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Projected shell model for nuclear structure and weak interaction rates

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The knowledge on stellar weak interaction processes is one of the most important ingredients for resolving astrophysical questions. Study of these rates is essentially a nuclear structure problem, in which the actual decay rates are determined by the microscopic inside of nuclear many-body systems. For many astrophysically-interested questions, information on detailed nuclear level structure at low excitations is important. It has been suggested that the nuclear shell model, i.e. a full diagonalization of an effective Hamiltonian in a chosen model space, is the most preferable method for calculations of these rates. However, performing a shell-model calculation for heavy nuclei is itself a long-standing problem in nuclear physics. This is particularly true for deformed mass regions where the conventional shell-model method cannot be applied.

The Projected Shell Model (PSM) treats the problem in a different way. The PSM starts with a deformed single-particle basis instead of the spherical one. The many-body configurations are constructed by superimposing the angular-momentum-projected multi-quasiparticle states, and nuclear wave functions are obtained by digonalizing the two-body interactions in these projected states. Thus, it follows exactly the shell model philosophy and is a multi-major-shell shell model defined in the deformed basis.

A method for calculation of Gamow-Teller transition rates is developed in the framework of the PSM. With this method, it may become possible to perform a state-by-state calculation for β-decay and electron-capture rates in heavy, deformed nuclei at finite temperatures. A preliminary example indicates that, while experimentally known Gamow-Teller transition rates from the ground state of the parent nucleus are reproduced, stronger transitions from some low-lying excited states are predicted to occur, which may considerably enhance the total decay rates once these nuclei are exposed to hot stellar environments. Possible applications of this method are discussed.

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Except for a few nuclei lying in the vicinity of shell closures, most medium to heavy nuclei are difficult to describe in a spherical shell model framework because of the unavoidable problem of dimension explosion. Therefore, the study of nuclear structure in heavy nuclei has relied mainly on mean-field approximations, in which the concept of spontaneous symmetry breaking is applied [1]. However, there has been an increasing number of compelling evidences indicating that the nuclear many-body correlations are important. For many astrophysical problems, a nuclear shell model can generate well-defined wave functions in the laboratory frame, allowing one to compute, without further approximations as often assumed in the mean-field approaches, quantities such as transition probabilities, spectroscopic factors, and β-decay and electron-capture rates. Results of shell model calculations could strongly modify the expectation of nuclear astrophysics, as demonstrated in [2].

In the long history of shell-model development, tremendous effort has been devoted to extending the shell-model capacity from its traditional territory to heavier shells. Despite the progress made in recent years, it is impossible to treat an arbitrarily large nuclear system in a spherical shell model framework. One is thus compelled to seek judicious schemes to deal with large nuclear systems. Since most nuclei in the nuclear chart are deformed, it is natural for a shell model calculation to use a deformed basis to incorporate the physics in large systems. That is the idea that the Projected Shell Model (PSM) [3] is based on.

The PSM works with the following scheme. It begins with the deformed Nilsson single particle basis, with pairing correlations incorporated into the basis by a BCS calculation. The Nilsson-BCS calculation defines a deformed quasiparticle (qp) basis. Then angular-momentum projection is performed on the qp basis to form a shell model space in the laboratory frame. Finally a two-body Hamiltonian is diagonalized in this projected space. The PSM uses an energy-dictated shell-model truncation. It has a large single-particle space, which ensures that the collective motion and the cross-shell interplay are defined microscopically. It usually includes three (four) major harmonic-oscillator shells each for neutrons and protons in a calculation for deformed (superdeformed or superheavy) nuclei.

If $|\Phi\rangle$ is the qp vacuum and $a^\dagger_\nu$ and $a^\dagger_\pi$ the qp creation operators, with the index ν (π) denoting the neutron (proton) quantum numbers and running over selected single-qp states for each configuration, the multi-qp configurations are given for even-even and odd-odd nuclei as follows:

$$\begin{aligned}
&\text{e-e: } \{|\Phi\rangle, a^\dagger_{\nu_i}a^\dagger_{\nu_j}|\Phi\rangle, a^\dagger_{\pi_i}a^\dagger_{\pi_j}|\Phi\rangle, a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle, \cdots\},\\
&\text{o-o: } \{a^\dagger_{\nu_i}a^\dagger_{\pi_j}|\Phi\rangle, a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\pi_l}|\Phi\rangle, a^\dagger_{\nu_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle, a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\pi_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}|\Phi\rangle, \cdots\}.
\end{aligned} \tag{1}$$

The indices ν and π in (1) are general; for example, a 2-qp state can be of positive parity if both quasiparticles i and j are from major N-shells that differ by $\Delta N = 0, 2, \ldots$, or of negative parity if i and j are from N-shells differing by $\Delta N = 1, 3, \ldots$. The PSM wave-function can be written as

$$|\Psi^\sigma_{IM}\rangle = \sum_{K\kappa} f^\sigma_{IK_\kappa}\hat{P}^I_{MK}|\Phi_\kappa\rangle, \quad \text{with} \quad \hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega D^I_{MK}(\Omega)\hat{R}(\Omega), \tag{2}$$

where $|\Phi_\kappa\rangle$ denotes the qp-basis given in (1), and $\hat{P}^I_{MK}$ is the angular momentum projection operator [1]. The energies and wave functions (given in terms of the coefficients $f^\sigma_{IK_\kappa}$ in Eq. (2)) are obtained by solving the following eigen-value equation:

$$\sum_{K'\kappa'}(H^I_{K\kappa,K'\kappa'} - E^\sigma_I N^I_{K\kappa,K'\kappa'})f^\sigma_{IK_{\kappa'}} = 0 \tag{3}$$

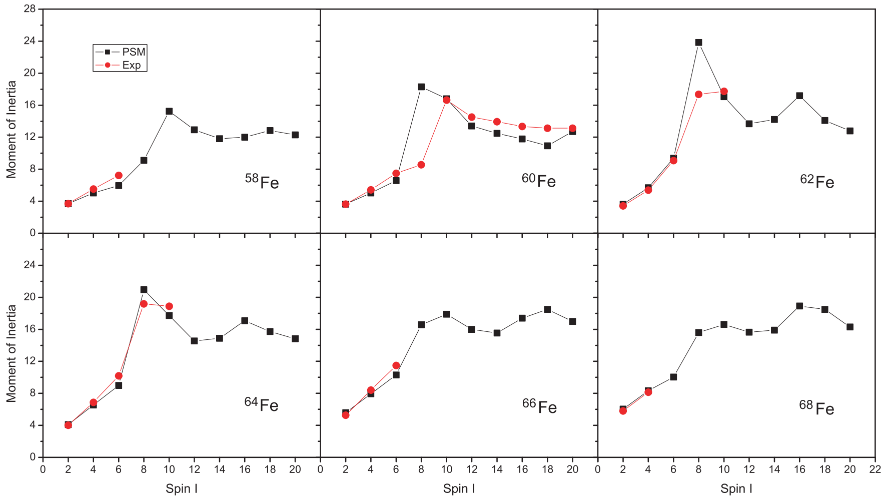

Figure 1: (Color online) Calculated moments of inertia $\mathscr{J}(I) = (2I-1)/[E(I)-E(I-2)]$ for neutron-rich Fe isotopes, compared with available data.

where $H^I_{K\kappa,K'\kappa'}$ and $N^I_{K\kappa,K'\kappa'}$ are respectively the matrix elements of the Hamiltonian and the norm.

The Hamiltonian in the present study consists of following separable forces

$$\hat{H} = \hat{H}_0 + \hat{H}_{QP} + \hat{H}_{GT}, \tag{4}$$

which represent different kinds of characteristic correlations between valence particles. The single-particle term $\hat{H}_0$ contains a set of properly adjusted single-particle energies in the Nilsson scheme [4]. The second force, $\hat{H}_{QP}$, is of the quadrupole+pairing type, and contains three terms [3]

$$\hat{H}_{QP} = -\frac{1}{2}\chi_{QQ}\sum_\mu \hat{Q}^\dagger_{2\mu}\hat{Q}_{2\mu} - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_\mu \hat{P}^\dagger_{2\mu}\hat{P}_{2\mu}. \tag{5}$$

The strength of the quadrupole-quadrupole force χ_{QQ} is determined in a self-consistent manner that it would give the empirical deformation as predicted in mean-field calculations [3]. The monopole-pairing strength is taken to be the form $G_M = [G_1 \mp G_2(N-Z)/A]/A$, where "+" ("$-$") is for protons (neutrons), and G_1 and G_2 are the coupling constants adjusted to yield the known odd-even mass differences. The quadrupole-pairing strength G_Q is taken to be about 20% of G_M, as is often assumed in the PSM calculations [3]. The last force, $\hat{H}_{GT}$ in Eq. (4), is the Gamow-Teller (GT) force

$$\hat{H}_{GT} = 2\chi_{GT}\sum_\mu \hat{\beta}^-_{1\mu}(-1)^\mu\hat{\beta}^+_{1-\mu} - 2\kappa_{GT}\sum_\mu \hat{\Gamma}^-_{1\mu}(-1)^\mu\hat{\Gamma}^+_{1-\mu}. \tag{6}$$

This is a charge-dependent interaction with both particle-hole (ph) and particle-particle (pp) channels, which act between protons and neutrons. This type of force was used by several authors [5, 6] in the study of single- and double-β decay. The pp interaction, which was introduced by Kuz'min and Soloviev [5], is a neutron-proton pairing force in the $J^\pi = 1^+$ channel, with the interaction strengths $\chi_{GT} = 23/A$ and $\kappa_{GT} = 7.5/A$ from Ref. [5]. It should be mentioned that the Hamiltonian in Eq. (4) may need to be extended when specific quantities or transition processes are studied. For example, the spin-dipole force would be necessary to reproduce first-forbidden transitions.

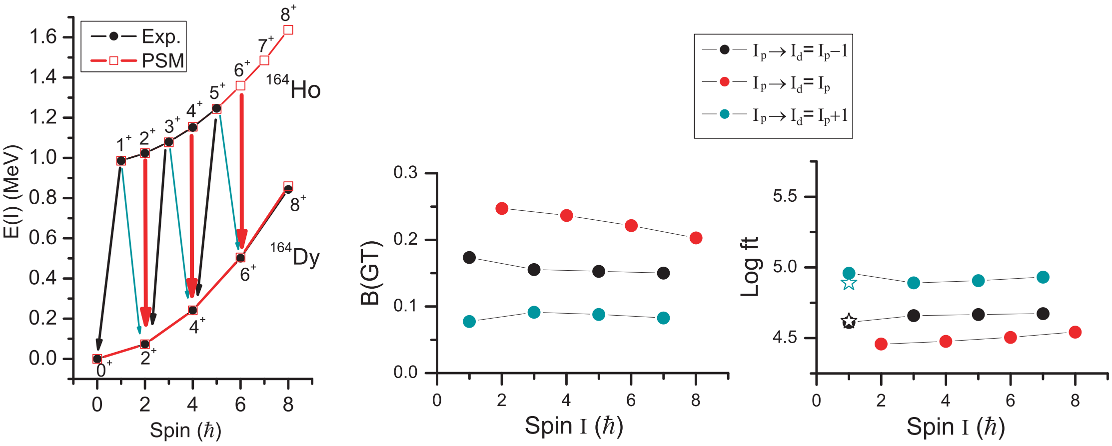

Figure 2: (Color online) (Left panel) Possible transition paths from the ground band of ^{164}Ho to that of ^{164}Dy. Experimentally known energies for these states are also displayed. (Middle panel) Calculated B(GT) and (right panel) $\log ft$ values for the ^{164}Ho $\rightarrow$ ^{164}Dy electron-capture process. Available $\log ft$ data are shown by stars, which are taken from Ref. [7]. These figures are reproduced from Ref. [8].

As an example, calculations for some neutron-rich Fe isotopes are shown in Fig. 1. Overall, the PSM results reproduce the data well. It has been known that around ^{66}Fe with the neutron sub-shell 40, the isotopes are well deformed with the neutron $g_{9/2}$ orbital lying close to the Fermi level. For such nuclei, it is thus the best applicable place for the PSM.

Within the framework of the PSM, a computer code for GT rates has recently been developed and tested [8]. Here, we present a preliminary example published in [8]. In the left panel of Fig. 2, possible allowed GT transitions from the ground band of ^{164}Ho to that of ^{164}Dy are illustrated. Here, energy levels up to about 800 keV of excitation in both parent and daughter nuclei are considered. They are $\Delta I = I_p - I_d = +1$ transitions (in black), $\Delta I = 0$ transitions (in red), and $\Delta I = -1$ transitions (in green). The calculated B(GT) values are shown in the middle panel, and the corresponding $\log ft$ values in the right panel. The experimentally measured decay probabilities are those of the $I_p = 1 \rightarrow I_d = 0$ and $I_p = 1 \rightarrow I_d = 2$ transitions, with which our calculation agrees well. The other transition probabilities associated with the excited states are our prediction. It is very interesting to note that the decay rates with $\Delta I = 0$ (in red) are predicted to have larger B(GT) and smaller $\log ft$ values than the measured $\Delta I = \pm 1$ transitions. Such rates should be included as part of the total rate when these states are thermally populated in hot stellar environments.

We mention a few attractive features in our approach for studying weak interactions rates, which may be relevant for future astrophysical applications.

(1) The PSM utilizes single particle bases generated by deformed mean-field models yet carries out a shell-model diagonalization like the conventional shell model. Conceptually, the PSM bridges two important nuclear structure methods: the deformed mean-field approach and the conventional shell model, and takes the advantages of both. As a shell model, the PSM can be applied to any heavy, deformed nuclei without a size limitation. The PSM wave functions contain correlations beyond mean-field and the states are written in the laboratory frame having definite quantum numbers such as angular-momentum and parity. These are needed properties when the wave functions are employed in transition calculations.

(2) Because of the way the PSM constructs its basis, the dimension of the model space is small (usually in the range of $10^2 - 10^4$). With this size of basis, a state-by-state evaluation of GT transition rates is computationally feasible. This feature is important because in stellar environments with finite temperatures, the usual situation is that the thermal population of excited states in a parent nucleus sets up connections to many states in a daughter by the GT operator [9].

(3) The calculation of forbidden transitions involves nuclear transitions between different harmonic oscillator shells and thus requires multi-shell model spaces. Such a calculation is not feasible for most of the conventional shell models working in one-major shell bases. The PSM is a multi-shell shell model. This feature is desired particularly when forbidden transitions are dominated.

(4) Isomeric states belong to a special group of nuclear states because of their long half-lives. The existence of isomeric states in nuclei could alter significantly the elemental abundances produced in nucleosynthesis. There are cases in which an isomer of sufficiently long lifetime can change the paths of reactions taking place and lead to a different set of elemental abundances [10]. The PSM is capable of describing the detailed structure of isomeric states.

In conclusion, the method described in the present paper can be applied to various fields such as nuclear astrophysics and fundamental physics, where weak interaction processes take place in nuclear systems. In particular, one may find interesting applications to cases where a laboratory measurement for certain weak interaction rates is difficult and where the conventional shell model calculations are not feasible. Potential applications in nuclear astrophysics are calculations of β-decay rates for the r-process [11] and the rp-process [12] nucleosynthesis, and electron-capture rates for the core collapse supernova modelling [13]. In the double-β decay theory, theoretical calculations for the nuclear matrix elements are needed, for which one has relied on the Quasiparticle Random Phase Approximation [6], particularly when heavy nuclei are involved. We expect that the method presented here can make important contributions to all these studies.

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