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Perturbative QCD analysis of exclusive $e^+e^- \longrightarrow J/\psi + \eta_c$

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We analyze the exclusive charmonium $J/\psi + \eta_c$ pair production in e^+e^- annihilation using the nonfactorized perturbative QCD and the light-front quark model(LFQM) that goes beyond the peaking approximation. We effectively include all orders of higher twist terms in the leading order of QCD coupling constant and compare our nonfactorized analysis with the usual factorized analysis in the calculation of the cross section. Our nonfactorized result enhances the NRQCD result by a factor of $3 \sim 4$ at $\sqrt{s} = 10.6$ GeV.

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1. Introduction

The exclusive pair production of heavy mesons has been known to be reliably predicted within the framework of perturbative quantum chromodynamics (PQCD), since the wave function is well constrained by the nonrelativistic consideration [1]. The nonrelativistic QCD (NRQCD) [2] factorization approach [3] for charmonium production assumes that the constituents are sufficiently nonrelativistic so that the relative motion of valence quarks can be neglected inside the meson. This leads to the sharply peaked quark distribution amplitude (DA) with the shape of the δ function (the so-called peaking approximation). One of the most puzzling problems in heavy quarkonium physics is the large discrepancy between the NRQCD prediction and the experimental data of the double charm production in e^+e^- annihilation at B-factories. For instance, the exclusive production cross sections of double charmonium in $e^+e^- \to J/\psi + \eta_c$ at $\sqrt{s} = 10.6$ GeV measured by Belle [4] and BABAR [5] were larger by an order of magnitude than the theoretical predictions [3] based on the NRQCD factorization at leading order in the QCD coupling constant α_s and the heavy-quark (or antiquark) velocity v in the quarkonium rest frame. This large discrepancy between the theoretical predictions based on the NRQCD factorization approach and the experimental results for the exclusive $J/\psi + \eta_c$ production in e^+e^- annihilation has triggered the need of better understanding both in the available calculational tools and the appreciable relativistic effects.

In order to reduce the discrepancy between theory and experiment, many theoretical efforts have been made. In particular, the authors in [6] considered a rather broad quark DA instead of δ-shaped quark DA. However, as we pointed out in [7, 8], if the quark DA is not an exact δ function, i.e. $\mathbf{k}_\perp$ in the soft bound state light-front (LF) wave function can play a significant role, the factorization theorem is no longer applicable. To go beyond the peaking approximation, the invariant amplitude should be expressed in terms of the LF wave function $\Psi(x_i, \mathbf{k}_{\perp i})$ rather than the quark DA. In going beyond the peaking approximation, we stressed [7, 8] a consistency of the formulation by keeping the transverse momentum $\mathbf{k}_\perp$ both in the wave function part and the hard scattering part together before doing any integration in the amplitude. Such non-factorized analysis should be distinguished from the factorized analysis [6] where the transverse momenta are separately integrated out in the wave function part and in the hard scattering part. In this paper, we extend our previous works [7] of pseudoscalar meson pair production to the case of $e^+e^- \to J/\psi + \eta_c$ process at leading order of α_s including effectively all orders of higher twist terms. More details about our nonfactorized LF PQCD approach for charmonium production can be found in our journal publication [9].

The paper is organized as follows. In Sec. 2, we describe the formulation of our light-front quark model (LFQM), which has been quite successful in describing the static and non-static properties of the pseudoscalar and vector mesons [10, 11]. Some special features of our nonfactorized analysis for $e^+e^- \to J/\psi + \eta_c$ transition are also given in the leading order of α_s. In Sec.3, we present the numerical results for the $e^+e^- \to J/\psi + \eta_c$ cross section. Summary and conclusions follow in Sec. 4.

2. Model Description

In our LFQM [10, 11], the momentum space light-front wave function of the ground state pseudoscalar and vector mesons is given by

$$\Psi^{JJ_z}_{100}(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \mathscr{R}^{JJ_z}_{\lambda_1\lambda_2}(x_i, \mathbf{k}_{i\perp})\phi_R(x_i, \mathbf{k}_{i\perp}), \tag{2.1}$$

where $\phi_R(x_i, \mathbf{k}_{i\perp})$ and $\mathscr{R}^{JJ_z}_{\lambda_1\lambda_2}$ are the radial- and the spin-orbit wave functions, respectively. For the radial wave function ϕ_R, we use the same Gaussian wave function for both pseudoscalar and vector mesons $\phi_R(x_i, \mathbf{k}_{i\perp}) = (4\pi^{3/4}/\beta^{3/2})\sqrt{\partial k_z/\partial x}\exp(-\vec{k}^2/2\beta^2)$, where β is the variational parameter. Since the longitudinal component k_z is given by $k_z = (x - 1/2)M_0$, the Jacobian of the variable transformation $\{x, \mathbf{k}_\perp\} \to k = (\mathbf{k}_\perp, k_z)$ is obtained as $\partial k_z/\partial x = M_0/(4x_1x_2)$.

The quark distribution amplitude (DA) of a hadron can be found from the hadronic wave function by integrating out the transverse momenta of the quarks in the hadron. The quark DAs for η_c and J/ψ mesons are constrained by

$$\int_0^1 \phi_{\eta_c(J/\psi)}(x, \mu)dx = \frac{f_{\eta_c(J/\psi)}}{2\sqrt{6}}, \tag{2.2}$$

where $f_{\eta_c(J/\psi)}$ is the decay constant for $\eta_c(J/\psi)$. The explicit forms of the decay constants can be found in [9].

For the exclusive process $e^+e^- \to \gamma^*(q) \to J/\psi(P_V) + \eta_c(P_P)$, the form factor is defined as

$$\langle J/\psi(P_V, h)\eta_c(P_P)|J^\mu_{\rm em}|0\rangle = \varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu P_{V\rho}P_{P\sigma}\mathscr{F}(q^2), \tag{2.3}$$

where $\varepsilon^*_V(P_V, h)$ is the polarization vector of the vector meson with four momentum P_V and helicity h. The cross section can be calculated as

$$\sigma(e^+e^- \to J/\psi\eta_c) = \frac{\pi\alpha^2}{6}|\mathscr{F}(q^2)|^2\left(1 - \frac{4M_h^2}{s}\right)^{3/2}, \tag{2.4}$$

where we neglect the small mass difference between J/ψ and η_c, i.e. $M_h \approx M_{J/\psi} \approx M_{\eta_c}$.

To obtain the timelike form factor $\mathscr{F}(q^2)$ for the process $e^+e^- \to \gamma^* \to J/\psi\eta_c$ at leading order of α_s, we first calculate the radiative decay process $\eta_c(P) + \gamma^*(q) \to J/\psi(P')$ using the Drell-Yan-West ($q^+ = q^0 + q^3 = 0$) frame, where the four momentum transfer is spacelike ($\mathbf{q}^2_\perp = Q^2 < 0$). We then analytically continue the spacelike form factor $\mathscr{F}(Q^2)$ to the timelike region by changing Q^2 to $-q^2$ in the form factor.

In the calculations of the form factor $\mathscr{F}(q^2)$, we use the '+'-component of the current and the transverse ($h = \pm 1$) polarization for J/ψ. In the energy region where PQCD is applicable, the hadronic matrix element $\langle J/\psi|J^+_{\rm em}|\eta_c\rangle$ can be calculated within the leading order PQCD by means of a homogeneous Bethe-Salpeter (BS) equation for the meson wave function. Taking the perturbative kernel of the BS equation as a part of hard scattering amplitude T_H, one thus obtains

$$\langle J/\psi|J^+_{\rm em}|\eta_c\rangle = \sum_{\lambda,\lambda'}\int[d^3k][d^3l]\Psi^{11\dagger}_{100}(y, \mathbf{l}_\perp, \lambda)T_H(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp; \mathbf{q}_\perp; \lambda, \lambda')\Psi^{00}_{100}(x, \mathbf{k}_\perp, \lambda'), \tag{2.5}$$

where $[d^3k] = dxd^2\mathbf{k}_\perp/16\pi^3$ and T_H contains all two-particle irreducible amplitudes for $\gamma^* + q\bar{q} \to q\bar{q}$ from the iteration of the LFQM wave function with the BS kernel. On the other hand, the right-hand-side of Eq. (2.3) for the matrix element of J^+ is obtained as $\mathscr{F}(q^2) = \sqrt{2}\langle J/\psi|J^+_{\rm em}|\eta_c\rangle/q^L$.

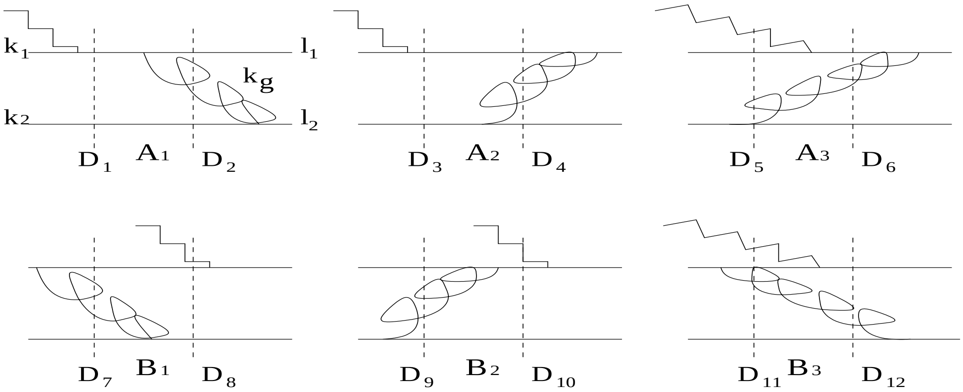

Figure 1: Leading order (in α_s) light-front time-ordered diagrams of the hard scattering amplitude for $\eta_c(P) \to \gamma^*(q) + J/\psi(P')$ process.

The leading order light-front time-ordered diagrams for the meson form factor are shown in Fig. 1, where the explicit forms of the energy denominators $D_i (i = 1, \cdots, 12)$ are given in Ref. [9]. Using the light-front gauge $\eta \cdot A = A^+ = 0$, we obtain the hard scattering amplitudes for the diagrams A_i and $B_i (i = 1, 2, 3)$ in Fig. 1. Since the gluon propagators for A_i and $B_i (i = 1, 2)$ have instantaneous parts ($\eta^\mu \eta^\nu / (k_g^+)^2$ in the light-front gauge), we absorb these instantaneous contributions into the regular propagators (see [9] for more details). If one includes the higher twist effects such as intrinsic transverse momenta and the quark masses, the LF gauge part proportional to $1/k_g^+$ leads to a singularity although the Feynman gauge part $g_{\mu\nu}$ gives the regular amplitude. This is due to the gauge-invariant structure of the amplitudes. The covariant derivative $D_\mu = \partial_\mu + igA_\mu$ makes both the intrinsic transverse momenta, $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$, and the transverse gauge degree of freedom $g\mathbf{A}_\perp$ be of the same order, indicating the need of the higher Fock state contributions to ensure the gauge invariance [12]. However, we can show that the sum of six diagrams for the LF gauge part ($1/k_g^+$ terms) vanishes in the limit that the LF energy differences Δ_x and Δ_y go to zero, where Δ_x and Δ_y are given by

$$\Delta_x = M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x_1 x_2} = M^2 - M_{0x}^2, \;\; \Delta_y = M^2 - \frac{\mathbf{l}_\perp^2 + m^2}{y_1 y_2} = M^2 - M_{0y}^2. \tag{2.6}$$

Details of the proof can be found in our previous work [7]. Following the same procedure presented in Ref. [7], we calculate the higher twist effects in the limit of $\Delta_x = \Delta_y = 0$ to avoid the involvement of the higher Fock state contributions. Our limit $\Delta_x = \Delta_y = 0$ (but $\sqrt{\langle \mathbf{k}_\perp^2 \rangle} = \beta \neq 0$) may be considered as a zeroth order approximation in the expansion of a scattering amplitude. That is, the scattering amplitude T_H may be expanded in terms of LF energy difference Δ as $T_H = [T_H]^{(0)} + \Delta [T_H]^{(1)} + \Delta^2 [T_H]^{(2)} + \cdots$, where $[T_H]^{(0)}$ corresponds to the amplitude in the zeroth order of Δ. This approximation should be distinguished from the zero-binding (or peaking) approximation that corresponds to $M = m_1 + m_2$ and $\mathbf{k}_\perp = \beta = 0$. The point of this distinction is to note that $[T_H]^{(0)}$ includes the binding energy effect (i.e. $\mathbf{k}_\perp, \mathbf{l}_\perp \neq 0$) that was neglected in the peaking approximation. We should also note that all higher orders of $\mathbf{k}_\perp^2/\mathbf{q}_\perp^2$, $\mathbf{l}_\perp^2/\mathbf{q}_\perp^2$ and $\mathbf{k}_\perp \cdot \mathbf{l}_\perp/\mathbf{q}_\perp^2$ are included in $[T_H]^{(0)}$. This corresponds to keep effectively all higher orders of the relative quark velocity beyond the second order $\langle v^2 \rangle$.

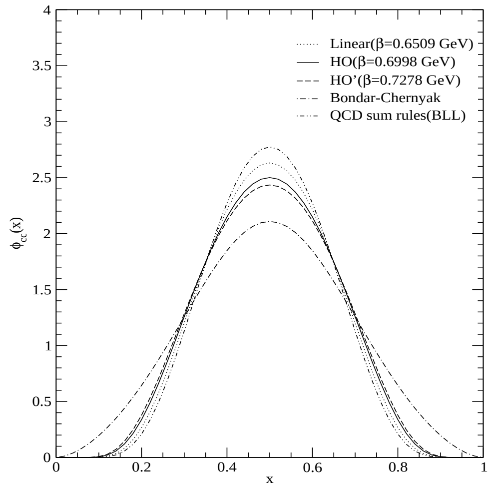

Figure 2: The distribution amplitudes $\phi(x)$ for $\phi_{\eta_c}(x) \approx \phi_{J/\psi}(x)$.

Using this approximation, we obtain the following non-factorized form of the form factor

$$\mathscr{F}(q^2) \simeq \int [d^3k][d^3l]\phi_R(x,\mathbf{k}_\perp)[T_H]^{(0)}\phi_R(y,\mathbf{l}_\perp), \tag{2.7}$$

where the explicit form of $[T_H]^{(0)}$ can be found in [9]. We note that the leading twist (LT)(i.e. neglecting transverse momenta $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$) contributions come from two leading helicity $\Delta H = |\lambda_{J/\psi} - \lambda_{\eta_c}| = 1$ components, i.e. $\uparrow\downarrow\to\uparrow\uparrow$ and $\downarrow\uparrow\to\uparrow\uparrow$ as in the nonrelativistic spin case. In the LT limit, the form factor factorizes into the convolution of the nonperturbative valence quark DAs with the perturbative hard scattering amplitude. Furthermore, in the NRQCD limit equivalent to the peaking approximation, the quark DAs for both η_c and J/ψ mesons become δ-type functions.

3. Numerical Results

In our numerical calculations, we use our LFQM [10, 11] parameters (m_c, β_{cc}) obtained from the meson spectroscopy with the variational principle for the QCD motivated effective Hamiltonian. In our LFQM, we have used the two interaction potentials $V_{Q\bar{Q}}$ for η_c and J/ψ mesons: (1) Coulomb plus harmonic oscillator (HO) potential, and (2) Coulomb plus linear confining potential. In addition, the hyperfine interaction essential to the distinction between J/ψ and η_c mesons is included for both cases (1) and (2), viz., $V_{Q\bar{Q}} = a + V_{\rm conf} - 4\alpha_s/3r + (32\pi\alpha_s/9m_c^2)\vec{S}_Q\cdot\vec{S}_{\bar{Q}}\delta^3(\vec{r})$, where $V_{\rm conf} = br^2$ for the HO potential and br for the linear confining potential, respectively. For the linear confining potential, we use the well-known string tension $b = 0.18$ GeV^2. The model parameters obtained from our variational principle are ($m_c = 1.8$ GeV, β_{cc} =0.6509 GeV) for the linear potential and ($m_c = 1.8$ GeV, β_{cc} =0.6998 GeV) for the HO potential, respectively. From the linear[HO] potential parameters, we obtain the decay constants $f_{\eta_c} = 326[354]$ MeV and $f_{J/\psi} = 360[395]$ MeV, which are quite comparable with the current experimental data [13], $(f_{\eta_c})_{\rm exp} = 335 \pm 75$ MeV and $(f_{J/\psi})_{\rm exp} = 416 \pm 6$ MeV.

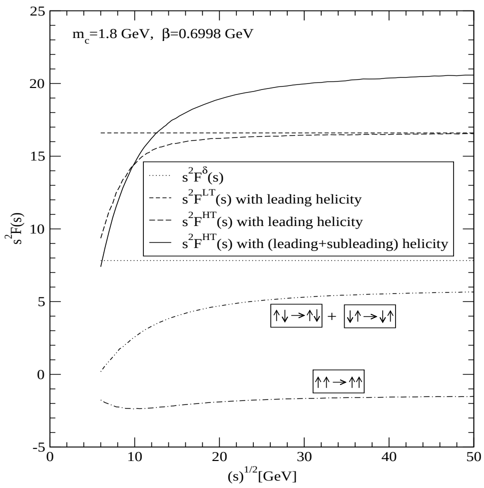

Figure 3: The form factor $s^2\mathscr{F}(s)$ for $e^+e^- \to J/\psi+\eta_c$.

In Fig. 2, we show the normalized quark DA for $\phi_{c\bar{c}}(x) = \phi_{\eta_c}(x) \approx \phi_{J/\psi}(x)$ obtained from linear (dotted line) and HO (solid line) potentials. As a sensitivity check of our variational parameters, we also include another Gaussian parameters ($m_c = 1.8$ GeV, β_{cc} =0.7278 GeV) denoted as HO$'$ (dashed line) to fit the central value of the experimental J/ψ decay constant. Our results are compared with the ones obtained from Bondar and Chernyak (BC) [6] (dot-dashed line) and from QCD sum rules [14] (doubledot-dashed line). Our results of the quark DA are wider than the delta function-type (i.e. the limit $\beta_{cc} \to 0$) of the NRQCD results [3] which do not take into account the relative motion of valence quark-antiquark pair. Within our model calculation, the relative quark velocity square is obtained as $\langle v^2\rangle_{c\bar{c}} = 0.30^{+0.02}_{-0.04}$, where the central, upper, and lower values are from the HO, HO$'$, and linear potential parameters, respectively.

In Fig. 3, we show $s^2\mathscr{F}(s)$ for $e^+e^- \to J/\psi+\eta_c$ process obtained from the central value (HO model) of our model parameters displaying both leading and subleading helicity contributions. The dotted and short-dashed lines represent the results obtained from the non-relativistic peaking approximation $\mathscr{F}^{\delta}(s)$ and the LT factorized form factor $\mathscr{F}^{\rm LT}(s)$ taking into account the relative motion of valence quarks, respectively. The long-dashed line represents the higher twist (HT) nonfactorized form factor $\mathscr{F}^{\rm HT}(s)$ obtained by including the transverse momenta $(\mathbf{k}_\perp, \mathbf{l}_\perp)$ both in the wave function and the hard scattering part. Note that $\mathscr{F}^{\delta}(s)$ (dotted line), $\mathscr{F}^{\rm LT}(s)$ (short-dashed line), and $\mathscr{F}^{\rm HT}(s)$ (long-dashed line) are obtained from the leading helicity contributions. The solid line represents our full solution $\mathscr{F}^{\rm HT}_{(\Delta{\rm H}=0+\Delta{\rm H}=1)}(s)$ including all (leading plus subleading) helicity contributions.

Our predictions for the cross section at $\sqrt{s} = 10.6$ GeV obtained from peaking approximation (σ_δ), leading twist ($\sigma_{\rm LT}$) and higher twist ($\sigma_{\rm HT}$) are given by $2.34^{+0.50}_{-0.69}$[fb], $10.57^{+3.15}_{-4.02}$[fb], and $8.76^{+1.61}_{-2.84}$[fb], respectively. Here, the central, upper and lower values of each prediction are obtained from HO, HO$'$ and linear potential parameters, respectively. On the other hand, the experimental results for $\sigma(J/\psi+\eta_c)\times B^{\eta_c}[\geq 2]$ are $(25.6\pm 2.8\pm 3.4)$[fb] by Belle [4] and $(17.6\pm 2.8^{+1.5}_{-2.1})$[fb] by BABAR [5], where $B^{\eta_c}[\geq 2]$ is the branching fraction for η_c decay into at least two charged particles.

4. Summary and Conclusion

We investigated the transverse momentum effect on the exclusive charmonium $J/\psi + \eta_c$ pair production in e^+e^- annihilation using the nonfactorized PQCD and LFQM that goes beyond the peaking approximation. In going beyond the peaking approximation, we stressed a consistency of the calculation by keeping the transverse momentum $\mathbf{k}_\perp$ both in the wave function part and the hard scattering part simultaneously before doing any integration in the amplitude. Such non-factorized analysis should be distinguished from the factorized analysis where the transverse momenta are separately integrated out in the wave function part and in the hard scattering part. We found that the higher twist contributions including all helicity contributions enhanced the NRQCD result by a factor of $3 \sim 4$ at $\sqrt{s} = 10.6$ GeV while it reduced that of the leading twist result by 20%.

Considering an enhancement by the factor of 1.8 from the corrections of next-to-leading order (NLO) of α_s[15] in the NRQCD approach, it might be conceivable to raise our leading α_s order result $\sigma_{\rm HT} = 8.76^{+1.61}_{-2.84}$[fb] by this factor and get a value close to the experimental data. However, it would be necessary to make detailed NLO investigation within the LF PQCD framework before we can make any firm conclusion.

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