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Indirect methods for astrophysical reaction rates

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Capture reactions can be a technical challenge in the energy region of astrophysical interest. Alternative experimental techniques were developed making use of beams available at higher energies. Although they have been widely applied, these indirect methods have long called for theoretical validation. In this work we discuss the use of transfer and breakup reactions as astrophysical tools. We discuss the models used to analyze these reactions and the level of accuracy that can be obtained in the capture cross section. We also consider the various sources of uncertainties that need to be under control.

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1. Introduction

There have been many advances toward understanding the origin of the elements, yet many open questions remain without an answer. A number of specific capture rates are important ingredients in the puzzle. In the nucleosynthesis of light elements and the rp-process there are a number of charged particle reaction rates that need to be known with good accuracy. These of course tend to be strongly hindered in the energy window of astrophysical interest due to the Coulomb barrier. Neutron capture rates relevant for the r-process, on the other hand, are a technical challenge, specially as one moves away from the stability valley, as then one would need beam on beam experiments. For these reasons, alternative methods for measuring reactions of astrophysical interest have become an important area of research. In this contribution we focus on using transfer reactions and breakup reactions to extract capture rates for astrophysics.

With the new technical developments at radioactive beam facilities it is now possible to measure reactions involving exotic species, in complete kinematics with good statistics. In many rare isotope laboratories around the world, transfer measurements are now part of the main program (e.g. [1, 2, 3, 4, 5]). Many of the nuclei of interest are at the limits of stability and therefore have large breakup cross sections. Breakup observables can now be measured in detail thanks to the large effort in the developments of efficient detector arrays, both for charged particles and neutrons (e.g [6, 7, 8, 9, 10, 11]). Given this present scenario, transfer or breakup reactions have realistically become useful tools for extracting astrophysical information. Whilst these nuclear reactions can provide a viable experimental alternative to the direct measurements, they also require a reliable reaction model. Here we focus mainly on this aspect.

The ANC method (asymptotic normalization coefficient) was first introduced in the eighties [12] and later implemented at Texas A&M [13, 14]. It consists of the following: for the proton capture $\mathrm{A}(p,\gamma)\mathrm{B}$, the zero energy S-factor is solely determined by the asymptotic normalization coefficient (ANC) of the overlap function $\langle A|B\rangle$. On the other hand, the cross section for the peripheral transfer reactions A(a,b)B, is proportional to the square product of the ANCs of $\langle A|B\rangle$ and $\langle a|b\rangle$. As long as the ANC of $\langle a|b\rangle$ can be determined independently, then the ANC of $\langle A|B\rangle$ can be extracted from the transfer A(a,b)B thus determining the direct $\mathrm{A}(p,\gamma)\mathrm{B}$ cross section. Amongst many other applications, this method was successfully used to determined the $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ reaction rate with 10% accuracy. It was also tested experimentally with $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$: cross sections obtained from transfer were in good agreement with direct capture data [15, 16]. The method has since been expanded to include resonant states by connecting the ANC of a resonant state with its width [17]. The ANC method can now handle complex reactions where several resonances interfere with the direct capture process [2].

Transfer reactions can also be used to extract neutron capture rates. One has first to understand whether the capture rate of astrophysical interest contains an important contribution from the nuclear interior. If that is the case then both the spectroscopic factor (SF) and the ANC are needed and should be determined through independent measurements. An experimental program at Oak Ridge has been initiated to measure a series of neutron capture rates through (d,p) reactions ([3, 4, 5]). On the theoretical side there have been a number of studies aiming at validating the method [18, 19, 20]. Here we review the standard transfer theories presently used and discuss uncertainties and ambiguities associated with the models. We look at the case of $^{48}\mathrm{Ca}(d,p)^{49}\mathrm{Ca}$

in particular, and compare with the corresponding (n,γ) reaction. The ^{48}Ca$(n,\gamma)^{49}$Ca capture reaction has an impact on the s-process. Because the capture is from an incoming s-wave neutron, the reaction is not peripheral. This type of capture reactions is particularly interesting since it can provide a large array of information concerning the structure of the final nucleus, as well as the incoming scattering potential.

Another important indirect method is the so called Coulomb Dissociation method [21]. Let us consider the direct capture rate for $A+x \rightarrow B$ is needed. In the Coulomb dissociation method, one uses the reaction $B+T \rightarrow A+x+T$ instead. The composite nucleus B is excited through the virtual photons generated by a heavy target T and breaks up into $A+x$. From the Coulomb dissociation cross section, and factoring out all kinematic effects, the photo-dissociation cross section is extracted. The desired capture cross section can then be obtained from the photo-dissociation cross section using detailed balance. The method has been applied to a variety of cases including the controversial ^{7}Be$(p,\gamma)^{8}$B [6, 7]. Theoretically, there have been many efforts to describe breakup reactions accurately. This is particularly relevant for unstable beams; with breakup being such an important part of the reaction process, perturbative approaches usually fail [22, 23, 24, 25]. Breakup reactions are very peripheral, so when the reaction mechanisms are well accounted for, these data can be used to extract the ANC. In [25] the ANC for ^{8}B is obtained from RIKEN data. The corresponding ^{7}Be$(p,\gamma)^{8}$B cross section obtained from Coulomb dissociation data compares well with the direct measurement [25]. A systematic method of extracting ANCs from Coulomb dissociation and using them to extract the peripheral direct capture cross section has recently been suggested [26]. Results for ^{14}C$(n,\gamma)^{15}$C obtained from breakup measurements at intermediate energy [8] provide cross sections in very good agreement with the latest direct capture data [27]. Here we include a further discussion of this systematic approach and present a critical assessment of the errors introduced in some approximations.

In Section II we present some general considerations concerning direct capture and nuclear reactions. Transfer reactions are discussed in Section III breakup reactions in Section IV. In Section V, we conclude with final remarks.

2. General considerations on direct capture and nuclear reactions

Direct capture processes at low energy are very sensitive to barriers, Coulomb or centrifugal. Let us then consider the process $A(x,\gamma)B$. The matrix element describing the process can be written as

$$M^{x\gamma} = < \Phi_{Ax} | \hat{O} | \varphi^{(+)}_{scatt} > \tag{2.1}$$

where $\hat{O}$ is the well known electromagnetic transition operator, and φ_{scatt} is the incoming scattering wave, usually calculated with the same potential that generates the final overlap function Φ_{Ax}. If there is a barrier present, the matrix element tends to be dominated by the asymptotic region and therefore becomes essentially proportional to the ANC of the bound state. This is the case of (p,γ) reactions.

It is only for incoming s-wave neutrons that low energy direct capture can occur at short distances, because then the penetrability is much larger. In those cases, one is sensitive to the overlap function in the nuclear interior and not only to its asymptotic tail. It is then useful to have independent experiments that can pin down both the SF and the ANC.

In addition to the bound state, the transition element $M^{x\gamma}$ depends on the incoming scattering state and the nuclear interaction generating it. Ideally one could determine this interaction from measured phase shifts. In practice, we use the same form for the interaction as that for the bound state, and adjust the depths to known states in the same partial wave. Note that if no states are known, this can introduce large uncertainties.

Nuclear reactions are intrinsically sensitive to specific parts of the overlap function. Sub-Coulomb reactions occur at large distances and therefore can only probe the tail of the overlap function. These reactions are good to extract ANCs. Higher energy (d,p) reactions have significant contributions from the surface region and, when the ANC is established independently, can be used to determined the SF [18, 19, 20].

Whereas the direct capture process is straightforward to calculate, nuclear reaction calculations depend on the nuclear distortion and consequently are subject to optical model ambiguities. One also needs to investigate other mechanisms as higher order processes may need to be taken into account.

3. Transfer (d,p) reactions

Transfer reactions have been used as a spectroscopic tool since the early days of nuclear physics. The standard theory for calculating transfer cross section is the Distorted Wave Born approximation (DWBA). The DWBA amplitude for the transfer reaction $A(d,p)B$ is given by:

$$M^{dp} = < \psi_f^{(-)} \varphi_{An} | \Delta V | \varphi_{pn} \psi_i^{(+)} > \tag{3.1}$$

Here, the transition operator is written in post-form $\Delta V = V_{pn} + V_{pA} - U_{pB}$ with V_{ij} the interaction potential between i and j and U_{pB} the optical potential in the final-state. The distorted waves in the initial and final states are $\psi_i^{(+)}$ and $\psi_f^{(-)}$, and φ_{pn} is the deuteron wavefunction. φ_{An} is the neutron single-particle wave function, that depends on the single particle parameters (usually radius and diffuseness). The single particle wavefunction has well known asymptotics: $\varphi_{An}(r) \rightarrow b\, i\, \kappa\, h_l(i\kappa r)$, where l is the orbital angular momentum and κ is the wave number. The constant b is the single particle ANC. From the ratio of data to the DWBA angular distribution cross sections at the forward angle peak, experimental spectroscopic factors are standardly obtained S_{exp}.

As deuteron breakup can play an important role, it is important to go beyond the 1-step Born approximation. One way of including deuteron breakup approximately is within the adiabatic model (ADWA) [28]. Although the adiabatic approach was originally developed within the zero range approximation, finite range corrections can be effectively included as in [29]. In ADWA, the distorting potential in the entrance channel no longer fits deuteron elastic scattering, but rather connects to the nucleon optical potentials, the neutron U_{nA} and the proton U_{pA} optical potentials, generally much better known.

Implicit in the DWBA procedure is the assumptions that the many body overlap function of nucleus B relative to A has the same radial dependence as $\varphi_{An}(r)$. Then the many-body ANC C relates to the single particle normalization coefficient b through the spectroscopic factor $C^2 = SF b^2$.

Within ADWA we benchmark the use of (d,p) reactions as an indirect astrophysical tool. Our test case is ^{48}Ca$(n,\gamma)^{49}$Ca and we analyze both the ground state and the first excited state [20]. The

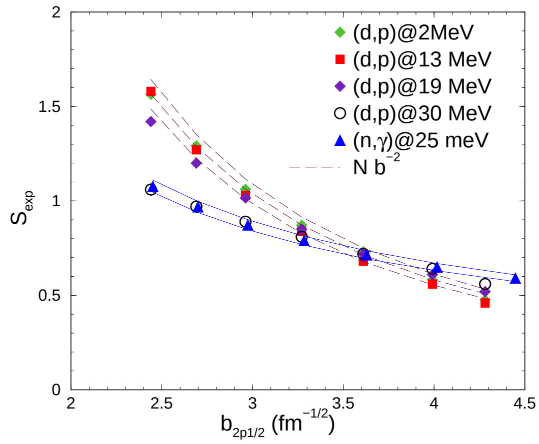

Figure 1: $S_{exp}(b)$ from ^{48}Ca$(d,p)^{49}$Ca(exc) at $E_d = 1.99$ MeV (green dots), $E_d = 13$ MeV (red squares), $E_d = 19$ MeV (purple diamonds) and $E_d = 30$ MeV (open circles) and from ^{48}Ca$(n,\gamma)^{49}$Ca(exc) at 25 meV (blue triangles). Also shown are the experimental uncertainties in the (d,p) reaction at 1.99 MeV (dashed lines) and the (n,γ) reaction (solid lines). The calculations done at $E_d = 30$ MeV are arbitrarily normalized because there is no data at this energy.

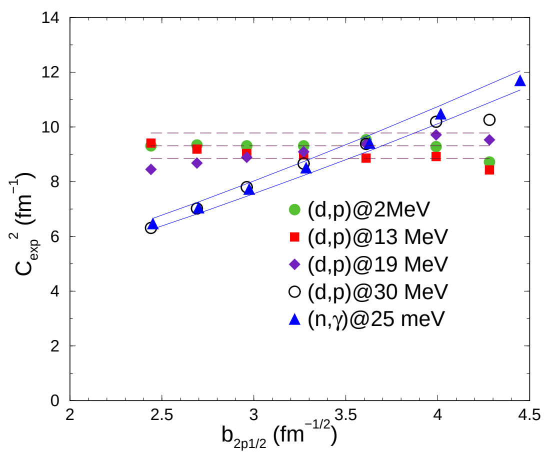

Figure 2: Same as Fig.(1) but now referring to $C^2_{exp}(b)$.

choice of ^{48}Ca is based on the very high quality data for thermal neutron capture, the well known neutron scattering length and the number of (d,p) data sets on ^{48}Ca which offer a significant test for the reaction model used.

All details for the calculations are described in [20]. Single particle parameters for the bound states in ^{49}Ca are varied, corresponding to a range of single particle ANCs (b) and for each case the transfer cross section is calculated. From the comparison to (d,p) data at 2, 13 and 19 MeV [30, 31] we extract SFs as a function of b. Results for the ground state are presented in [20]. Here we show the results for the first excited state Fig.1 and Fig.2. The ANCs are obtained from the relation $C^2 = S_{exp} b^2$. When the reaction is completely peripheral, it becomes proportional to the square of the ^{49}Ca ANC and is insensitive to the details of the overlap function in the interior. In other words, $C(b)$ is independent of the single particle parameters and the graph $C^2(b)$ is a horizontal line. This is illustrated in Fig.2 for the sub-Coulomb (d,p) reaction. The sub-Coulomb reaction is insensitive to the optical potentials, so that for this case one can determine an ANC very accurately $C^2_{p_{1/2}} = 9.30 \pm 0.93$ fm^{-1}. Contrary to our expectation, the (d,p) reactions at 13 MeV and 19.3 MeV are also rather peripheral. If we extract a SF from these data, it will suffer from large ambiguities due to the single particle parameters. Instead, an ANC can be extracted but one would need to evaluate the uncertainty due to the nucleon optical potential parameterization. Nevertheless, it is reassuring to verify that the ANCs extracted from the three (d,p) data sets are consistent.

For the same range of single particle parameters, we perform the capture calculations and extract SFs from the thermal data [32]. Using these results we are also able to reproduce the neutron capture cross section in the astrophysically relevant energy region from a few keV to about 200 keV [32, 33]. By looking at Fig.2 and the slope of $C^2(b)$ obtained from (n,γ), it becomes clear that the (n,γ) is sensitive to the nuclear interior and in conjunction with the sub-Coulomb (d,p) can be used to determine the SF rather accurately. Our result for the SF, taking into account the experimental errors of the data, is $S_{exp} = 0.71^{+0.20}_{-0.12}$.

Which energy would be appropriate for the (d,p) experiment in order to have the same level of sensitivity to the nuclear interior as the (n,γ)? We have explored this in detail and concluded that, for this particular case, $E_d = 30$ MeV would provide a similar dependence on the single particle parameters. The conclusion could change somewhat with different parameterizations of the nucleon optical potentials. Unfortunately no (d,p) data is available between 20 and 56 MeV. For the calculations at $E_d = 30$ MeV shown in Figs.1 and 2, we have used ficticious data as normalization just to illustrate the parameter dependence (data at this energy would be very useful). The angular distribution for the 56 MeV data is not well described by our model and therefore we do not include it here.

4. Breakup reactions

Coulomb dissociation has been suggested as an alternative method to extract direct capture cross sections [21]. Even though the breakup of a loosely bound light nucleus on a heavy target is dominated by the Coulomb interaction it has been proven that nuclear effects can contribute significantly [34] and should not be subtracted from the data [23]. The continuum discretized coupled channel method (CDCC) [35] provides a non-perturbative framework in which to describe Coulomb dissociation, treating Coulomb and nuclear effects on the same footing. Multipole excitations are fully taken into account as well as final state interaction effects (e.g. [34]). The idea is then to start with a single particle structure model for the projectile and adjust the parameters to obtain a good description of the breakup data. The reaction model then already takes into account nuclear and Coulomb effects in a coherent manner. That same potential model for the projectile which fits the breakup data would then provide the corresponding neutron capture cross section.

As mentioned before, when the direct capture is completely peripheral, the only information needed from the bound state is the ANC. In [26] we introduce a methodology for extracting the ANC from Coulomb dissociation data and from it calculating the capture reaction. We apply the method to ^{14}C(n,γ)^{15}C, a reaction that has been subject to long debate [36]. A number of single particle parameters are used to span a range of single particle ANCs. Results for the corresponding cross sections, calculated within CDCC, are shown in Fig.3. All details of the calculations are

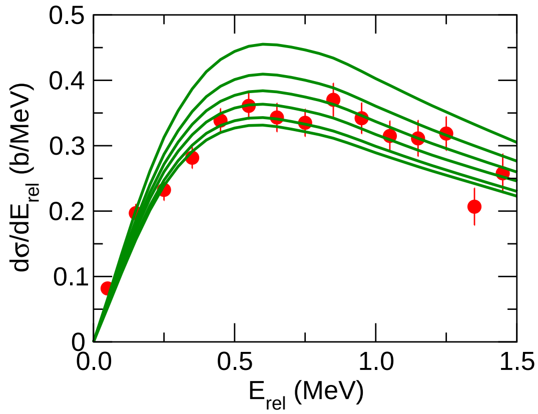

Figure 3: CDCC cross sections for the Coulomb dissociation of ^{15}C on ^{208}Pb at 68 MeV/nucleon. Data from [8].

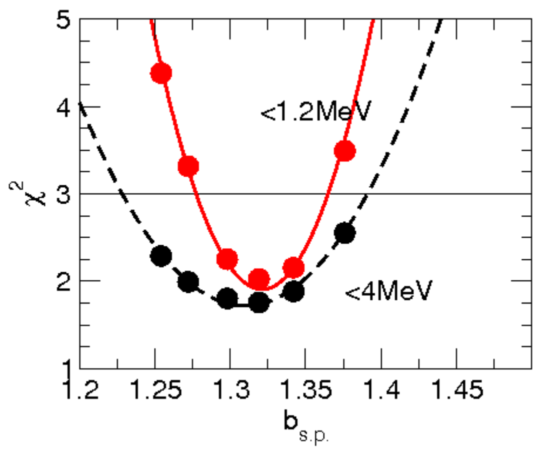

Figure 4: χ^2 per degree of freedom extracted from comparing CDCC cross sections to the Coulomb dissociation data.

presented in [26]. For each curve, a χ^2 is determined from the comparison with the RIKEN data up to 4 MeV relative energy (dashed black curve) and up to 1.2 MeV relative energy (red solid curve) [8] These χ^2 per degree of freedom are plotted in Fig.4 where a clear minimum appears. Following [37], we use this minimum to extract the ANC and from the intersection of $(\chi^2_{min}+1)$ and our χ^2 curve we extract an error bar. When including all the data up to 4 MeV relative energy, we obtain $C_0 = 1.31 \pm 0.07$ fm$^{-1/2}$. When including data up to 1.2 MeV relative energy, which is the region where the theoretical cross section is sensitive to the ANC, we obtain $C_0 = 1.32 \pm 0.04$ fm$^{-1/2}$. Additional errors may come from the systematic errors in the data, optical potentials used in the CDCC calculation and the p-wave potential used for the ^{14}C-n continuum.

With this ANC it becomes possible to determine the capture cross section. Our results show there is very good agreement between the capture determined from the Coulomb dissociation data [8] and that obtain through the direct measurement [27].

5. Concluding remarks

In conclusion, nuclear reactions, when appropriately used, can provide alternatives to direct capture experiments of astrophysical interest, which may be to difficult due to very small cross sections or the unstable nature of the target. In this contribution we focus on transfer and breakup reactions as indirect tools with particular emphasis in validating the theoretical model used to extract the astrophysical information.

A study on the consistency between ^{48}Ca$(d,p)^{49}$Ca and the corresponding (n,γ) measurement is discussed. In this case, the (n,γ) reaction has a significant contribution from the nuclear interior as there are no barriers for the incoming neutron. Unfortunately all the (d,p) reactions studied are essentially peripheral. Our calculations indicate that (d,p) data at energies higher than 20 MeV are necessary to be able to extract the spectroscopic factor from transfer data alone. Although there is a wide spread of (d,p) data available up to around 20 MeV, there is only one set of data above this energy and it is not well described by our model. A new (d,p) measurements at $E_d = 30$ MeV would be very desirable.

In this contribution we also discuss the use of Coulomb dissociation as an indirect measurement of peripheral direct capture. We show for ^{14}C$(n,\gamma)^{15}$C that the capture cross section obtained from Coulomb dissociation data at 68 MeV/nucleon is in very good agreement with direct measurements, as long as an adequate reaction theory is used to model the breakup process.

Many different theoretical aspects remain to be studied and here we mention but a few.

- An important step forward is to extend these indirect methods to study unbound states. The treatment of resonances as bound states may well be inadequate and theoretical studies are needed to better understand the implication of final states embedded in the continuum.

- So far we have assumed the projectile to be well described by a single particle state. In many cases, this is not a good approximation and we need to better understand how results may change when including mixed configurations in the projectile.

- These indirect methods are partly developed to handle (n,γ) reactions of neutron rich nuclei. The implications of particle instability, in particular the loosely bound nature of the ground state and the proximity to threshold needs to be further studied in particular in the region of the nuclear chart relevant to the r-process.

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