PROCEEDINGS OF SCIENCE

Innovative SETI by the KLT (part 2)

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SETI searches are, by definition, the extraction of very weak radio signals out of the cosmic background noise. When SETI was born in 1959, it was "natural" to attempt this extraction by the only detection algorithm well known at the time: the Fourier Transform (FT). In fact:

1) SETI radio astronomers had adopted the viewpoint that a candidate ET signal would necessarily be a sinusoidal carrier, i.e. a very narrow-band signal. Over such a narrow band, the background noise is white. And so, the basic assumption behind the FT that the background noise must be white was "perfectly matched" to SETI for the next fifty years!
2) In addition, the Americans, J. W. Cooley and J. W. Tukey discovered in April 1965 that all the FT computations could be speeded up to N*ln(N) (rather than N^2) (N is the number of numbers to be processed) by their own Fast Fourier Transform (FFT). Then, SETI radio astronomers all over the world gladly and unquestioningly adopted the new FFT forever.

In 1983, however, the French SETI radio astronomer, François Biraud, pointed out that we only can make guesses about ET's telecommunication systems, and that the trend on Earth was shifting from narrow-bands to wide-bands. Thus, a new transform, other than the FFT, was needed. Such a transform had actually been discovered as early as 1946 by the Finn, Kari Karhunen, and the French, Michel Loève, and is thus named KLT for them. So, François Biraud suggested to "look for the unknown in SETI" by adopting the KLT rather than the FFT. The same ideas were reached independently by this author also, and starting 1987, he too was "preaching the KLT": first at the SETI Institute, then (since 1990) at the Italian CNR (now called INAF) SETI facilities at Medicina, near Bologna. Their director, Stelio Montebugnoli, had bright students succeeding in programming the KLT algorithm and the KLT for SETI is now a reality at the SETI-Italia facilities and for the first time in history.

This paper describes the KLT exceptional capability to extract a sinusoidal carrier embedded in noise having a Signal-to-Noise Ratio (SNR) of 10^{-3} or less. The first part of this paper (8 pages) already was posted in the POS archive as the paper presented by this author at the Conference:

"Bursts, Pulses and Flickering: Wide-field monitoring of the dynamic radio sky" held in Kerastari, Tripolis, Greece, 12-15 June 2007. That paper has the same title as this one: "Innovative SETI by the KLT". The present 3-pages paper is just the second and part of that first paper, and is published here for the first time.

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1. KLT of a STATIONARY PROCESS *X(t)* found by virtue of the "BAM" METHOD

We confine ourselves to a ***stationary X(t) only*** over a ***discrete*** set of instants $t = 0, \ldots, N$.
In this case, it can be proven that the autocorrelation of $X(t)$ becomes the Toeplitz matrix

$$Toeplitz = \begin{bmatrix} R_{XX}(0) & R_{XX}(1) & R_{XX}(2) & \ldots & \ldots & R_{XX}(N) \\ R_{XX}(1) & R_{XX}(0) & R_{XX}(1) & \ldots & \ldots & R_{XX}(N-1) \\ R_{XX}(2) & R_{XX}(1) & R_{XX}(0) & \ldots & \ldots & R_{XX}(N-2) \\ \ldots & \ldots & \ldots & R_{XX}(0) & \ldots & \ldots \\ R_{XX}(N) & R_{XX}(N-1) & \ldots & \ldots & R_{XX}(1) & R_{XX}(0) \end{bmatrix}$$

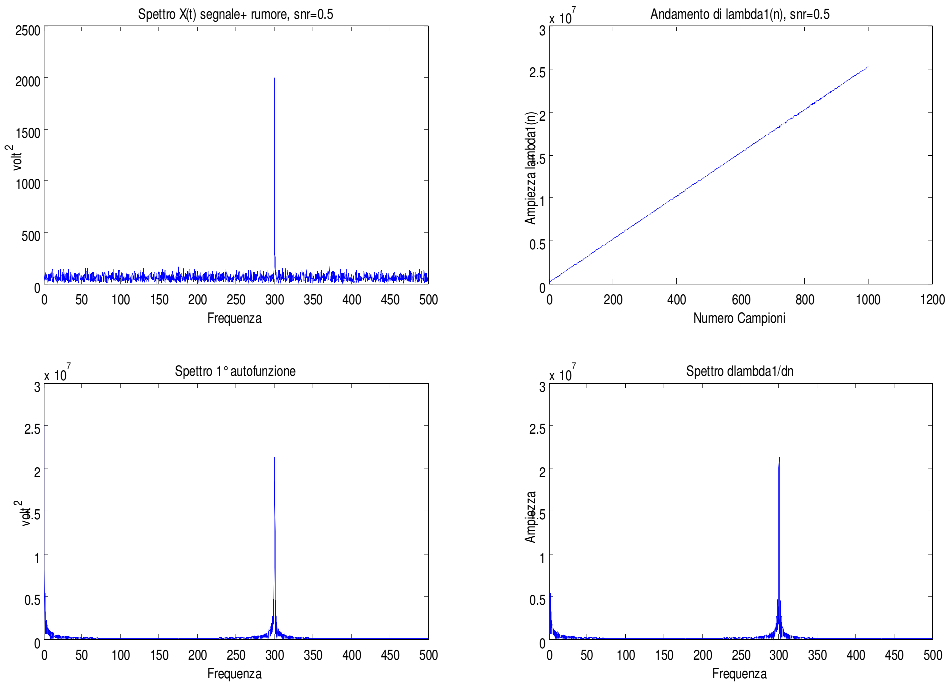
We may now choose N at will but, clearly, the higher is N, the more accurate the KLT of *X(t)* is. On the other hand, the final instant T in the KLT can be chosen at will and now is T=N. So, we can regard T=N as a sort of "new time variable" and even take derivatives with respect to it!

Let us now go back to the above correlation. If we let N vary as a new free variable, that amounts to ***bordering the autocorrelation,*** i.e. adding one (last) column and one (last) row to the previous correlation. Whence the name BAM (= Bordered Autocorrelation Method) given to this procedure. In practice, we have to solve the system of linear algebraic equations of the KLT once more for N+1, rather than for N. So, for each different value of N, we get a new value of the first eigenvalue $\lambda_1(N)$ now regarded as a function of N. ***But now a surprise comes! This*** $\lambda_1(N)$ ***was mathematically proven by the author to be proportional to N. And so the derivative*** $\frac{\partial \lambda_1(N)}{\partial N}$ ***is of course a CONSTANT with respect to N. And the ordinary Fourier Transform of this constant is a Dirac delta function, i.e. a neat peak in the frequency domain. These unusual facts pave the way to the applications of the KLT to all STATIONARY X(t).***

2. Finding a pure tone (= a sinusoidal carrier from ETs, as is typical in SETI) embedded in STATIONARY *X(t)* by virtue of the FIRST KLT EIGENVALUE λ_1

The first eigenvalue $\lambda_1(N)$ also is the (by far) largest, i.e. the most important one, since it is the variance of the most largely-scattered random variable Z_1. In other words, even if you study the first eigenvalue of the KLT only, you actually study the ***bulk*** of your data! That's why $\lambda_1(N)$ is called "dominant". Well, numeric simulations performed by Francesco Schilliro` and Salvatore Pluchino (ref. 2) in December 2006 and January 2007 lead to the results shown in 4 plots below. The first plot is the ordinary Fourier spectrum of a pure tone at 300 Hz buried in noise with SNR=0.5 (this is about the lowest SNR beyond which the FFT starts failing to denoise a signal). The second plot shows the first KLT dominant eigenvalue $\lambda_1(N)$ over 1200 time samples. Clearly, this $\lambda_1(N)$ is proportional to N. So, its derivative $\frac{\partial \lambda_1(N)}{\partial N}$, is a constant with respect to N. But then we may take the Fourier transform of such a constant and clearly we get a Dirac delta function, i.e. a peak just at 300 Hz. In other words, we have KLT-recovered the original tone by virtue of the BAM. The fourth plot below is such a BAM-recovered peak, and it is of course identical to the third plot, that is the ordinary FFT of first KLT eigenfunction (obtained by solving the full and long system of N algebraic first-degree equations).

Let us now do the same again… but with an incredibly low SNR of 0.005. Poor Fourier here is in a mess! Just look at the first plot below! No classical FFT spectrum can be identified at all!
But for the KLT… no problem! The second plot clearly shows that $\lambda_1(N) \propto N$. The third plot (KLT FAST way) is the NEAT KLT spectrum of the 300 Hz tone obtained by the FFT of $\frac{\partial \lambda_1(N)}{\partial N}$ and this is just the same as the third plot (KLT SLOW way). This proves the superior behavior of the KLT over the FFT once and for all!



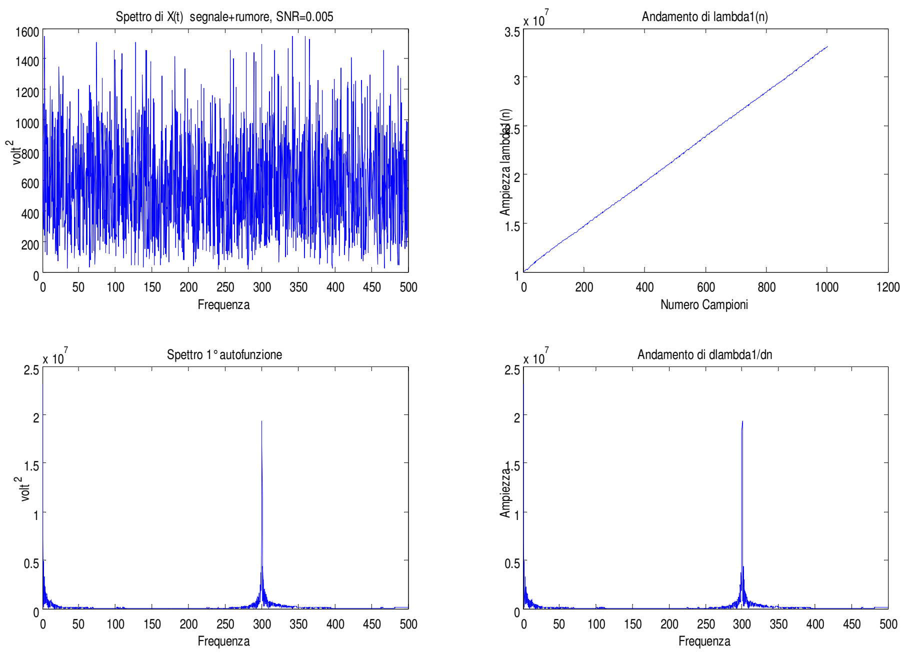
Conclusions

Let us summarize the results mathematically described in this paper.
When the stochastic process *X(t)* is stationary (i.e. it has both mean value and variance constant in time), then there are two alternative ways to compute the first KLT dominant eigenfunction (that is the roughest

approximation to the full KLT expansion, that may be “enough” for practical applications!):

1) Either you compute the first eigenvalue and Fourier-transform it to get the first eigenfunction or
2) (BAM) You compute the derivative of the first eigenvalue with respect to $T=N$ and then Fourier-transform it to get the first eigenfunction. In practical numerical simulations of the KLT it may be less time-consuming to choose option 2) rather than option 1).

In either case, the KLT of a given stationary process can retrieve a sinusoidal carrier out of the noise for values of the signal-to-noise ratio (SNR) that are three orders of magnitude lower than those that the FFT can still filter out. In other words, while the FFT (at best) can filter out signals buried in a noise that as a SNR of about 1 or so, the KLT can, say, filter out signals that have a SNR of, say, 0.001 or so. This is the superior achievement of the KLT over the FFT.



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[2] F. Schilliro`, S. Pluchino, C. Maccone, S. Montebugnoli: Istituto Nazionale di Astrofisica (INAF) – Istituto di Radioastronomia (IRA) – Rapporto Tecnico - *“La KL Transform: considerazioni generali sulle metodologie di analisi ed impiego nel campo della Radioastronomia”*, Technical Report (available in Italian only) published in January 2007.