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Neutrino effects in cosmology with A primordial magnetic field

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If a primordial magnetic field was present during photon decoupling and afterward, a finite neutrino mass could affect the Cosmic Microwave Background (CMB). In previous work [1], we studied the effect of a neutrino mass on the vector and tensor modes of the CMB power spectra. In this work, we also calculated the scalar mode and found that a neutrino mass also has a large effect on it. We decomposed the scalar, vector and tensor modes into Integrated Sachs-Wolfe (ISW), polarization (Pol), doppler (Dop) and Sachs-Wolfe (SW) components, and studied which components are affected strongly by the neutrino mass. We found that a cancellation among neutrino effects on these components offsets each other and the total effect becomes small.

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1. Introduction

The primordial magnetic field is presumed to have existed in the early universe as a source of the magnetic field in clusters of galaxies today [2, 3]. Although some astrophysical processes, such as the dynamo mechanism, can generate the magnetic field on small scales, a primordial magnetic field can explain the origin of the large scale magnetic field. A primordial magnetic field could affect the CMB spectrum in two ways: 1) Baryons are affected by the Lorentz force and then influence the CMB spectrum indirectly through Thomson scattering, 2) The anisotropic stress of the magnetic field sources a shear as shown in Ref. [5, 1].

In our previous work [1], we examined the neutrino effect on the CMB vector and tensor mode in the presence of a primordial magnetic field. We showed that a finite neutrino mass enhances the effective wave number which increases the CMB power spectrum at lower multipoles $\ell < \ell_{m_\nu}$. This arises from the cancellation of the anisotropic stresses between the magnetic field and the massive neutrinos. In this work, we calculate the scalar mode in a similiar way and decompose the scalar, vector and tensor modes into ISW, Pol, Dop and SW components. This decomposition is important to study the details of the neutrino mass effect.

2. Equations and Initial conditions

To begin, we choose metric perturbations in terms of the scale factor a,

$$\delta g_{ij} = 2a^2\left(H_L^{(0)}Q^{(0)}\gamma_{ij} + \sum_m H_T^{(m)}Q_{ij}^{(m)}\right) , \tag{2.1}$$

where $m=0$ is the scalar mode, $m=1$ is the vector mode and $m=2$ is the tensor mode. Here, $Q^{(0)}$ and $Q_{ij}^{(m)}$s are the harmonic modes for the scalar, vector and tensor components. $H_L^{(0)}$ and $H_T^{(m)}$s are synchronous metric perturbations. The scalar mode has two parameters $H_L^{(0)}$ and $H_T^{(0)}$, although the vector and tensor modes have only transverse perturbations.

We write the distribution functions for neutrinos as

$$f_h(q,n^i,k^i,\tau) = \bar{f}(q) + \delta f_h(q,n^i,k^i,\tau) \equiv \bar{f}(1+\Theta_h) \ , \tag{2.2}$$

where δf_h is the perturbed part, q is the comoving momentum, n^i is its direction, τ is the conformal time, and Θ_h is the normalized perturbation. The subscript h means hot dark matter (i.e. massive neutrinos). We use the subscript ν to denote massless neutrinos, i.e. X_ν. In order to calculate the CMB power spectrum, we need the linearized Boltzmann equation for massive particles [1]. Expanding the perturbation in spherical harmonics as $\Theta_h = \sum_{\ell,m}(-i)^\ell\sqrt{4\pi/(2\ell+1)}Y_\ell^m\Theta_{h\ell}^{(m)}$, we obtain the Boltzmann hierarchial equations:

$$\Theta_{h\ell}^{(m)\prime} = \frac{qk}{\varepsilon}\left(\frac{\sqrt{\ell^2-m^2}}{2\ell-1}\Theta_{h\ell-1}^{(m)} - \frac{\sqrt{(\ell+1)^2-m^2}}{2\ell+3}\Theta_{h\ell+1}^{(m)}\right) + S_{h\ell}^{(m)} \ , \tag{2.3}$$

$$S_{h0}^{(0)} = H_L^{(0)\prime}\frac{\partial\ln\bar{f}}{\partial\ln q} \ , \ S_{h2}^{(0)} = \frac{2}{3}H_T^{(0)\prime}\frac{\partial\ln\bar{f}}{\partial\ln q} \ , \ S_{h2}^{(1)} = \frac{1}{\sqrt{3}}H_T^{(1)\prime}\frac{\partial\ln\bar{f}}{\partial\ln q} \ , \ S_{h2}^{(2)} = H_T^{(2)\prime}\frac{\partial\ln\bar{f}}{\partial\ln q} \ , \tag{2.4}$$

where a prime denotes the derivative with respect to the conformal time τ. The evolution of the perturbation variables is given by the Einstein equations [4] to be:

$$H_T^{(m)\prime\prime} + 2\mathscr{H}H_T^{(m)\prime} + k^2\tau^2 S_E^{(m)} = 8\pi G a^2(\rho_\nu\pi_\nu^{(m)} + \rho_\gamma\pi_{\rm B}^{(m)}) \ , \tag{2.5}$$

where π_ν is the neutrino anisotropic stress normalized by its energy density ρ_ν and $\pi_{\rm B}$ is the anisotropic stress of the magnetic field normalized by the photon energy density ρ_γ. Here $S_E^{(0)} = -H_L^{(0)} - \frac{1}{3}H_T^{(0)}$, $S_E^{(1)} = 0$ and $S_E^{(2)} = H_T^{(2)}$. On superhorizon scales, $k\tau \ll 1$, the $S_E^{(m)}$ terms can be neglected.

In order to obtain the power spectrum of the CMB and its polarization, we need to expand the temperature perturbation Θ with spherical harmonics, and expand the polarization fluctuation $Q \pm iU$ with spin-2 harmonics. The expansion coefficients are $\Theta_\ell^{(m)}$, and $E_\ell^{(m)} \pm iB_\ell^{(m)}$ respectively. Decomposing the integral solutions for Θ into the polarization (Pol) effect, the Integrated Sachs-Wolfe (ISW) effect, the doppler (Dop) effect, and the Sachs-Wolfe (SW) effect, the Θ's are given as follows,

$$\begin{aligned}\frac{\Theta_\ell^{(0)}(\tau_0,k)}{2\ell+1} = \int_0^{\tau_0} d\tau e^{-\tau_c}\Big(&\tau_c{}'(\Theta_0^{(0)} - H_T^{(0)\prime\prime}/k^2)j_\ell^{(00)} && \text{(SW)}\\ &+\tau_c{}'(v_b^{(0)} - H_T^{(0)\prime}/k)j_\ell^{(10)} && \text{(Dop)}\\ &-(H_T^{(0)\prime\prime\prime}/k^2 + H_T^{(0)\prime}/3 + H_L^{(0)\prime})j_\ell^{(00)} && \text{(ISW)}\\ &+\tau_c{}'P^{(0)}j_\ell^{(20)}\Big)\,, && \text{(Pol)}\end{aligned} \tag{2.6}$$

$$\begin{aligned}\frac{\Theta_\ell^{(1)}(\tau_0,k)}{2\ell+1} = \int_0^{\tau_0} d\tau e^{-\tau_c}\Big(&\tau_c{}'(v_b^{(1)} - H_T^{(1)\prime}/k)j_\ell^{(11)} && \text{(Dop)}\\ &-(H_T^{(1)\prime\prime}/k)j_\ell^{(11)} && \text{(ISW)}\\ &+\tau_c{}'P^{(1)}j_\ell^{(21)}\Big)\,, && \text{(Pol)}\end{aligned} \tag{2.7}$$

$$\begin{aligned}\frac{\Theta_\ell^{(2)}(\tau_0,k)}{2\ell+1} = \int_0^{\tau_0} d\tau e^{-\tau_c}(&-H_T^{(2)\prime}j_\ell^{(22)} && \text{(ISW)}\\ &+\tau_c{}'P^{(2)}j_\ell^{(22)})\,, && \text{(Pol)}\end{aligned} \tag{2.8}$$

where τ_0 is the present conformal time, while the radial temperature function $j_\ell^{(\ell' m)}(x)$, the radial E function $\varepsilon_\ell^{(m)}(x)$ and the radial B function $\beta_\ell^{(m)}(x)$ are evaluated at $x = k(\tau_0 - \tau)$. Here, we have used the anisotropic scattering source $P^{(m)} \equiv (\Theta_2^{(m)} - \sqrt{6}E_2^{(m)})/10$ and denote the baryon velocity by $v_b^{(m)}$. We defined the optical depth as $\tau_c(\tau) \equiv \int_\tau^{\tau_0} a n_e \sigma_T d\tilde{\tau}$, where n_e is the number density of free electrons and σ_T is the Thomson cross section. From Eqs. (2.6) - (2.8), we can then derive the CMB power spectrum of temperature anisotropies.

Using the perturbed Einsteins and Boltzmann equations, we can derive initial conditions for massless neutrinos as follows [5, 6]:

$$\pi_\nu^{(m)} = -\pi_{\rm B}^{(m)}\frac{R_\gamma}{R_\nu}(1 - \pi_2^{(m)}k^2\tau^2) \tag{2.9}$$

where $\pi_2^{(1)} = \frac{45}{14(4R_\nu+15)}$ and $\pi_2^{(2)} = \frac{15}{14(4R_\nu+15)}$. For the scalar mode, we need to study the matter contributions carefully and obtain $\pi_2^{(0)} = -\frac{1}{42}\frac{R_\gamma}{R_\nu}\frac{14R_\nu\Delta_{\rm B}/\pi_{\rm B}^{(0)} - 55}{4R_\nu+5}$ as shown in Ref. [6], where $\Delta_{\rm B}$ is the energy density of the magnetic field normalized by the photon energy density.

If neutrinos are massive, the initial conditions are changed. As shown in [1], initial conditions for massive neutrinos become:

$$\rho_h\pi_h^{(m)} \simeq \rho_\nu\pi_\nu^{(m)}(1 - \frac{1}{2}\frac{5}{7\pi^2}H_0^2\Omega_R m_\nu^2\tau^2)\ . \tag{2.10}$$

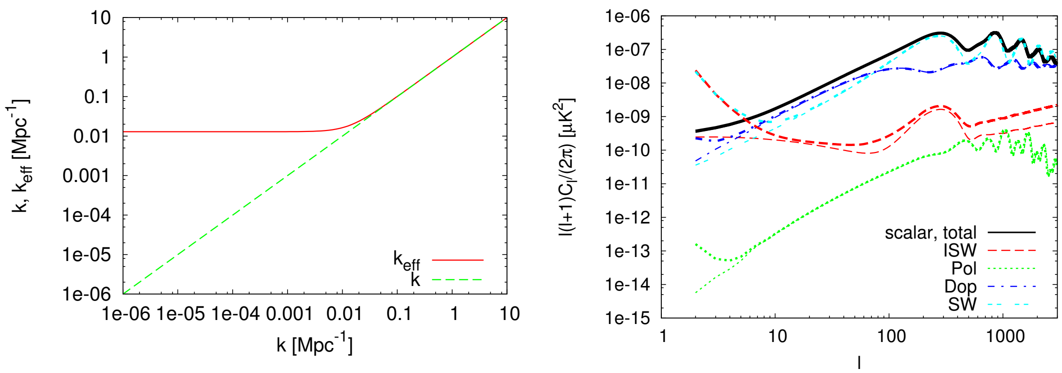

Figure 1: Left: The effective wave number $k_{\rm eff}$ with $\pi_2^{(m)} = 1$. Right: The scalar type CMB spectrum from a primodial magnetic field. The thin lines are for the massless case, and the thick lines are for the massive case.

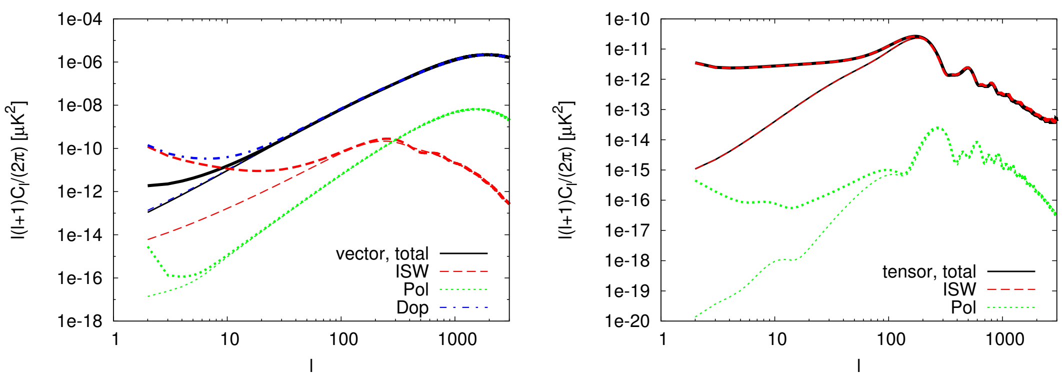

Figure 2: Vector (left) and tensor (right) type CMB spectra from a primodial magnetic field. The thin lines are for the massless case, while the thick lines are for the massive case.

We here define the effective wave number $k_{\rm eff}^{(m)}$ by

$$k_{\rm eff}^{(m)2} = k^2 + k_{m_\nu}^{(m)2} \quad , \; k_{m_\nu}^{(m)} = \sqrt{\frac{1}{2}\frac{5}{7\pi^2} H_0^2 \Omega_R m_\nu^2 / \pi_2^{(m)}} \quad . \tag{2.11}$$

Replacing k with $k_{\rm eff}$ in Eq. (2.9), we obtain similar initial conditions for massive neutrinos. On small scales, $k > k_{\rm eff}$, the neutrino mass effect is negligible. However, if k is smaller than $k_{\rm eff}$, a neutrino mass changes the time evolution of the perturbation dramatically. In the CMB spectrum, this effect is clear for $k < k_{m_\nu}^{(m)}$, i.e. $\ell < \ell_{m_\nu}^{(m)}$ where $\ell_{m_\nu}^{(m)} \equiv k_{m_\nu}^{(m)} \tau_0$. The exact values of $k_{m_\nu}^{(m)}$ and $\ell_{m_\nu}^{(m)}$ are:

$$k_{m_\nu}^{(m)} \sim 1.3 \times \frac{m_\nu}{\rm eV} \times \sqrt{1/\pi_2^{(m)}} \times 10^{-2}\ {\rm Mpc}^{-1} \quad , \; \ell_{m_\nu}^{(m)} \sim 180 \times \frac{m_\nu}{\rm eV} \times \sqrt{1/\pi_2^{(m)}} \ . \tag{2.12}$$

3. Calculation and Discussion

We have modified the CAMB code [7] to calculate neutrino mass effects in two cases, $\sum m_\nu = 0{\rm eV}$ and $\sum m_\nu = 1{\rm eV}$ using the best-fit parameters from the WMAP-5yr analysis [8]. The spectrum used in this calculation is the same as in Ref. [9] with a spectral index $n_B = -2.5$ and an amplitude $B_\lambda = 1$nG. Figs. 1 and 2 are the results.

If the neutrino mass is finite, all modes are enhanced at lower multipoles. In the scalar mode, the ISW and SW effects are enhanced, but they cancel each other. The reason for the increase of the ISW and SW effects is the increased effective wave number. Similarly, in the vector mode, the ISW and Dop effects become large at lower ℓ, though the total effect is small. The tensor mode has ISW and Pol components. The increase of the ISW effect directly enhances the total spectrum. The multipole $\ell_{m_\nu}^{(m)}$ is $\ell_{m_\nu}^{(0)} = 103$, $\ell_{m_\nu}^{(1)} = 136$ and $\ell_{m_\nu}^{(2)} = 233$ for $\sum m_\nu = 1$eV. These scales are shown on Fig. 1.

We note that the Pol effect becomes very large at $\ell \sim 2$. This is because of the radial temperature functions $j_\ell^{(2m)}$ shown in Ref. [4]. These radial functions behave as $j_\ell^{(2m)}(x) \propto x^{\ell-2}$ at $x \sim 0$. Then the Pol component of the CMB power spectrum $C_{\ell,Pol}^{TT(m)}$ is calculated as

$$(2l+1)^2 C_{\ell,Pol}^{TT(m)} \propto \int dk k^{2n_B+5} k_{m_\nu}^4 j_\ell^{(2m)2}(k(\eta_0 - \eta_{rec})) \propto \int dk k^{2(n_B+3)+2\ell-5} \quad . \tag{3.1}$$

If the spectrum index is nearly scale invariant, $n_B \sim -3$, this integral has a logarithmic infrared divergence for the quadrapole $\ell = 2$ term, although it is regular for the higher multipoles $\ell \geq 3$.

4. Summary

We studied a neutrino mass effect on the CMB spectra generated by the primordial magnetic field. If neutrinos are massive, the effective wave number becomes large because of the compensation mechanism of the anisotropic stress between massive neutrinos and the magnetic field. This new effect changes the specra in large scales, $\ell < \ell_{m_\nu}^{(m)}$. We found that there is large neutrino mass effect on the ISW and SW effects in the scalar mode, the ISW and Dop effects in the vector mode, and the ISW component in the tensor mode. However, these large effects cancel one another and the total effect becomes small, except for the tensor mode. As a result, the neutrino mass effect become large in tensor TT mode.

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