_1 y) + N_\gamma \xi_1 y/(1+\xi_1 y)$, respectively. The parameters N_{cl}, N_γ, and ξ_1 are described by the independent beam parameters $\alpha = (E_0, \sigma_z, N/(\sigma_x+\sigma_y))$, where E_0 is the nominal energy and N is the number of particles in a bunch with the size of $(\sigma_x, \sigma_y, \sigma_z)$. (For the detail of parametrization, please see the references [7][8]). Since the Yokoya-Chen function is described by two parts, the nominal energy part, $\phi_1(E,\tau;\alpha) := e^{-N_\gamma\tau}\hat{\delta}(E-E_0)$, and the beamstrahlung part, $\phi_2(E,\tau;\alpha) := e^{-N_\gamma\tau}\frac{e^{-y}}{E_0-E}h(\tau N_1 y^{1/3})$, eq(3.14) can be described by mixed distribution;

$$\tilde{Y}(E;\alpha) = \sum_{k=1}^{2} \lambda_k(\alpha)\tilde{\phi}_k(E;\alpha), \tag{3.17}$$

$$\tilde{\phi}_k(E;\alpha) := \int d\tau \phi_k(E,\tau;\alpha) \Big/ \int d\tau dE\, \phi_k(E,\tau;\alpha), \tag{3.18}$$

$$\lambda_{1,2}(\alpha) := \int d\tau dE\, \phi_{1,2}(E,\tau;\alpha) \Big/ \int d\tau dE \left[\phi_1(E,\tau;\alpha) + \phi_2(E,\tau;\alpha)\right]. \tag{3.19}$$

Whereinsoever the mixed distribution is defined so as to satisfy the normalization condition, $\int dE\, \tilde{\phi}_k(E;\alpha) = 1$ and $\lambda_1 + \lambda_2 = 1$, thus $\int dE Y(E;\alpha) = 1$, such that Yokoya-Chen function eq(3.17) and both $\lambda_k(\alpha)$ and $\tilde{\phi}_k(E;\alpha)$ are treated as probability functions. Therefore the luminosity function eq(3.12) is rewritten into decomposed formula as follows;

$$L(E^+,E^-;\alpha_+,\alpha_-) = \sum_{k,l=1}^{2} \lambda_k^{(+)}(\alpha_+)\lambda_l^{(-)}(\alpha_-)\tilde{\phi}_k(E^+;\alpha_+)\tilde{\phi}_l(E^-;\alpha_-). \tag{3.20}$$

Substituting the above equation into eq(3.11), and extracting the arguments of summations by l,k, the universal distribution of Bhabha events under the beamstrahlung is obtained in terms of $(q_+,q_-;k,l)$;

$$\begin{aligned}\tilde{N}(q_+,q_-;k,l) =& \tilde{C}^{-1} d\sigma(q_+,q_-;p_+,p_-)\\ &\times \lambda_k^{(+)}(\alpha_+)\lambda_l^{(-)}(\alpha_-)\tilde{\phi}_k(E^+;\alpha_+)\tilde{\phi}_l(E^-;\alpha_-).\end{aligned} \tag{3.21}$$

The Bhabha distribution function in the mixed distribution eq(3.21) has been obtained as well as the marginalized formula eq(3.11), the maximum likelihood method can be applied for the parameter estimation. Because of the efficiency the mixed distribution for parameter fitting is used in this paper. The EM algorithm is introduced as some preparations for the parameter estimation with missing observation (l, k).

3.3 EM algorithm

The EM algorithm, which is one of the popular algorithms in learning of neural network, parameter estimation Bayesian network and so on, is briefly reviewed. Consider the case where the parameters θ of the model $m(\mathbf{z};\theta)$ for the data set $D = \{\mathbf{z}^{(k)}; k = 1,..,N\}$ is estimated but the observed data is incomplete because of unobservable or missing. The data set can be represented by $D = \{\mathbf{z}^{(k)} = (\mathbf{x}^{(k)}, \mathbf{y}^{(k)});\ x^{(k)} \in D_o$ (observable), $y^{(k)} \in D_h$ (unobservable or hidden)$\}$, which is the case, for example, where the observable data is obtained as a marginal distribution of the full data: $\tilde{m}(\mathbf{x};\theta) = \sum_{\mathbf{y}} m(\mathbf{x},\mathbf{y};\theta)$. The EM algorithm calculates the conditional probability function of hidden variable $\mathbf{y}$ under the observed variable $\mathbf{x}$ and given parameter θ, $\Pr(\mathbf{y}|\mathbf{x},\theta)$. Using the Bayes' theorem $\Pr(\mathbf{y}|\mathbf{x},\theta)$ can be written in terms of the model function $m(\mathbf{x},\mathbf{y};\theta)$,

$$\Pr(\mathbf{y}|\mathbf{x},\theta) = \frac{\Pr(\mathbf{x},\mathbf{y}|\theta)}{\Pr(\mathbf{x}|\theta)} = \frac{m(\mathbf{x},\mathbf{y};\theta)}{\tilde{m}(\mathbf{x};\theta)} = \frac{m(\mathbf{x},\mathbf{y};\theta)}{\sum_{\mathbf{y}'} m(\mathbf{x},\mathbf{y}';\theta)}. \tag{3.22}$$

Algorithm 1. *Initiate θ^0, ε (the convergence condition), and set $i \leftarrow 0$*

do $i \leftarrow i+1$

***E-step**: compute*

$$Q(\theta';\theta^i) := \frac{1}{N} \sum_{\{(\mathbf{x},\mathbf{y})\}} \left[\frac{m(\mathbf{x},\mathbf{y};\theta)}{\tilde{m}(\mathbf{x};\theta)} \times \ln\left(m(\mathbf{x},\mathbf{y};\theta')\right)\right] \tag{3.23}$$

***M-step**:*

$$\theta^{i+1} \leftarrow \arg\max_{\theta} \left(Q(\theta;\theta^i)\right) \tag{3.24}$$

until convergence $Q(\theta^{i+1};\theta^i) - Q(\theta^i;\theta^{i-1}) \leq \varepsilon$

return $\hat{\theta} \leftarrow \theta^{i+1}$

It should be noticed that the EM algorithm is reduce to the ordinal maximum likelihood method in case that all variables are observable, since $Q(\theta;\theta^i)$ is reduced to the log likelihood function and M-step is nothing but maximization of log likelihood function. For more detail please see, for examples, references [9],[10] and [11].

4. Estimation of the luminosity spectrum

Our method determining the luminosity spectrum is examined by using a numerical simulation, since all experimental conditions can be controlled and the data under necessary condition can be easily generated. In this paper, only the data without detection errors are considered in order to investigate the potentialities of this method as well as to make everything as simple as possible. The simulation environments is set up to create Bhabha events in the ILC linear collider. The profile of the distributed data comes from the broad energy spectrum of colliding beams and the effect of the scattering using the analytic expression. For the luminosity function Yokoya-Chen function eq(3.12) is used for both event generation and parametric inference. One million Bhabha events are generated by simulation for parameter fittings as explained below subsection.

The parameter estimation requires the CPU time, i.e., the integral of hidden variables in evaluating the EM algorithm and the multidimensional integration in evaluate the total cross section.

		simulation	
	ILC	electron	positron
Beam energy	250 GeV	250 GeV	250 GeV
Charge per bunch	2×10^{10}	2.2×10^{10}	2×10^{10}
X beam size at IP	554nm	554nm	554nm
Y beam size at IP	5nm	4.5nm	5nm
Bunch-length at IP	300μm	330μm	300μm

Table 1: ILC Beam paramaters, and the parameters for simltations.

We introduce the parallel computing techniques based on the message passing interface (MPI) to accelerated them. Evaluation by the EM algorithm is independent for each event, pluralization is straightforward. The multidimensional integral is carried by using the pearlized version of the BASSES package.

4.1 Event generation

Bhabha events are generated using `GRACE` system[12]. The numerical integration of the Bhabha process cross section is calculated using `BASES` and the equal-weighted event generation are carried out using `SPRING`[13]. The `GRACE` system is an automatic system for generating Feynman diagram and a `FORTRAN` source-code to evaluate the amplitude. In the `GRACE` system, the matrix elements are calculated numerically using `CHANEL`[14] library based on the helicity amplitude formalism. `CHANEL` contains routines to evaluate such things as: wave-functions/spinors at external states, interaction vertices, and particle propagator. Immediately after the numerical integration of the matrix elements by `BASES`, event generation can be made using `SPRING` utility. The beamstrahlung effect is taken into account using `LUMINUS` [15]. The beam parameters are set at those values taken from the TESLA design report[16] as ILC design parameter, which is shown in table 1. An expected luminosity with these beam parameters is $2.94\times10^{38}/m^2/s$. The beam energy spread is assumed to be zero for simplicity. The initial state radiative correction is also omitted. The observed electrons and positrons are generated with the energy greater than10 GeV, and within the angular range between 20 degree to 160 degree. The total cross section under these conditions are calculated to be 28.68 ± 0.01 pb, which corresponds to about 3k events for one hour data taking.

4.2 Luminosity spectrum measurements

We first focus on the beam spectrum in the ILC beam parameters. The luminosity spectrum can be further decomposed into the mixed distribution function by the coefficients λ_k in eq(3.20). The contribution weights in eq(3.17) for the ILC beam parameters are obtained as $\lambda_1 = 0.725$ and $\lambda_2 = 0.275$, thus the effects of the beamstrahlung can be decomposed using coefficients as follows, the nominal-nominal energy part, $\lambda_1^{(+)}\lambda_1^{(-)} \simeq 0.525$, the nominal-beamstrahlung part $\lambda_1^{(+)}\lambda_2^{(-)} + \lambda_2^{(+)}\lambda_1^{(-)} \simeq 0.399$, and the beamstrahlung-beamstrahlung part $\lambda_2^{(+)}\lambda_2^{(-)} \simeq 0.076$. If high precision measurements were not required, the collision from the beamstrahlung–beamstrahlung part

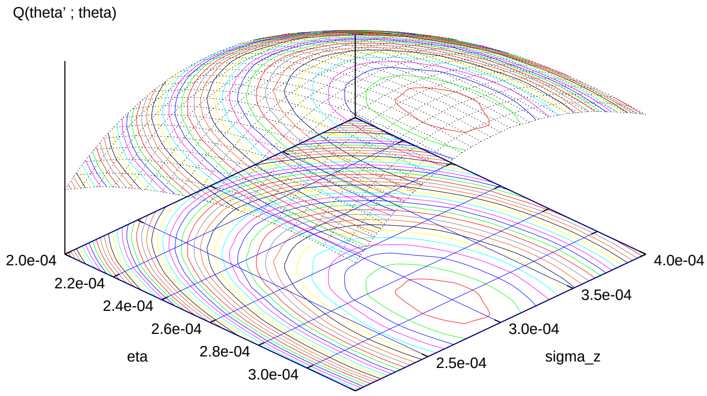

Figure 4: Contour plot of parameter θ in Q(θ'; θ)) using 10k events.

could be negligible and the luminosity spectrum could be estimated by measuring only the acolinearity angle. When we come back to the previous works, for example [5], it was assumed that most of Bhabha events are contribution from the nominal-nominal part and from the nominal-beamstrahlung part, i.e., at least one of the $\Delta\varepsilon_{\pm}$ is nearly zero[1]. Thus, more than 8% of estimation error is contained in the previous methods using the acolinearity angle.

Figure 1 shows the beam energy spectrum distribution used in the simulations, where distribution is parametrized by the Yokoya-Chen function listed in table 1. An asymmetric parameterization is selected to investigate whether our method can distinguish small difference between spectrums. The luminosity spectrum estimation is obtained by parameter fitting of independent beam parameters in Yokoya-Chen function, θ =(η^+, σ_z^+, η^-, σ_z^-), where η is defined by $\eta = N/(\sigma_x + \sigma_y)$ for electron and positron. As discussed in the previous section, EM algorithm is applied to maximize the likelihood function eq(3.21). In E-step, the estimation of the normalization constant C in eq(3.21) requires the 4-dimensional integration, and the adaptive Monte Carlo integration package BASES is used. Figure 4 shows the contour plot of θ' in σ_z^+–σ_z^-, and an single peak is obtained around the ILC beam parameters in the M-step in the EM algorithm. Since the marginal likelihood function $Q(\theta';\theta)$ gives a single smooth peak in each M-step, the steepest descent method in multi-dimensional space are used to obtain the peak of the likelihood function.

Thus, applying EM algorithm for one million Bhabha events, we have successfully obtained the beam parameter of the simulation. We next study the errors of estimated beam parameters to determine the required Bhabha events for luminosity measurements. The confidence level could be a good measure for a statistical test for the likelihood function of χ^2 fitting, however, the likelihood eq(3.21) is a even more general distribution function and the confidence level cannot be applicable. Here, we apply the bootstrap method[18]. In the bootstrap method the bootstrap samplings

[1] It should be remined that $\sqrt{s}$ and Δp values ware obtained by $\sqrt{s} \simeq \sqrt{s_0} - (\Delta\varepsilon_+ + \Delta\varepsilon_-)$ and $\Delta p = \Delta\varepsilon_+ - \Delta\varepsilon_-$, where $\Delta\varepsilon_{\pm}$ ware the energy loss of positron and electron by beamstrahlung, respectively.

$\mathscr{D}^{(b)}(b=1,..,B)$ are prepared by resampling, where each of data-set consists of a certain number of events sampled independently from the data. The bootstrap estimator such as variance of the estimated parameters is calculated;

$$V(\hat{\theta}) = \frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^{*(b)} - \hat{\theta}^{*(.)}\right)\left(\hat{\theta}^{*(b)} - \hat{\theta}^{*(.)}\right)^T, \tag{4.1}$$

using the estimated parameters $\hat{\theta}^{*(b)}$ for each $\mathscr{D}^{(b)}$.Where $\hat{\theta}^{*(.)}$ is the bootstrap average of the estimates of parameters; $\hat{\theta}^{*(.)} = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}^{*(b)}$. This $V(\hat{\theta})$ is identified the errors of estimated parameters.

Since the typical event rate of the ILC linear collider is about 3k events per hour, we study two kind of data-set; 1k Bhabha events and 10k events data. Two kind of hundred ($B = 100$) bootstrap samplings are prepared from 1M events generated by simulation. The estimated parameters $\hat{\theta}^{*(b)}$ are shown in Figure 5. Each plot shows the the scatter plots of estimated parameters, where the 4-dimensional estimated beam parameters are projected into η–σ_z plain. The left parameters is the plot for 1k Bhabha events. The resultant bootstrap estimator can be read as $\hat{\theta}^{*(.)}$ =($759\times10^{14}m^{-1}$, $317\mu m$, $688\times10^{14}m^{-1}$, $286\mu m$), standard deviation $\sigma^{(.)}$ =($29\times10^{14}m^{-1}$, $18\mu m$, $35\times10^{14}m^{-1}$, $23\mu m$) and the correlation:

$$\rho_{1k} = \begin{pmatrix} 1 & 0.46 & -0.092 & -0.053 \\ 0.46 & 1 & 0.12 & 0.08 \\ -0.092 & 0.12 & 1 & 0.54 \\ -0.053 & 0.08 & 0.54 & 1 \end{pmatrix},$$

where the indices of correlation $\rho_{i,j}$ stand for $(i,j = (e^-\eta)\,,\ (e^-z)\,,\ (e^+\eta)\,,(e^+z))$. The right panel shows the plot of the estimated parameter for 10k events data-set. The result of bootstrap estimator is $\hat{\theta}^{*(.)}$ =($765\times10^{14}m^{-1}$, $317\mu m$, $694\times10^{14}m^{-1}$, $290\mu m$), $\sigma^{(.)}$ =($13\times10^{14}m^{-1}$, $85\mu m$, $92\times10^{14}m^{-1}$, $49\mu m$),

$$\rho_{10k} = \begin{pmatrix} 1 & 0.70 & -0.40 & 0.10 \\ 0.70 & 1 & -0.50 & 0.020 \\ -0.40 & -0.50 & 1 & 0.15 \\ 0.10 & 0.020 & 0.15 & 1 \end{pmatrix}.$$

These result suggest that the beam parameter estimation is achieved even for small data-set of 1k events, and the difference of beam parameters are distinguished. By using 10k events, the beam parameters are estimated within a few percent.

5. Conclusion and discussion

We have proposed a statistical method to determine luminosity spectrum using Bhabha events. It was for the first time to introduce statistical method to measurement of luminosity. First and foremost the exact formula of the Bhabha event distribution function has been formulated in terms of the differential cross section, the error distribution function and the luminosity spectrum function. The luminosity function is expressed by Yokoya-Chen function, and is parametrized by beam parameters. Secondary by applying the EM algorithm to the parameter fitting the beam parameters

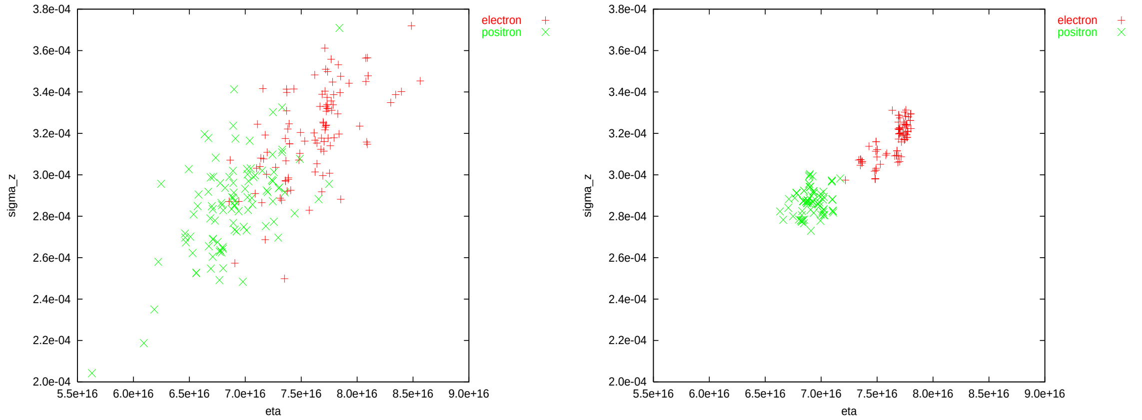

Figure 5: Estimated beam parameters 100 boostrap resampling data-set. The 4-dimensional estimated parameters $(\eta^+, \sigma_z^+, \eta^-, \sigma_z^-)$ are plotted in sigle (η, σ_z) plain. The left panel shows the plot for data-set of 1k Bhabha events, and the right panel for data-set of 10k events,

are successfully determined with an error of a few percents using 10k events of Bhabha process by the "ideal" detector with no detection error. This strongly suggests that our method can determine the luminosity spectrum in the future ILC linear collider.

In order to apply this method to a realistic experiment, several points in this study should be improved. First, the model have to be extended to include the detection errors in measurement of 4-momentum,which can be made straightforwardly as it has been shown in the formulation, and the EM algorithm should work well. This extension causes inclusion of estimation errors by several percent, and the expected number of Bhabha events for the parameter fitting will be increased. The second point to be studied further is the effect of the background processes against Bhabha process, especially two photon process, which may produce more fake events than Bhabha process. If the case the event distribution function must be extended to include the two photon process since no discrimination between Bhabha process and two photon process will be found in the event shape in the high luminosity e^+e^- linear collider. The third point is the inclusion of the other effects of beam dynamics such as an initial state radiation and a beam spread, which make the beam energy more broad spectrum and change the Bhabha events distribution. For the purpose of precision determination of luminosity and of luminosity monitoring, the modeling of the beam spectrum function in terms of the beam parameters will be a key issue in data modeling.

In addition to the extension and improvement of data modeling, study in computation algorithms are indispensable. For examples, inclusion of the detector errors causes to increase several hidden variables in EM algorithms, and expected high-dimensional integration for hidden variables. Within this study, we have introduced parallel processing algorithms, e.g., parallel Monte Carlo integration, however, further improvements is expected.

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