

## Measurement of CKM angle $\gamma/\phi_3$

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**Anton POLUEKTOV\***

*Budker Institute of Nuclear Physics, Novosibirsk*

*E-mail: A.O.Poluektov@inp.nsk.su*

This report summarizes the most recent progress in measuring the angle  $\gamma$  (or  $\phi_3$ ) of the Unitarity Triangle.

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\*Speaker.

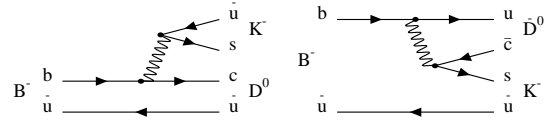
## 1. Introduction

Measurements of the Unitarity Triangle parameters allow to search for New Physics effects at low energies. Most of such measurements are currently performed at  $B$  factories — the  $e^+e^-$  machines operated with the center-of-mass energy around 10 GeV at  $\Upsilon(4S)$  resonance, which primarily decays to  $B$  meson pairs. One parameter, the angle  $\phi_1$  (or  $\beta$ )<sup>1</sup>, has been measured with high precision by BaBar experiment at PEP-II collider in SLAC [1] and Belle experiment at KEK-B machine in KEK (Japan) [2]. The measurement of the angle  $\phi_2/\alpha$  is more difficult due to theoretical uncertainties in calculation of the penguin diagram contribution. Precise determination of the third angle,  $\phi_3/\gamma$ , requires a lot more data than for the other angles, but it is theoretically clean due to the absence of loop contributions and can be used as a Standard Model reference for the searches of new physics effects. This report summarizes the recent progress in measuring the angle  $\phi_3/\gamma$ .

Two complimentary approaches are possible to extract  $\phi_3$ . As a complex phase of the  $V_{ub}$  CKM matrix element,  $\phi_3$  can be visible only in the interference of two different amplitudes. This is achieved either in the interference of final states (such as in  $B \rightarrow DK$  decays) or with neutral  $B$  decays in the interference of amplitudes with and without mixing. Here we concentrate only on the first case utilizing charged  $B$  decays, which now dominates the sensitivity.

## 2. GLW analyses

The technique of measuring  $\phi_3$  proposed by Gronau, London and Wyler [3]. GLW method makes use of  $D^0$  decays to  $CP$  eigenstates. Since both  $D^0$  and  $\bar{D}^0$  can decay into the same  $CP$  eigenstate ( $D_{CP}$ , or  $D_1$  for a  $CP$ -even state and  $D_2$  for a  $CP$ -odd state), the  $b \rightarrow c$  and  $b \rightarrow u$  amplitudes



**Figure 1:** Feynman diagrams for  $B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$ .

shown in Fig. 1 interfere in the  $B^\pm \rightarrow D_{CP} K^\pm$  decay channel. This interference may lead to direct  $CP$  violation. The observables sensitive to  $CP$  violation are the charge asymmetries  $\mathcal{A}_{1,2}$  and ratios  $\mathcal{R}_{1,2} = \mathcal{B}(B \rightarrow D_{1,2} K) / \mathcal{B}(B \rightarrow D_{\text{flavor}} K)$ . These variables can be used to extract  $\phi_3$ :

$$\mathcal{R}_{1,2} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \phi_3, \quad \mathcal{A}_{1,2} = \pm 2r_B \sin \delta_B \sin \phi_3 / \mathcal{R}_{1,2}, \quad (2.1)$$

where  $r_B \equiv |A(B^- \rightarrow \bar{D}^0 K^-) / A(B^- \rightarrow D^0 K^-)|$  is the ratio of the magnitudes of the two tree diagrams shown in Fig. 1,  $\delta_B$  is their strong-phase difference. The value of  $r_B$  is given by the ratio of the CKM matrix elements  $|V_{ub}^* V_{cs}| / |V_{cb}^* V_{us}| \sim 0.38$  and the color suppression factor. Here we assume that mixing and  $CP$  violation in the neutral  $D$  meson system can be neglected.

An alternative set of three parameters can also be used:

$$x_\pm = r_B \cos(\delta_B \pm \phi_3) = [\mathcal{R}_1(1 \mp \mathcal{A}_1) - \mathcal{R}_2(1 \mp \mathcal{A}_2)]/4, \quad \text{and} \quad r_B^2 = (\mathcal{R}_1 + \mathcal{R}_2 - 2)/2. \quad (2.2)$$

The use of these observables allows for a direct comparison with the methods involving Dalitz plot analyses of  $D^0$  (see Section 4), where the same parameters  $x_\pm$  are obtained.

Measurements of  $B \rightarrow D_{CP} K$  decays have been performed by both the BaBar and Belle collaborations. Recently, BaBar updated their GLW analysis using the data sample of 382M  $B\bar{B}$

<sup>1</sup>Two different notations of the Unitarity Triangle are used:  $\alpha$ ,  $\beta$ ,  $\gamma$  or  $\phi_2$ ,  $\phi_1$  and  $\phi_3$ , respectively. The second option (adopted by Belle collaboration) will be used throughout the paper except for the case when BaBar results are discussed.

pairs [4]. The analysis uses  $D^0$  decays to  $K^+K^-$  and  $\pi^+\pi^-$  as  $CP$ -even modes,  $K_S^0\pi^0$  and  $K_S^0\omega$  as  $CP$ -odd modes. BaBar measures the  $CP$  asymmetries to be  $\mathcal{A}_1 = +0.27 \pm 0.09 \pm 0.04$ ,  $\mathcal{A}_2 = -0.09 \pm 0.09 \pm 0.02$  and double ratios  $\mathcal{R}_1 = 1.06 \pm 0.10 \pm 0.05$ ,  $\mathcal{R}_2 = 1.03 \pm 0.10 \pm 0.05$ . The corresponding values of parameters  $x_{\pm}$  are  $x_+ = -0.09 \pm 0.05 \pm 0.02$  and  $x_- = +0.10 \pm 0.05 \pm 0.03$ . The signs of the  $\mathcal{A}_1$  and  $\mathcal{A}_2$  asymmetries should be opposite, which is confirmed by the experiment. The  $x_{\pm}$  values are in a good agreement with the ones obtained by Dalitz analysis technique.

The GLW measurement of with  $CP$ -even  $D$ -meson states ( $\pi^+\pi^-$  and  $K^+K^-$ ) is available also from CDF experiment at the Tevatron collider [5]. The result,  $\mathcal{R}_1 = 1.30 \pm 0.24 \pm 0.12$ ,  $\mathcal{A}_1 = 0.39 \pm 0.17 \pm 0.04$  is comparable by accuracy with the results from  $B$ -factories.

### 3. ADS analyses

The difficulties in the application of the GLW methods arise primarily due to the small magnitude of the  $CP$  asymmetry of the  $B^{\pm} \rightarrow D_{CP}K^{\pm}$  decay probabilities, which may lead to significant systematic uncertainties in the observation of the  $CP$  violation. An alternative approach was proposed by Atwood, Dunitz and Soni [6]. Instead of using the  $D^0$  decays to  $CP$  eigenstates, the ADS method uses Cabibbo-favored and doubly Cabibbo-suppressed decays:  $\bar{D}^0 \rightarrow K^-\pi^+$  and  $D^0 \rightarrow K^-\pi^+$ . In the decays  $B^+ \rightarrow [K^-\pi^+]_D K^+$  and  $B^- \rightarrow [K^+\pi^-]_D K^-$ , the suppressed  $B$  decay corresponds to the Cabibbo-allowed  $D^0$  decay, and vice versa. Therefore, the interfering amplitudes are of similar magnitudes, and one can expect the significant  $CP$  asymmetry.

Unfortunately, the branching ratios of the decays mentioned above are so small that they cannot be observed using the current experimental statistics. The observable that is measured in the ADS method is the fraction of the suppressed and allowed branching ratios:

$$\mathcal{R}_{ADS} = \mathcal{B}(B^{\pm} \rightarrow [K^{\mp}\pi^{\pm}]_D K^{\pm}) / \mathcal{B}(B^{\pm} \rightarrow [K^{\pm}\pi^{\mp}]_D K^{\pm}) = r_B^2 + r_D^2 + 2r_B r_D \cos \phi_3 \cos \delta, \quad (3.1)$$

where  $r_D = 0.060 \pm 0.002$  is the ratio of the doubly Cabibbo-suppressed and Cabibbo-allowed  $D^0$  decay amplitudes, and  $\delta$  is a sum of strong phase differences in  $B$  and  $D$  decays:  $\delta = \delta_B + \delta_D$ .

The update of the ADS analysis using 657M  $B\bar{B}$  pair was recently reported by Belle [7]. The ratio of the suppressed and allowed modes is  $\mathcal{R}_{ADS} = (8.0_{-5.7-2.8}^{+6.3+2.0}) \times 10^{-3}$ . Belle also reports the measurement of the  $CP$  asymmetry, which appears to be consistent with zero:  $\mathcal{A}_{ADS} = -0.13_{-0.88}^{+0.98} \pm 0.26$ . The ADS analysis currently does not give a significant constraint on  $\phi_3$ , but it provides an important information on the value of  $r_B$ . Using the conservative assumption  $\cos \phi_3 \cos \delta = -1$  one obtains the upper limit  $r_B < 0.19$  at 90% CL.

### 4. Dalitz plot analyses

A Dalitz plot analysis of a three-body final state of the  $D$  meson, such as  $K_S^0\pi^+\pi^-$ , allows one to obtain all the information required for determination of  $\phi_3$  in a single decay mode [8, 9]. Assuming no  $CP$  asymmetry in neutral  $D$  decays, the amplitude of the  $\bar{D}_+$  ( $\bar{D}_-$ ) decay from  $B^{\pm} \rightarrow DK^{\pm}$  as a function of Dalitz plot variables  $m_+^2 = m^2(K_S^0\pi^+)$  and  $m_-^2 = m^2(K_S^0\pi^-)$  is

$$f_{B^{\pm}} = f_D(m_{\pm}^2, m_{\mp}^2) + r_B e^{\pm i\phi_3 + i\delta_B} f_D(m_{\mp}^2, m_{\pm}^2), \quad (4.1)$$

where  $f_D(m_{\pm}^2, m_{\mp}^2)$  is the amplitude of the  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decay, which can be determined from a large sample of  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decays produced in continuum  $e^+e^-$  annihilation. Once  $f_D$  is

known, a fit of  $B^+$  and  $B^-$  data yields  $r_B$ ,  $\phi_3$  and  $\delta_B$ . The method has only a two-fold ambiguity:  $(\phi_3, \delta_B)$  and  $(\phi_3 + 180^\circ, \delta_B + 180^\circ)$  solutions cannot be distinguished. Both  $B$ -factory experiments reported recently the updates of the  $\phi_3(\gamma)$  measurements using Dalitz plot analysis.

The preliminary result obtained by Belle [10] uses the data sample of 657M  $B\bar{B}$  pairs and two modes,  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^*K^\pm$  with  $D^* \rightarrow D\pi^0$ . The neutral  $D$  meson is reconstructed in  $K_S^0\pi^+\pi^-$  final state in both cases. The description of the  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decay amplitude is based on the isobar model and is extracted from  $D$ -mesons produced in  $e^+e^- \rightarrow c\bar{c}$  continuum process. The model includes 18 two-body states: five Cabibbo-allowed and five doubly Cabibbo-suppressed amplitudes with excited  $K$ -meson and a pion, and eight amplitudes with  $K_S^0$  and a  $\pi\pi$  resonance. The Dalitz distributions of the  $B^+$  and  $B^-$  samples are fitted separately, using parameters  $x_\pm = r_\pm \cos(\pm\phi_3 + \delta_B)$  and  $y_\pm = r_\pm \sin(\pm\phi_3 + \delta_B)$ . Confidence intervals in  $r_B$ ,  $\phi_3$  and  $\delta_B$  are then obtained from the  $(x_\pm, y_\pm)$  using a frequentist technique. The combination of  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^*K^\pm$  modes gives  $\phi_3 = 76^\circ_{-13^\circ}^{+12^\circ} \pm 4^\circ \pm 9^\circ$ ,  $r_{DK} = 0.16 \pm 0.04 \pm 0.01 \pm 0.05$ ,  $r_{D^*K} = 0.21 \pm 0.08 \pm 0.02 \pm 0.05$ . Note that in addition to the detector-related systematic error (the second error), the result suffers from the uncertainty of the  $D$  decay amplitude description (the third error). The statistical confidence level of  $CP$  violation is  $(1 - 5.5 \times 10^{-4})$ , or 3.5 standard deviations.

In contrast to Belle analysis, BaBar [11] uses a smaller data sample of 383M  $B\bar{B}$  pairs, but analyses seven different decay modes:  $B^\pm \rightarrow DK^\pm$ ,  $B^\pm \rightarrow D^*K^\pm$  with  $D^0 \rightarrow D\pi^0$  and  $D\gamma$ , and  $B^\pm \rightarrow DK^{*\pm}$ , where the neutral  $D$  meson is reconstructed in  $K_S^0\pi^+\pi^-$  and  $K_S^0K^+K^-$  (except for  $B^\pm \rightarrow DK^{*\pm}$  mode) final states. Unlike Belle treatment, the K-matrix formalism is used by default to describe the  $\pi\pi$   $S$ -wave in  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  amplitude. The description of  $\bar{D}^0 \rightarrow K_S^0K^+K^-$  amplitude uses an isobar model with eight contributions including  $a_0$ ,  $\phi$ ,  $f_0$  and  $f_2$  states. The fit to signal samples is performed similarly to Belle analysis. The combination of all modes yields  $\gamma = (76_{-24}^{+23} \pm 5 \pm 5)^\circ \pmod{180^\circ}$ . The values of the amplitude ratios are  $r_B = 0.086 \pm 0.035 \pm 0.010 \pm 0.011$  for  $B^\pm \rightarrow DK^\pm$ ,  $r_B^* = 0.135 \pm 0.051 \pm 0.011 \pm 0.005$  for  $B^\pm \rightarrow D^*K^\pm$ , and  $\kappa r_s = 0.163_{-0.105}^{+0.088} \pm 0.037 \pm 0.021$  for  $B^\pm \rightarrow DK^{*\pm}$  (here  $\kappa$  accounts for nonresonant  $B^\pm \rightarrow DK_S^0\pi^\pm$  contribution). The significance of the direct  $CP$  violation is 99.7%, or 3.0 standard deviations.

## 5. Impact of $c\tau$ -factory

The decays of  $\psi(3770)$  to  $D$ -meson pairs available at charm factory can provide important information for future  $\phi_3$  analyses. Principal limitation of the Dalitz plot analysis method of  $\phi_3$  determination is uncertainty due to  $D^0$  decay model. Model description is needed to obtain the strong phase dependence, which is unknown from decays of flavor-specific  $D^0$  states. Using quantum correlations of the  $D^0$  mesons from  $\psi(3770)$  one can obtain experimentally the unknown strong phase difference  $\Delta\delta$  between the symmetric Dalitz plot points  $(m_+^2, m_-^2)$  and  $(m_-^2, m_+^2)$ , and thus to perform model-independent measurement [8, 12]. Preliminary measurement of the parameters needed for such analysis were reported by CLEO collaboration [13]. The CLEO result should reduce the uncertainty due to unknown strong phase in  $D^0$  decay to a 1–2° level.

Another application of the charm data is ADS analysis, where  $\delta_D$  phase can be measured. CLEO measured  $\delta_D$  in  $D^0 \rightarrow K^-\pi^+$  decay to be  $(22_{-12}^{+11}{}_{-11}^{+9})^\circ$  [14]. In ADS analyses with multibody  $D^0$  decays CLEO can measure the unknown coherence factor that enters the expression for  $\mathcal{R}_{ADS}$  which accounts for the amplitude structure.

## 6. Conclusion

Recently, many new measurements related to determination of  $\phi_3/\gamma$  have appeared. As a result, strong evidence of a direct  $CP$  violation in  $B^\pm \rightarrow DK^\pm$  decays is obtained for the first time in a combination of B-factories results. The world average  $\phi_3$  results that include the latest measurements presented in 2008, are available from UTFit and CKMfitter groups [15]. Their world average values range from  $\phi_3/\gamma = (78 \pm 12)^\circ$  (UTFit) to  $\phi_3/\gamma = (70_{-29}^{+27})^\circ$  (CKMfitter), the difference being due to variations of the statistical procedure used for averaging. Essential is the fact that the value of  $r_B$  is shown to be significantly non-zero. In previous measurements, poor  $r_B$  constraint made it difficult to predict the future sensitivity of  $\phi_3$ . Now that  $r_B$  is constrained to be of the order 0.1, one can confidently extrapolate the current precision to future measurements at LHCb and Super-B facilities.

Current world average is dominated by the measurements based on Dalitz plot analyses of  $D$  decay from  $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$  processes. Although these analyses currently include a hard-to-control uncertainty due to the  $D$  decay model, there are ways of dealing with this problem using charm data samples from CLEO-c and BES-III facilities, that should allow for a degree-level precision of  $\phi_3/\gamma$  to be reached at the next generation  $B$  factories and LHCb.

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