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The physics of strong magnetic fields and activity of magnetars

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A phase transition from paramagnetism to ferromagnetism in neutron star interior is explored. Since there is $^3P_2$ neutron superfluid in neutron star interior, it can be treated as a system of magnetic dipoles. Under the presence of background magnetic field, the magnetic dipoles tend to align in the same direction. Below a critical temperature, there is a phase transition from paramagnetism to ferromagnetism. And this gives a convenient explanation of the strong magnetic field of magnetars. In our point of view, there is an upper limit for the magnetic field strength of magnetars. The maximum field strength of magnetars is about $(3.0-4.0) \times 10^{15}$ G. This can be tested directly by further investigations.

Magnetars are instable due to the ultra high Fermi energy of electrons. The Landau column becomes a very long cylinder along the magnetic field, but it is very narrow and the Fermi energy of electron gas is given as $E_F(e) \approx 40(B/B_{cr})^{1/4}$ when $B \gg B_{cr}$. $E_F(e) \approx 90MeV$ When $B \sim 10^{15}$ G. Hence, the electron capture process $e^- + p \rightarrow n + \nu_e$ will be happen rapidly. Thus the $^3P_2$ Cooper pairs will be destroyed quickly by the outgoing neutrons with high energy. It will cause the isotropic superfluid disappear and then the magnetic field induced by the $^3P_2$ Cooper pairs will be also disappear. These energy will immediately be transmitted into thermal energy and then transformed into the radiation energy with X-ray - soft γ-ray. We may get a conclusion that the activity of magnetars originates from instability caused by the high Fermi energy of electrons in extra strong magnetic field.

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1. Introduction

A puzzle is what is the origin of strong magnetic field of magnetars and it is still open question, although it has been investigated by many authors.

In this paper we propose a new idea for the origin of the magnetars. The strong magnetic fields of the magnetars may originate from the induced magnetic moment of the $^3P_2$ neutron Cooper pairs in the anisotropic neutron superfluid.

2. Induced paramagnetic moment of the $^3P_2$ neutron superfluid and the Upper limit of the magnetic field of magnetars

A magnetic moment of the $^3P_2$ neutron Cooper pair is twice that of the abnormal magnetic moment of a neutron, $2\mu_n$ in magnitude, and its projection on the external magnetic field (z-direction) is $\sigma \times (2\mu_n)$, $\sigma_z = 1, 0, -1$, where μ_n is the absolute value of the magnetic moment of a neutron, $\mu_n = -0.966 \times 10^{-23}$ erg/G.

A magnetic dipole tends to align in the direction of the external magnetic field. The $^3P_2$ neutron Cooper pair has energy $\sigma_Z \times \mu_n B$ in the applied magnetic field due to the abnormal magnetic moment of the neutrons. B is the total magnetic field including the background one.

The difference of the number density of $^3P_2$ neutron Cooper pairs with paramagnetic and diamagnetic moment is

$$\Delta n_{\mp} = n_{-1} - n_{+1} = n_n(^3P_2) f\left(\frac{\mu_n B}{kT}\right) \tag{2.1}$$

$$f(x) = \frac{2\sinh(2x)}{1 + 2\cosh(2x)} \tag{2.2}$$

The Brillouin function, $f(\mu_n B/kT)$, is introduced to take into account the effect of thermal motion We note that f is an increasing function, in particular, $f(x) \approx 4x/3$, for $x \ll 1$ and $f(x) \to 1$, when $x \gg 1$. $f(\mu_n B/kT)$ increase with decreasing temperature. And this is the mathematical formula for the B-phase of the $^3P_2$ superfluid.

A relevant question is how many neutrons have been combined into the $^3P_2$ Cooper pairs? The total number of neutrons is given by $N = 2V \int_0^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{V}{3\pi^2} k_F^3$ (Here k is a wave vector). The neutrons combined into the $^3P_2$ Cooper pairs are just in a thin layer in the Fermi surface with thickness k_Δ[1], $\hbar k_\Delta = \sqrt{2m_n \Delta_n(^3P_2)}$ (We would like to emphasize that the energy gap, Δ, is the binding energy of the Cooper pair rather than a variation of the Fermi energy due to the variation of particle number density). Then we have $\delta N \approx \frac{V}{\pi^2} k_f^3 \frac{k_\Delta}{k_F} (k_\Delta << k_F)$. Thus, the fraction of the neutrons that combined into the $^3P_2$ Cooper pairs is

$$q = \frac{\delta N}{N} \approx 3\frac{k_\Delta}{k_F} \tag{2.3}$$

The fraction of the neutrons that combined into the $^3P_2$ Cooper pairs is ($p = \hbar k$)

$$q = \frac{4\pi p_F^2 [2m_n \Delta(^3P_2)]^{1/2}}{(4\pi/3)p_F^3} = 3\left[\frac{\Delta(^3P_2)}{E_F}\right]^{1/2}. \tag{2.4}$$

The Fermi energy of the neutron system is $E_F \approx 60\left(\frac{\rho}{\rho_{nuc}}\right)^{2/3}$ Mev. The energy gap of the anisotropic neutron superfluid is $\Delta(^3P_2) \sim 0.05$ MeV [2], $q \sim 8.7\%$. Thus, the total number of the

$^3P_2$ Cooper pairs is $n_n(^3P_2) \approx qN_A m(^3P_2)/2$. Therefore, the total difference of the $^3P_2$ neutron Cooper pair number with paramagnetic and diamagnetic moment is

$$\Delta N_\mp = n_n(^3P_2)f(\frac{\mu_n B}{kT}) = \frac{1}{2}N_A m(^3P_2)qf(\frac{\mu B}{kT}) \tag{2.5}$$

The total induced magnetic moment, of the anisotropic neutron superfluid is

$$\mu^{(in)}(^3P_2) = 2\mu_n \times \Delta N_\mp = \mu_n N_A m(^3P_2)qf(\mu_n B/kT) \tag{2.6}$$

Where $m(^3P_2)$ is the mass of the anisotropic neutron superfluid in the neutron star, N_A is the Avogadro constant. The induced magnetic moment is just the fully magnetized quantity $\mu_n N_A m(^3P_2)$, with two modification factors. The factor q takes into account the Fermi surface effect. While $f(\mu_n B/kT)$ is the thermal factor taking into consideration the finite temperature effect. For a dipolar magnetic field [3] $|\mu_{NS}| = B_\gamma R_{NS}^3/2$. Here B_p is the polar magnetic field strength and R_{NS} is the radius of the neutron star. The induced magnetic field is then

$$B^{(in)} = \frac{2\mu^{(in)}(^3P_2)}{R_{NS}^3} = \frac{2\mu_n N_A(^3P_2)}{R_{NS}^3}qf(\mu_n B/kT) \tag{2.7}$$

$$B_{max}^{(in)}(^3P_2) = \frac{2\mu_n N_A(^3P_2)}{R_{NS}^3}q \ G \approx 2.02 \times 10^{14}\eta \ G \tag{2.8}$$

$$\eta = \frac{m(^3P_2)}{0.1M_{sun}}R_{NS,6}^{-3}[\frac{\Delta_n(^3P_2)}{0.05MeV}]^{1/2} \tag{2.9}$$

Here η is the dimensionless factor describes both the macroscopic and microscopic properties of neutron stars.

We note that the induced magnetic field for the anisotropic neutron superfluid increases with decreasing temperature as the Brillouin function $f(\mu_n B/kT)$ which tends to 1 when the temperature decreases low enough. Actually, $f(\mu_n B/kT) \sim 1$ as long as $\mu_n B/kT \gg 1$. For example, this is true when $T \leq 10^7$K if $B = 10^{15}$ G.

There is an upper limit for the induced magnetic field of the $^3P_2$ superfluid according to eq.(2.8). It corresponds to the maximum value unity of the temperature factor $f(\mu_n B/kT)$. This upper limit can be realized when all the magnetic moments of the $^3P_2$ neutron Cooper pairs are arranged with the paramagnetic direction as the temperature become low enough.

The maximum magnetic field for magnetars depends on the total mass of the anisotropic neutron superfluid of the neutron star. The upper limit of the mass for the neutron stars is more than $2M_{sun}$[5] . It is therefore possible that the mass of the anisotropic neutron superfluid of the heaviest neutron star may be about $(1-1.5)M_{sun}$. Hence, the maximal magnetic field for the heaviest magnetar may be estimated to be $(3.0-4.0) \times 10^{15}$ G. This can be tested directly by magnetar observations.

3. From paramagnetism to ferromagnetism

We have $\mu_n B \ll kT$ when $B < 10^{13}$ G and $T > 3 \times 10^6$ K, therefore we could make Taylor series

$$B^{(in)} \approx \frac{8}{3}\frac{\mu_n N_A m(^3P_2)}{R_{NS}^3}q\frac{\mu_n B}{kT} \approx \frac{1.9}{T_7}\eta B \tag{3.1}$$

$$B = B^{(in)} + B^{(0)} \tag{3.2}$$

Where $B^{(0)}$ is the applied magnetic field which includes both the initial magnetic field of the collapsed supernova core and the induced magnetic field produced by the Pauli paramagnetization of the highly degenerate relativistic electron gas in the neutron star interior [6].

We may solve the induced magnetic field by combining eq.(3.1) and eq.(3.2):

$$B^{(in)} = (\frac{T_7}{1.9\eta} - 1)^{-1} B^{(0)} \tag{3.3}$$

The induced magnetic field, $B^{(in)}$, of the anisotropic neutron superfluid is much weaker than the applied magnetic field , $B^{(0)}$, when the temperature is very high, $T_9 \gg 1.9\eta$. However, the induced magnetic field of the anisotropic neutron superfluid would exceed the applied magnetic field when the temperature decreases down to $1 < T_7/(1.9\eta) < 2$. Therefore $B^{(in)}$ should be calculated by both eq.(2.8) and eq.(3.2) when $T_7 \sim 1.9\eta$. This belongs to the domain of ferromagnetism (see [4][7]).

In view of the above discussions we may get a very important conclusion: the strong magnetic field of magnetars may originate from the induced magnetic field by the ferromagnetic moments of the $^3P_2$ Cooper pairs of the anisotropic neutron superfluid at a moderate low temperature about 10^7K.

4. Activity of Magnetars

The core temperature of the magnetars is about 10^7 K in our model. While observations show that some SGR's and AXP's have high thermal-type-spectrum X-ray flux, being among the hottest neutron stars. We discuss this question and give reasonable and consistent explanation in this paragraph.

Energy of an electron under the super strong magnetic field is quantized. It is

$$E_e^2(p_z, B, n, \sigma) = m_e^2 c^4 + p_z^2 c^2 + (2n+1+\sigma)2m_e c^2 \mu_e B \tag{4.1}$$

(The magnetic moment of an electron is $0.92710^{-20} erg/G$) The overwhelming majority of electrons congregate in the lowest levels $n = 0$ or $n = 1...$ when $B \gg B_{cr}$. The Landau column is a very long cylinder along the magnetic field and it is very narrow.

We may find the Fermi energy of the electrons by solving the following complicated integral equation

$$N_{total} = N_A Y_e \rho = \frac{2\pi}{h^3} \int_0^{p_F} dp_z \sum_{\sigma=\pm 1} \int \delta((p_\perp/m_e c) - [(2n+1+\sigma)b]^{1/2}) p_\perp dp_\perp \tag{4.2}$$

$$n_{max}(p_z, b, \sigma) = Int\{\frac{1}{2b}[(\frac{E_F}{m_e c^2})^2 - 1 - (\frac{p_z}{m_e c})^2] - 1 - \sigma\} \tag{4.3}$$

$$b = B/B_{cr} \tag{4.4}$$

N_{total} is the total occupied number of electrons in a unit volume and Int{...} is the integer function. After some calculation, we may get the approximation expression

$$E_F(e) \approx 40(B/B_{cr})^{1/4} \qquad \text{when } B \gg B_{cr} \tag{4.5}$$

We have $E_F(e) \approx 90 MeV$, when B $\sim 10^{15} G$.

Magnetars are instable due to the ultra high Fermi energy of electrons. Hence, the electron capture process $e^- + p \rightarrow n + \nu_e$ will be happen rapidly as long as the Fermi energy of electrons is much greater than 60 Mev which is the Fermi energy of neutrons. Energy of the resulting neutrons will be rather high and they will react with the neutrons in the $^3P_2$ Cooper pairs and thus the $^3P_2$ Cooper pairs will be destroyed quickly by the process $n + (n \uparrow, n \uparrow) \rightarrow n + n + n$. It will cause the isotropic superfluid disappear and then the magnetic field induced by the $^3P_2$ Cooper pairs will be also disappear.

The remaining energy per outgoing neutron after the process above will be approximately

$$\bar{\varepsilon} \approx \frac{1}{3}[E_F(e) + E_F(p) - E_F(n) - (m_n - m_p - M_e)c^2] \tag{4.6}$$

These energy will immediately be transmitted into thermal energy and then transformed into the radiation energy with X-ray - soft γ-ray.

The total released energy may be approximately estimated as

$$E^{(tot)} \approx \bar{\varepsilon} \times n(^3P_2) = 3\bar{\varepsilon} \times qN_A m(^3P_2) \approx 2.0 \times 10^{51}[(\frac{B}{B_{cr}})^{1/4} - 1.5]\frac{m(^3P_2)}{0.1 M_{sun}} ergs \tag{4.7}$$

The x-ray Luminosity of AXPs is $L_x \approx 10^{34} - 10^{36}\ erg/sec$. It will be enough to maintain the luminosity of AXPs over 10^8yr.

We may get a conclusion that the activity of magnetars originates from instability caused by the high Fermi energy of electrons in extra strong magnetic field. However, we have to calculate rate of the electron capture process for given magnetic field to see if our idea is reasonable for the x-ray luminosity of AXPs and their surface temperature. This is our next work.

Acknowledgments

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