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Nuclear Many-Body Approach for Supernova EOS: Thomas-Fermi Calculations of Non-Uniform Nuclear Matter

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An equation of state (EOS) for uniform nuclear matter is constructed at zero and finite temperatures with the variational method starting from the realistic nuclear Hamiltonian composed of the AV18 two-body potential and the UIX three-body potential. The maximum mass of the cold neutron star with this EOS is 2.2 $M_{\odot}$. Making use of uncertainty of the three-body nuclear force, adjustable parameters in the EOS are tuned so that the Thomas-Fermi calculations for β-stable nuclei reproduce the empirical data. The obtained EOS is appropriate for constructing a new nuclear EOS table for supernova simulations.

10th Symposium on Nuclei in the Cosmos
Mackinac Island, Michigan, USA
27 July – 1 August, 2008

[1] Speaker

1. Introduction

Recently the importance of the nuclear equation of state (EOS) is increasing in the study of supernova (SN) explosions and related astrophysical phenomena. At present, however, there are only few nuclear EOS's available for SN simulations. One is constructed by Lattimer and Swesty[1], and another is by Shen et al.[2]; the latter is extended recently to take into account hyperon mixing[3] and hadron-quark phase transition[4]. Since these EOS's are based on phenomenological methods, nuclear EOS's constructed with microscopic many-body theories starting from the realistic nuclear Hamiltonian are desirable.

In this study, we undertake to construct the nuclear EOS for SN simulations with the variational method. First, we construct the EOS for uniform nuclear matter at zero and finite temperatures[5]. Then, we apply the obtained EOS to the Thomas-Fermi calculation of atomic nuclei, and tune parameters in the EOS so as to reproduce the empirical data of β-stable nuclei.

2. Uniform matter at zero and finite temperatures

In this section, we calculate the energy for uniform nuclear matter at zero and finite temperatures. The nuclear Hamiltonian H is decomposed into two parts: the two-body Hamiltonian H_2 including the isoscalar part of the AV18 two-body potential, and the three-body Hamiltonian H_3 composed of the UIX three-body potential.

At zero temperature, the expectation value of H_2 per nucleon with the Jastrow-type wave function, $<H_2>/N$, is evaluated in the two-body cluster (TBC) approximation, and denoted by E_2/N. Then, E_2/N is minimized with respect to various correlation functions in the Jastrow-type wave function. In the minimization, two constraints are imposed: the extended Mayer's condition and the healing-distance condition. The latter includes an adjustable parameter, the value of which is chosen so that E_2/N is close to the result obtained with the Fermi Hypernetted Chain method by Akmal, Pandharipande and Ravenhall (APR)[6].

The contribution from the three-body force (TBF) is expressed in the form

$$\frac{E_3}{N} = \alpha \frac{\left\langle H_3^{\mathrm{R}} \right\rangle_{\mathrm{F}}}{N} + \beta \frac{\left\langle H_3^{2\pi} \right\rangle_{\mathrm{F}}}{N} + \gamma \rho^2 e^{-\delta\rho}. \tag{1}$$

Here, H_3^{R} and $H_3^{2\pi}$ are the repulsive and two-π-exchange parts of H_3, respectively. In Eq. (1), the brackets with the subscript F represent the expectation values with the Fermi-gas wave function, and ρ is the nucleon number density. The adjustable parameters, α, β, γ and δ, are determined so that the total energy $E/N = E_2/N + E_3/N$ reproduces the empirical saturation density $\rho_0 = 0.16$ fm^{-3}, saturation energy $E_0/N = -15.8$ MeV, incompressibility $K = 250$ MeV and symmetry energy $E_{\mathrm{sym}}/N = 30$ MeV. It is noted that α and β are common to symmetric nuclear matter and neutron matter, while $\gamma = 0$ for neutron matter. The obtained E/N for symmetric nuclear matter and neutron matter are in fair agreement with those by APR.

At finite temperatures, the free energy for uniform nuclear matter is calculated with the variational method proposed by Schmidt and Pandharipande [7]. In this method, the free

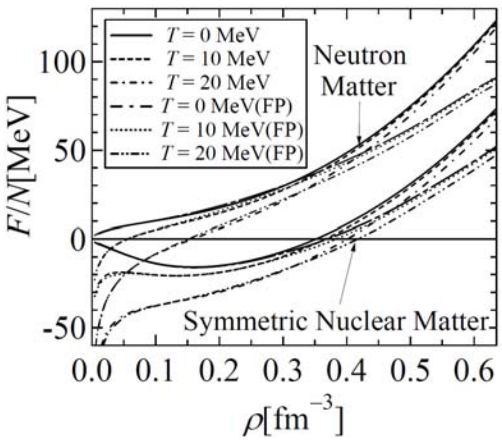

FIGURE 1. Free energies per nucleon for symmetric nuclear matter and neutron matter. The free energies obtained by FP are also shown.

energy per nucleon F/N at temperature T is given by $F/N = E_T/N - T\,S/N$. The approximate internal energy E_T/N is the sum of E_{2T}/N and E_3/N, in which E_{2T}/N is the TBC approximation of $\langle H_2\rangle/N$ with the Jastrow-type wave function at finite temperature, while E_3/N is the TBF contribution and assumed to be the same as at zero temperature. Here, the Jastrow-type wave function at finite temperature is specified by the averaged quasi-particle occupation probability $n(k)$, which includes the effective mass of the quasi-nucleon m^*. The approximate entropy S can also be written in terms of $n(k)$ as in the case of the Fermi gas. Then, the total free energy F/N is minimized with respect to m^*.

In Fig. 1, the obtained energies at zero and finite temperatures are compared with the results by Friedman and Pandharipande (FP)[8]. At high densities, the present result is higher than that by FP, which implies that the obtained EOS is stiffer than that by FP. The maximum mass of the cold neutron star obtained with the present EOS is 2.2 $M_\odot$ (See Fig. 3), and the critical temperature is about 18 MeV.

3. Thomas-Fermi calculation of β-stable nuclei

We are planning to calculate the EOS for nonuniform SN matter at low densities in the Thomas-Fermi (TF) approximation. As a preparation for these calculations, we treat atomic nuclei in the TF calculation, and tune the parameters in the EOS so as to reproduce the empirical data of β-stable nuclei.

In the simplified TF approximation used by Oyamatsu,[9] the binding energy $B(N, Z)$ of a nucleus with the proton number Z and the neutron number N is given by

$$-B(N,Z) = \int d\boldsymbol{r}\,\varepsilon(\rho_{\rm n}(r),\rho_{\rm p}(r)) + F_0 \int d\boldsymbol{r} \left|\nabla\rho(r)\right|^2 + \frac{e^2}{2}\int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{\rho_{\rm p}(r)\rho_{\rm p}(r')}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}. \quad (2)$$

The first term on the right-hand side of Eq. (2) is the bulk energy, where the energy density of uniform nuclear matter $\varepsilon(\rho_{\rm n}, \rho_{\rm p})$ for the neutron number density $\rho_{\rm n}$ and the proton number density ρ_p is obtained from E/N calculated in Section 2. The second term represents the gradient energy with $\rho = \rho_p + \rho_n$, and the third term is the Coulomb energy. The nucleon density distribution, $\rho_i(r)$ (i = p, n) is parameterized, and then $-B(N, Z)$ is minimized with respect to the parameters in $\rho_i(r)$.

The obtained masses, RMS charge radii and proton numbers of several β–stable nuclei are compared with the empirical values given in Ref. [9]. The deviation of the calculated mass M_{cal} from the smoothed empirical mass M_{emp} i.e., $\Delta M = M_{cal} - M_{emp}$ is about 10 ~ 80 MeV, while the deviations of the RMS charge radii r_{RMS} and the proton numbers of the β-stable nuclei Z_β are $|\Delta r_{RMS}| < 0.01$ fm and $|\Delta Z_\beta| \sim 1$, respectively.

In order to reduce the deviations, we tune the parameters α, β, γ, δ in Eq. (1) and F_0 in Eq. (2). After tuning, $|\Delta M| < 2$ MeV is achieved, mainly due to the slight lowering of the saturation energy, $E_0/N = -16.2$ MeV. Figure 2 shows the deviation of the calculated mass from the experimental data for mass-measured nuclei[10]. It is seen that the gross feature of the nuclear mass is well reproduced in the present TF calculation.

It is noted that the EOS is altered by the parameter tuning only slightly around the saturation density. Therefore, it has little influence on the EOS at high densities. The maximum mass of the neutron star with the parameter-tuned EOS is very close to that obtained in Section 2, as shown in Fig. 3.

4. Summary

In this paper, we constructed the EOS for uniform nuclear matter at zero and finite temperatures starting from the realistic Hamiltonian. Then, we performed the TF calculation of atomic nuclei, and tuned the parameters in the EOS so as to reproduce the empirical data of the β–stable nuclei. Using this parameter-tuned EOS, we are planning to calculate the energy for nonuniform nuclear matter with the TF approximation, toward the nuclear EOS table for SN simulations.

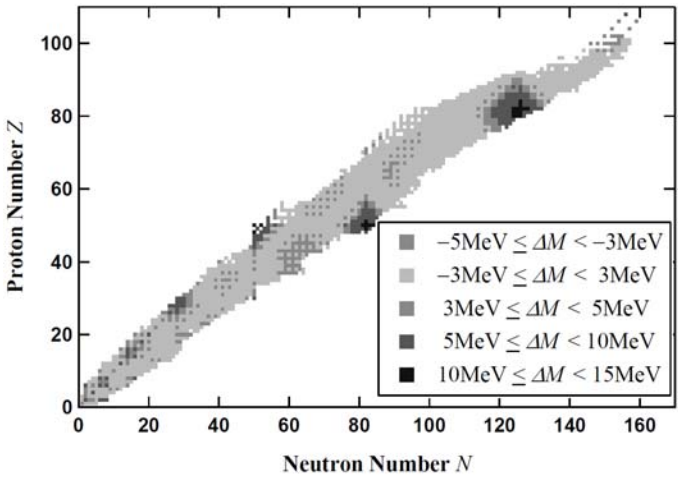

FIGURE 2. The deviation of the calculated masses from the experimental data for mass-measured nuclei. The calculated masses are obtained with the EOS after parameter tuning.

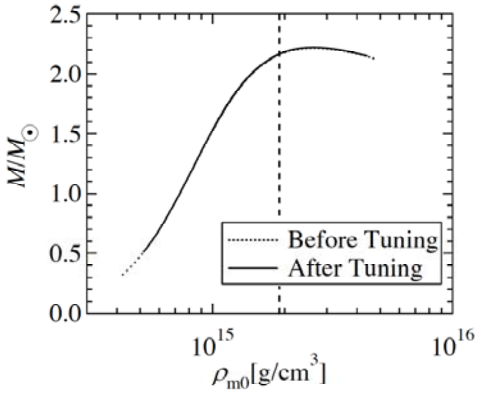

FIGURE 3. Masses of the neutron stars with the EOS's before and after tuning as functions of the central mass density. The vertical dashed line shows the critical density, above which the causality is violated.

Acknowledgments

This study is supported by Grants-in-Aid from the Scientific Research Fund of the JSPS (Nos. 18540291, 18540295 and 19540252). Some of the numerical calculations were performed with SR8000/MPP and SR11000/J2 at the Information Center of the University of Tokyo.

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