

Microlensing in binary systems

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We present an easy way to simulate and fit light curves of microlensing events for the particular case of binary stellar systems with occultation, using semi-automatic graphical interfaces written in IDL language. Gravitational microlensing effects are known to induce light amplifications of the background source and, until now, photometry has been the main technique used to study this phenomenon. A good software, capable to fit multiple parameters of the observed light curve of the microlens event, would allow astronomers to obtain important informations about the physical characteristics of the objects studied. The developpement of such a software is focused to some peculiar case, briefly treated in Sec. 1. The software, written following the parametrization described in Sec. 2, is presented in Sec. 3 and 4.

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1. Introduction

Within the wide field of gravitational lensing, two situations are particularly interesting: simple microlensing events and binary systems with occultation. In the first situation, a foreground point-like deflector acts as a lens on a background source. The expression of the light curve only depends on a few parameters (see Sec. 2), and its simulation and fit do not present any particularly difficult aspect. The second situation is somewhat more problematic. When two stars come into close alignment on the sky, the background star may be either eclipsed or microlensed by the foreground star. As the stars get aligned, if the angular size of the foreground star is larger than its gravitational deflection angle, then the foreground star can eclipse the background star; if the contrary is true, then it can magnify it. The microlens effect in this case may be quite appreciable for binaries including one or two degenerated stars (WD, BH, NS)[2]. Furthermore, the microlens effect in a binary system is the only case where the gravitational lens action may lead to well reproductive effects. The expression of the light curve in this situation is more complex than in the simple microlensing event, and depends also on the angular sizes of the two components. Secondary effects (limb darkening) may also be considered.

2. Parametrization

The parameters that describe the light curves in the different situations of Sec. 1 are generally normalized to the angular radius θ_E of the Einstein Ring:

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ds}}{D_{od}D_{os}}}, \quad (2.1)$$

where D_{ds} , D_{od} and D_{os} are respectively the distance deflector–source, observer–deflector and observer–source; M is the mass of the deflector; G is the constant of gravitation and c is the velocity of light. In the situation of simple microlensing event, illustrated in Fig. 1(a), the angular separation between the two images is

$$\Delta\theta = \sqrt{\theta_S^2 + 4\theta_E^2}, \quad (2.2)$$

and the total amplification due to the lens effect is

$$\mu_s = \frac{1}{2} \left(\frac{\Delta\theta}{\theta_S} + \frac{\theta_S}{\Delta\theta} \right) = \frac{1}{2} \left(\frac{\sqrt{\theta_S^2 + 4\theta_E^2}}{\theta_S} + \frac{\theta_S}{\sqrt{\theta_S^2 + 4\theta_E^2}} \right) = \frac{1}{2} \left(\sqrt{1 + \frac{4}{u^2}} + \frac{1}{\sqrt{1 + 4/u^2}} \right), \quad (2.3)$$

obtained substituting (2.2) in (2.3), and where $u = \theta_S/\theta_E$. Always referring to Fig.1(a), θ_S can be written as

$$\theta_S = \sqrt{\theta_{min}^2 + \dot{\theta}_x^2} = \sqrt{\theta_{min}^2 + \dot{\theta}_x^2(t - t_0)^2}, \quad (2.4)$$

where $\dot{\theta}_x$ is the constant relative angular speed of the source; θ_{min} the minimum angular approach; and t the time[3]. So, substituting θ_S in the expression of u , and normalizing θ_{min} and $\dot{\theta}_x$ to θ_E , we find

$$u = \frac{\theta_S}{\theta_E} = \sqrt{\Theta_{min}^2 + \dot{\Theta}_x^2(t - t_0)^2}. \quad (2.5)$$

Resuming, in the case of both a point like source and a deflector, only two parameters are involved:

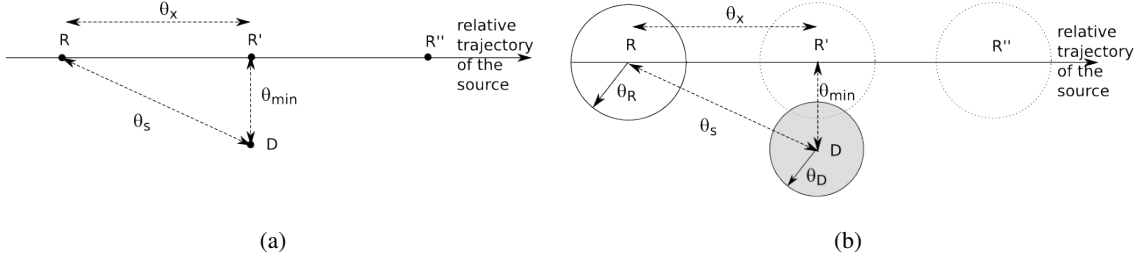


Figure 1: (a): Observer’s view of the situation “point-like source R passing behind a point-like deflector D”. (b): Observer’s view of the situation “extended background source R with an angular radius θ_R passing behind an extended deflector D with an angular radius θ_D ”. In both figures, θ_s is the angular distance between the observer and the deflector; θ_{min} is the angular distance at minimal approach and θ_x is the angular distance between source and point of minimal approach.

- Θ_{min} , the angular separation between the center of the source and the center of the deflector at the minimum distance, normalized to the Einstein’s angular radius. If $\Theta_{min} = 0$ there will be a perfect alignment.
- $\dot{\Theta}_x$, the angular speed normalized to the Einstein’s angular radius.

The global light curve used with these parameters in the simulation and fit software is that of Eq. 2.3.

In the case of binaries with occultation, illustrated in Fig. 1(b), the light curve is more complex and depends also on the size of the source and of the deflector, whose radii are respectively θ_R and θ_D . So, the following parameters are added to the previous two:

- Θ_R , the angular radius of the source, normalized to the Einstein’s angular radius;
- Θ_D , the angular radius of the deflector, normalized to the Einstein’s angular radius.

Also, there are several situations to be considered. We shall distinguish between values of Θ_R equal or greater than 0, combined with values of Θ_D smaller or greater than 1. This distinction must be operated because the total amplification of the light curve is the sum of two separated contributions of two images produced by the lens effect. One of these two images is always internal to θ_E , the other is external. So, for example, depending on the angular size of the deflector, the inner image can be occulted ($\Theta_R > 0$, $0 < \Theta_D < 1$) or totally occulted ($\Theta_R > 0$, $\Theta_D > 1$). Several global light curves used with these parameters in the simulation and fit software have already been studied in the literature [1]¹.

Then, a term describing the limb darkening is required to obtain a better representation of the phenomenon. Limb darkening leads to a larger magnification during microlensing and a larger dimming during occultation, and is introduced in the fit using the following quadratic law, where I

¹In particular, the expressions of the light curves are discussed in paragraph 3 and summarized in Tables 1 (magnification of the inner image) and 2 (magnification of the outer image).

is the intensity of light that variates with the radius θ :

$$\frac{I(\theta)}{I(0)} = 1 - \gamma_1 \left(1 - \sqrt{1 - \left(\frac{\theta}{\Theta_R} \right)^2} \right) - \gamma_2 \left(1 - \sqrt{1 - \left(\frac{\theta}{\Theta_R} \right)^2} \right)^2, \quad (2.6)$$

The limb darkening parameters in the previous expression are

- γ_1 , the first (linear) term;
- γ_2 , the second (quadratic) term.

The quadratic law was used for simplicity because of the complex dependence of the magnification with the radius. For this matter, the computation of the limb darkening effect is done numerically. The global light curve used with these parameters in the simulation and fit software is

$$\mu_{limb} = \frac{\int_0^{\Theta_R} I(\theta) (d\mu_0 \theta^2 / d(\theta)) d\theta}{\int_0^{\Theta_R} 2\theta I(\theta) d\theta} \quad (2.7)$$

where μ_0 is the light curve obtained using the parameters Θ_x , Θ_{min} , Θ_R and Θ_D .

Finally, to give more “elasticity” to the software, two other parameters are added in each of the situations presented, in order to be ready to work with curves that are not normalized in phase and/or flux:

- N , the number of photons in the unit of time. The photons arrive randomly following the Poisson statistics with an error $\sigma_N = \sqrt{N}$.
- $cent$, the position of the central peak (positive or negative) on the x axis, that represents the term $t - t_0$.

The global light curve used with these parameters in the simulation and fit software is

$$\mu(\Theta_x, \Theta_{min}, N, t - t_0) = N \cdot \mu_s(t - t_0) \quad (2.8)$$

With this set of parameters it is possible to proceed and simulate the light curves.

3. Simulating the light curves

An IDL software composed of several procedures and routines was prepared for simulations. The first procedure, simplified to reproduce the situation of simple microlensing event, allows to variate the parameters N , $cent$, Θ_{min} and Θ_x ; and finally to save an ASCII file that contains the data. For what concerns the binary system with occultation, a routine that utilizes Algol’s `microccult.pro` IDL procedure^[1] was built. If the limb darkening effect is disregarded, the software produces an exact analytical solution²; else the solution is numerical³. Because of the large number of parameters, A GUI (Graphical User Interface) has been designed as a facility to

²This procedure uses the parameters itemized in Sec. 2 except γ_1 and γ_2

³This procedure uses all the parameters itemized in Sec. 2

Parameter		Simulated data	Parameter		Fitted data
N	=	8517	N	=	8517 ± 0.103
Θ_R	=	1.0	Θ_R	=	1.038 ± 0.081
Θ_D	=	1.5	Θ_D	=	1.542 ± 0.041
<i>cent</i>	=	-2.3	<i>cent</i>	=	-2.307 ± 0.008
Θ_{min}	=	0.7	Θ_{min}	=	0.749 ± 0.124
$\dot{\Theta}_x$	=	0.7	$\dot{\Theta}_x$	=	0.746 ± 0.033

Table 1: Example of data simulated with the GUI and fitted using PAN.

easily control all the input values (see Fig. 3(c)). The GUI has two goals: the first one is to simulate the light curves and save them in a format ready to be used in the fit software (Sec. 4). So several curves can be produced varying only the noise, to test the reproducibility. The second goal is to provide an helpful tool to select them looking at the shapes of the light curves, and knowing about the variation of the shape with the parameters. This could be very helpful as the fit program needs an initial parameter guess. The usage of the GUI is very simple. The scroll “Curves” allows to choose the number of curves that will be generated and saved. The Check button “Show noise” enables or disables the visualization of the photon noise. All the curves will be always saved with the respective noise, in an ASCII format importable in the fit software described in Sec. 4. The other scrolls allow to vary the parameters itemized in Sec. 2. The check button “limb darkening” enable the numerical computation of limb darkening effect, and can be disabled to increase the speed and obtain the exact analytical solution. The check buttons “variate” perform a number of variations of the parameter chosen equal to the number of curves selected. Finally, the buttons “Call PAN” and “Generate and save!” respectively open the fit software environment and save the curves with a given name.

Fig. 2(a) shows a typical light curve where both the effects of occultation and microlensing are present. The values of the parameters are listed by the software directly on the corresponding figure. The evolution of the shape of the light curves (Figs. 2(b)–2(f)) was obtained using the “variate” check buttons of the GUI.

4. Fitting the light curves

Once that the light curves have been generated and saved, a curve-fitting software can be used to retrieve all the parameters. The tool used to perform this operation is a utility called PAN (Peak ANalysis), a general-purpose curve-fitting utility with a graphical user interface, written in IDL language by Robert Dimeo, in the framework of the DAVE software package[4] (Figs. 3(a) and 3(d)). The software can fit ASCII data to some preset functions (gaussian, lorentzian...) and provides the possibility to add custom functions; so it was improved adding the fit functions of occultation and microlensing (with and without the limb darkening parameters). Once that the ASCII file is imported in PAN, a curve from the “select” function" menu has to be chosen: `nolimb`, if the effect of limb darkening can be disregarded; `lensing`, to fit also the two limb darkening parameters (slower). A window popup (Fig. 3(b)) allows to choose the initial parameter guesses, and the “Fit current group” button performs the fit. Thanks to the GUI, if the shape of the light curve is well

characterized, it is easy to choose good parameters. For example, the presence of two lateral peaks or two lateral inflection points for a value of Θ_D greater than 1.0; Θ_x only acts on the width of the curve; if around the central minimum there is a weak peak, this means that Θ_{min} is very close to 0 or that the limb darkening effect is very low. This last point presents a degeneracy that may be solved using additional information to set limits to the value of one of the two parameters. Fig. 3 shows an example of fit and the importance of choosing good initial guesses, the simulated and fitted parameters are listed in Tab. 1

Resuming, the simulation and fit softwares offer the possibility to easily control all the initial parameters, set upper or lower limits or force some values as constants. The disadvantages to be solved concern the less performances on numerical mode (limb darkening), and some degeneracies which are sometimes difficult to solve.

References

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- [4] DAVE developpement page, <http://www.ncnr.nist.gov/dave>

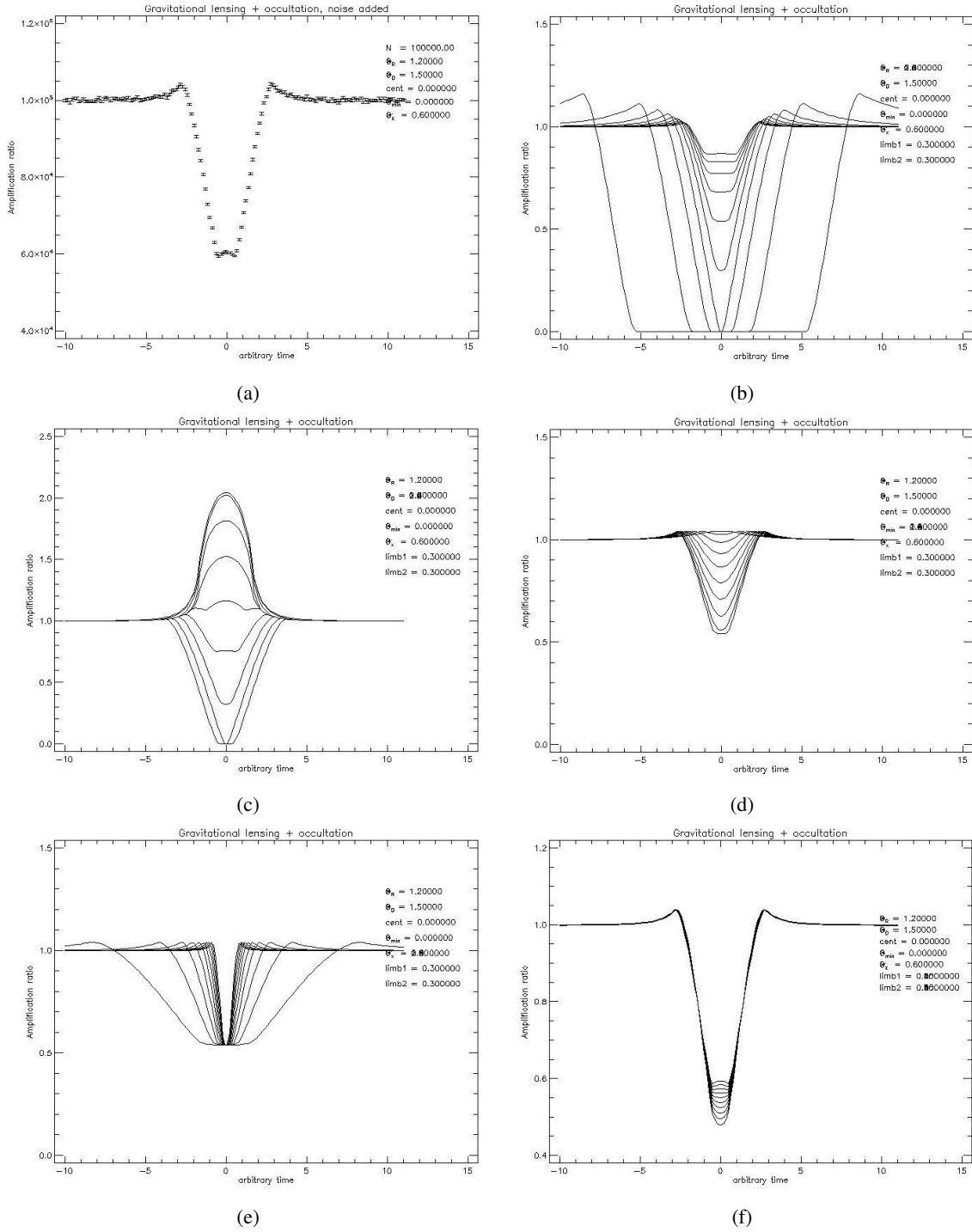
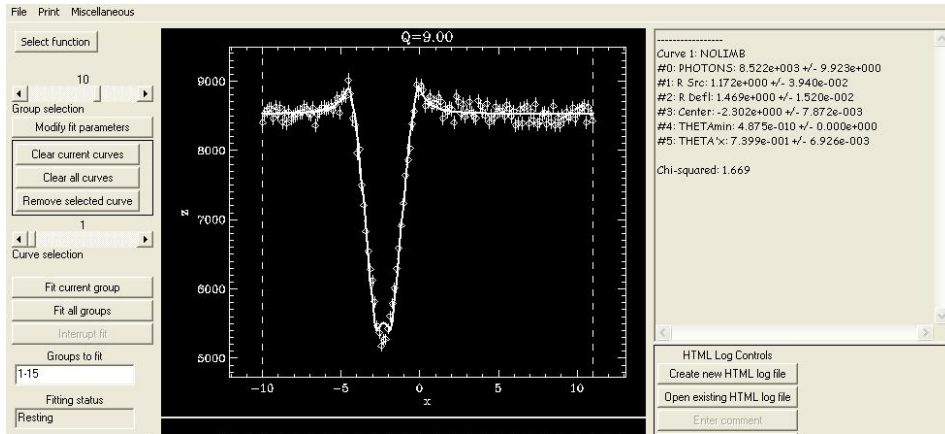
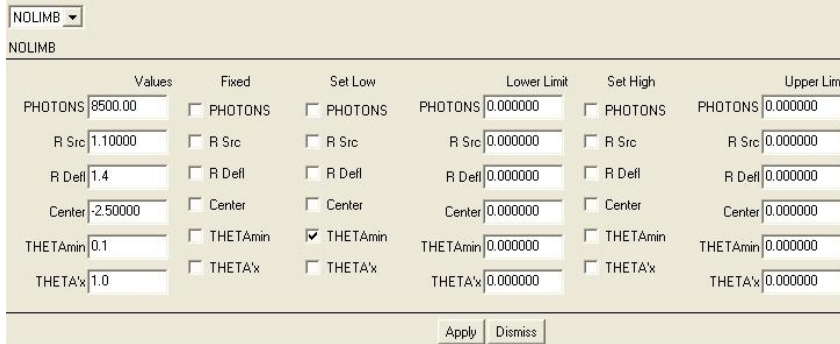


Figure 2: (a): Generated curve with an average number of $\approx 1.0 \times 10^5$ photons per unit of time, without limb darkening effects. (b) – (e): Variation of the parameters on the light curve. The parameters variate within the range 0.2–2 by steps of 0.2. In particular: (b): Increasing the value of Θ_R , the minimum grows up. (c): Increasing the value of Θ_D , the peak decreases. (d): Increasing the value of Θ_{min} , the minimum grows up. (e): Increasing the value of Θ_x , the curve becomes narrower. (f): Variation of limb darkening within the range 0.05–0.5 by steps of 0.05. Increasing the value of both $limb1$ and $limb2$, the central minimum decreases.



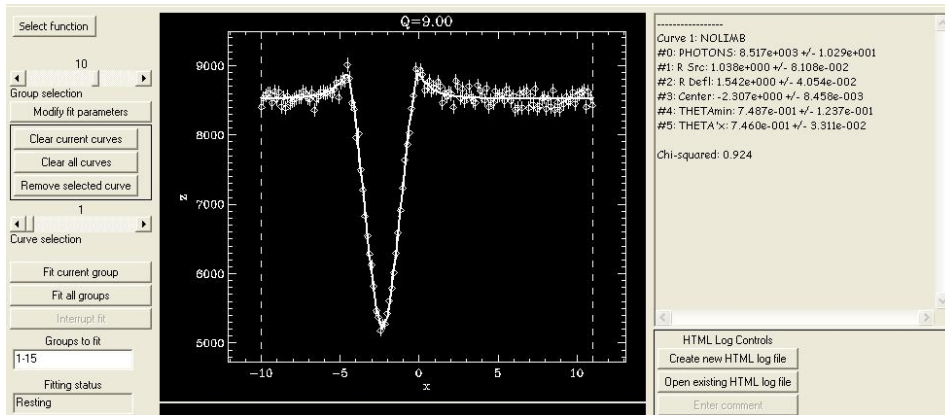
(a)



(b)



(c)



(d)

Figure 3: (a): Bad fit performed by PAN. The little central peak is absent, but the Θ_{min} value is fitted ≈ 0 . (b): Manual adjustment of the initial parameter guesses in PAN to resolve the problem in the previous figure: Θ_{min} is forced to be larger than 0, and the initial value is selected to be 0.1 (helping with the shape of the light curve produced by the GUI). (c): Graphical User Interface (GUI). (d): Good fit performed by PAN: the user “suggested” that no central peak means a value of Θ_{min} sensibly larger than 0, and the system converges to a solution visibly better and with a better χ^2 .