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Nonperturbative calculation of the anomalous magnetic moment in the Yukawa model

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Within the covariant formulation of light-front dynamics, we calculate the state vector of a fermion coupled to identical scalar bosons (the Yukawa model). The state vector is decomposed in Fock sectors and we consider the first three ones: a single fermion, a fermion coupled to one boson, and a fermion coupled to two bosons. This last three-body sector generates nontrivial and nonperturbative contributions to the state vector, and these contributions are calculated with no approximations. The divergences of the amplitudes are regularized using Pauli-Villars fermion and boson fields. Physical observables can be unambiguously deduced using a systematic renormalization scheme we developed. This renormalization scheme is a necessary condition in order to avoid uncancelled divergences when Fock space is truncated. As an example, we present preliminary numerical results for the anomalous magnetic moment of a fermion in the Yukawa model.

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1. Bound state systems in light-front dynamics

1.1 Fock representation of the state vector

Light-front dynamics enables a very convenient representation of the state vector, $\phi(p)$, for any relativistic bound state system with total four-momentum p. In the standard formulation of light-front dynamics, the state vector is defined on a given light front plane $t+z/c=0$. It is solution of the eigenvalue equation $\hat{P}^2\ \phi(p)=M^2\ \phi(p)$, where $\hat{P}$ is the momentum operator and M is the bound state mass. One of the main advantages of light-front dynamics is that, due to kinematical constraints, the vacuum state of a system of interacting particles coincides with the free vacuum, and all intermediate states result from fluctuations of the physical system. One can thus construct the state vector in terms of combinations of free fields, i.e. decompose it in a series of Fock sectors: $\phi(p) \equiv |1\rangle + |2\rangle + \ldots + |N\rangle + \ldots$ with its subsequent truncation. This enables a systematic calculation of state vectors of physical systems and their observables. We call N the maximal number of Fock sectors considered in a given approximation, and n the number of constituents in a given Fock sector described by the many-body vertex function Γ_n, shown in Fig. 1.

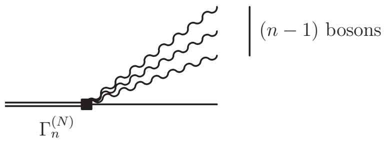

Figure 1: Vertex function of order n for the N-body Fock space truncation.

1.2 Strict control on explicit violation of rotational invariance

The standard formulation of light-front dynamics has however a serious drawback, since the equation of the light front plane is not invariant under spatial rotations. To avoid such an unpleasant feature, we use the Covariant formulation of Light-Front Dynamics (CLFD) [1], which provides a very powerful tool in order to describe physical systems. In this formulation, the state vector is defined on the plane determined by the equation $\omega{\cdot}x=0$, where ω is an arbitrary light-like four-vector. The covariance of our approach is due to the invariance of the light front plane. This implies that ω is not the same in any reference frame, but varies according to Lorentz transformation, like the coordinate x. It is not the case in the standard formulation where ω is fixed to $\omega=(1,0,0,-1)$.

This scheme is very convenient in order to parameterize the general spin structure of vertex functions. For a spin-1/2 system consisting of one fermion (with momentum k_1) and scalar bosons (with momenta k_2, k_3,...), the two- and three-body vertex functions are represented as:

$$\bar{u}(k_1)\Gamma_2^{(N)}u(p)=\bar{u}(k_1)\left[b_1+b_2\frac{m\not{\omega}}{\omega{\cdot}p}\right]u(p)\ , \tag{1.1}$$

$$\bar{u}(k_1)\Gamma_3^{(N)}u(p)=\bar{u}(k_1)\left[c_1+c_2\frac{m\not{\omega}}{\omega{\cdot}p}+C_{ps}\left(c_3+c_4\frac{m\not{\omega}}{\omega{\cdot}p}\right)\gamma_5\right]u(p)\ , \tag{1.2}$$

with

$$C_{ps}=\frac{1}{m^2\omega{\cdot}p}\ e^{\mu\nu\rho\lambda}\ k_{2\mu}\ k_{3\nu}\ p_\rho\ \omega_\lambda\ . \tag{1.3}$$

We identified the bound state mass M with the physical fermion mass m. Here $b_{1,2}$ and c_{1-4} are scalar functions determined by dynamics. Generally speaking, their explicit form depends on N, but the total number of irreducible spin components for each vertex function does not. The two-body vertex function has two, while the three-body one (as well as any of higher order) has four irreducible components. The one-body vertex function is just a constant which also depends on N.

In order to impart sense to divergent amplitudes, we choose here the Pauli-Villars (PV) regularization scheme which preserves rotational invariance [2] and extends the state vector to include both fermion and boson PV particles with very large masses. An alternative scheme, based on the use of specific test functions on which operator fields are defined, has been developed very recently [3]. Its application to CLFD is currently under study [4].

1.3 Fock sector dependent renormalization

In order to be able to make definite predictions for physical observables, one should also define a proper renormalization scheme. This should be done with care since the Fock decomposition of the state vector is truncated to a given order. Indeed, looking at Fig. 2 for the calculation of the fermion propagator in second order perturbation theory, one immediately realizes that the cancellation of divergences (or terms infinitely increasing as the PV masses tend to infinity) between the self-energy contribution (of order two in the Fock decomposition) and the fermion Mass Counterterm (MC) (of order one) involves two different Fock sectors [5]. This means that any MC and,

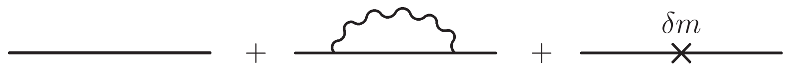

Figure 2: Renormalization of the fermion propagator in second order perturbation theory.

more generally, any Bare Coupling Constant (BCC) should be associated with the number of particles present (or "in flight") in a given Fock sector. In other words, all MC's and BCC's must depend on the Fock sector under consideration. The original MC δm and the fermion-boson BCC g_0 should thus be extended to a whole series:

$$g_0 \to g_0^{(i)} \,, \tag{1.4}$$

$$\delta m \to \delta m^{(i)} \,, \tag{1.5}$$

with $i = 1, 2, \ldots N$. The quantities $g_0^{(i)}$ and $\delta m^{(i)}$ are calculated by solving the systems of equations for the vertex functions in the $N = 1$, $N = 2$, $N = 3$, ... approximations successively. We shall illustrate this procedure in Section 2. The BCC $g_0^{(N)}$ is determined by demanding that the ω-independent part of the two-body vertex function Γ_2 at $s \equiv (k_1 + k_2)^2 = m^2$ coincides with the physical coupling constant g:

$$b_1(s = m^2) \equiv g \,. \tag{1.6}$$

Note that in perturbation theory, we also have to consider a whole series of bare parameters/counterterms $g_0^{(n)}$ and $\delta m^{(n)}$, where n denotes the order of the perturbative expansion. In light-front dynamics, the index n refers to the number of particles in "flight". A calculation of order N involves

$\delta m^{(1)} \ldots \delta m^{(N)}$ and $g_0^{(1)} \ldots g_0^{(N)}$. This procedure, which we shall call Fock Sector Dependent Renormalization (FSDR), is a well defined, systematic, and nonperturbative scheme [5]. The main difference of our procedure from that used in [6] consists in the use of CLFD and FSDR scheme.

2. The anomalous magnetic moment

2.1 QED in two-body truncated Fock space

The decomposition of the spin-1/2 electromagnetic vertex in CLFD is given by [7]:

$$\bar{u}(p')G^\rho u(p) = e\bar{u}(p') \left[F_1\gamma^\rho + \frac{iF_2}{2m}\sigma^{\rho\nu}q_\nu + B_1\left(\frac{\not\omega}{\omega\cdot p}P^\rho - 2\gamma^\rho\right) + B_2\frac{m\omega^\rho}{\omega\cdot p} + B_3\frac{m^2\not\omega\omega^\rho}{(\omega\cdot p)^2}\right] u(p) \; , \tag{2.1}$$

with $P = p' + p'$, $q = p' - p$. F_1 and F_2 are the physical form factors, while $B_{1,2,3}$ are spurious (nonphysical) contributions which appear if rotational invariance is broken, e. g. by Fock space truncation. The decomposition (2.1) enables to separate unambiguously the physical form factors from the nonphysical ones. Under the condition $\omega\cdot q = 0$, all $F_{1,2}$, B_{1-3} depend on $Q^2 \equiv -q^2$ only.

The simplest realistic physical system one can consider first is QED in two-body truncated Fock space. This approximation is equivalent to the summation, to all orders, of the second order perturbative correction to the electron self-energy.

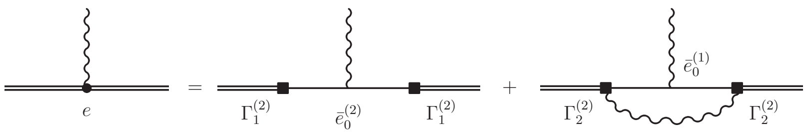

Figure 3: Electromagnetic vertex of the electron in the two-body approximation.

The electromagnetic form factors of the electron are given by the one- and two-body contributions shown in Fig. 3, where we have denoted by $\bar{e}_0^{(1)}$ and $\bar{e}_0^{(2)}$ the electron BCC's which, according to our FSDR scheme, depend on the Fock sector. Note that these BCC's describing the interaction of an electron with an external photon field do also differ from the "internal" photon-electron BCC's, denoted by $e_0^{(i)}$, which appear in the calculation of the state vector itself [5], since the external photon does not participate to the internal structure of the state vector. The calculation of the anomalous magnetic moment of the electron, given by $F_2(Q^2 = 0)$, is thus done according to the following steps:

- We first decompose the two-body vertex function in independent spin structures in a way very similar to (1.1), (1.2), including vector indices for the photon line. For the $N = 2$ truncation, the components b_1 and b_2 are constants.

- We solve the eigenvalue equation which is represented graphically in Fig. 4. Note the appearance in this figure of the Fock sector dependent MC's and BCC's.

- We determine the MC's and BCC's according to our FSDR scheme. We have from the very beginning $\delta m^{(1)} = 0$, while $\delta m^{(2)}$ is fixed from the compatibility condition of this system of two homogeneous equations and $e_0^{(2)}$ is fixed from the condition analogous to (1.6).

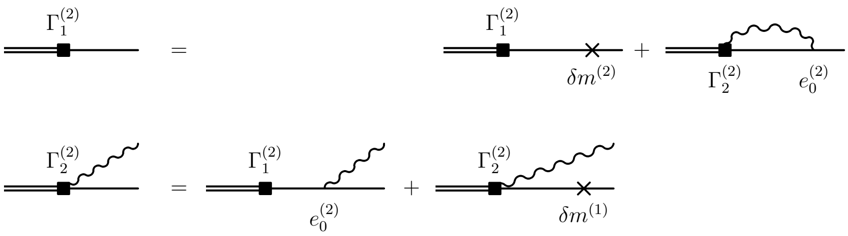

Figure 4: System of equations for the vertex functions in QED for the two-body Fock space truncation.

- Once the state vector is known, we calculate the electromagnetic form factors and demand that $F_1(Q^2=0)=1$. This defines $\bar{e}_0^{(2)}$, while $\bar{e}_0^{(1)}$ is equal to e, since it corresponds, by definition, to an external photon coupling to a single electron, with no radiative corrections at all. Because of the normalization of the state vector, and the counterterms which depend explicitly on the Fock sector, we find $\bar{e}_0^{(2)} \equiv e$, as dictated by the Ward identity.

We can thus predict analytically the value of F_2 without any perturbative expansion and find, in the limit of infinite PV particle masses, $F_2(Q^2=0)=\frac{\alpha}{2\pi}$ which coincides with the well-known perturbative Schwinger correction.

2.2 The Yukawa model in three-body truncated Fock space

In order to address the calculation of a true nonperturbative system, we investigate the system composed of a fermion coupled to scalar bosons for the three-body, $N=3$, Fock space truncation. The strategy to analyze this system is very similar to that outlined above for QED. The system

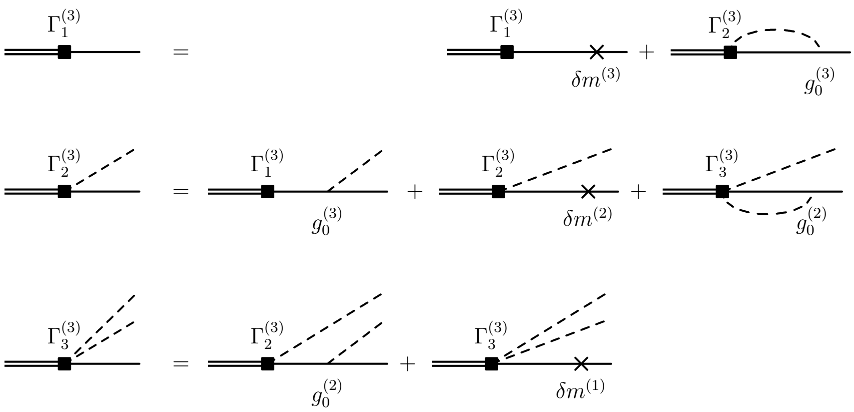

Figure 5: System of equations for the vertex functions in the Yukawa model for the three-body Fock space truncation. We do not show on this figure, for simplicity, the interchange of identical bosons in $\Gamma_3^{(3)}$. Dashed lines correspond to scalar bosons.

of equations one has to solve is given in Fig. 5. Note that the indices of MC's and BCC's in this figure are different from those present in Fig. 4 for the case of the two-body truncation, according

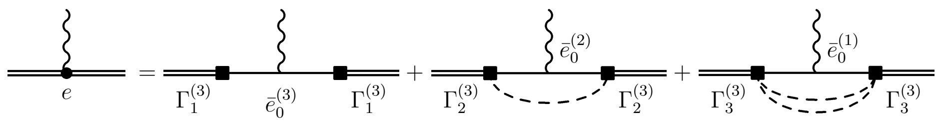

Figure 6: Fermion electromagnetic vertex in the Yukawa model for the three-body Fock space truncation.

to our FSDR scheme. The set of indices just corresponds to $i=1,2,3$ in (1.4) and (1.5). The electromagnetic vertex is given by Fig. 6. Since the state vector is normalized, and within the FSDR scheme, we find again $\bar{e}_0^{(3)}=\bar{e}_0^{(2)}=\bar{e}_0^{(1)}=e$ as it should be. Note that the system of equations shown in Fig. 5 has a structure very similar to the one shown in Fig. 4 for the $N=2$ truncation. The extension to higher order Fock space truncations is thus straightforward.

2.3 Numerical results

We solved numerically the system of linear integral equations for the vertex functions, shown graphicaly in Fig. 5, and calculated the anomalous magnetic moment of the fermion. The original system of homogeneous equations is reduced to a system of inhomogeneous equations by setting the one-body vertex function to a fixed value, since all Γ's are defined up to a normalization constant. After discretizing the integrals by means of the Gaussian procedure, the solution is found by standard matrix inversion methods. The vertex functions are finally normalized [5].

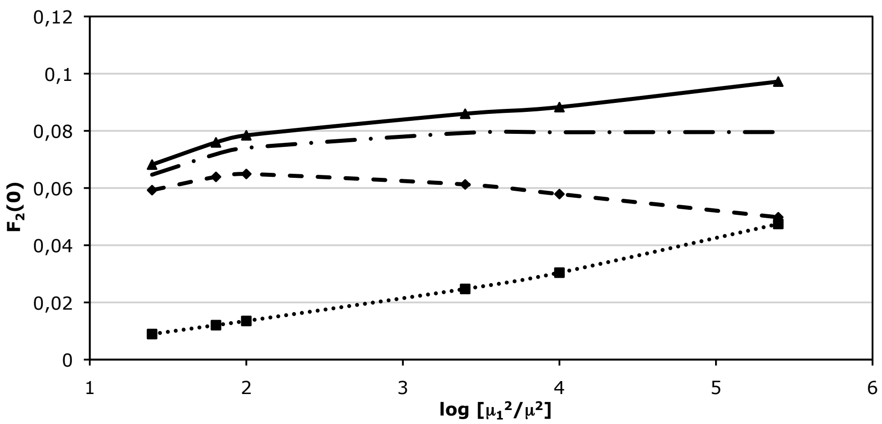

Figure 7: The fermion anomalous magnetic moment as a function of the Pauli-Villars boson mass μ_1 for the $N=3$ Fock space truncation. We separate the contributions from the two-body (dashed line) and three-body (dotted line) vertex function to the total result (solid line). The lines are just drawn to guide the eyes. The dash-dotted line represents the anomalous magnetic moment calculated in the N=2 approximation.

The solution depends, apart from the physical masses of the fermion (m) and the boson (μ) and the physical coupling constant g, on the regularization parameters, namely, the masses of the PV fermion (m_1) and the PV boson (μ_1). After calculating observables (say, the form factors), one should go over to the limit of infinite PV masses. The limit $m_1 \to \infty$ can be done analytically, already at the level of the equations for the vertex functions. This is not the case for the PV boson mass and we consider the value of the fermion anomalous magnetic moment as a function of μ_1, for large values of the latter.

For our first numerical study, we consider a typical set of physical parameters: $m = 1$ GeV, $\mu = 1$ GeV and $\alpha = \frac{g^2}{4\pi} = 1$. The results of our calculation are shown in Fig. 7. We separate on this figure the two- and three-body contributions to the anomalous magnetic moment. The first one is slightly decreasing with μ_1 while the second is increasing. The total contribution is rather stable, although it increases slightly with μ_1. We indicate also on this figure the fermion anomalous magnetic moment calculated in the lower order approximation (i.e. in $N = 2$ truncated Fock space). Similarly to the QED case, it has a finite limit when $\mu_1 \to \infty$.

It remains to investigate the origin of the residual dependence of the anomalous magnetic moment on μ_1. We have already shown that because of Fock state truncation, violation of rotational invariance may arise, leading, in particular, to a nonzero ω-dependent component in the two-body vertex function at $s = m^2$. This nonrenormalized b_2, in Eq. (1.1), may contain uncancelled μ_1-dependence (even at $\mu_1 \to \infty$) giving rise to analogous dependence of the anomalous magnetic moment. In perturbation theory, we can show that the incorporation, in the state vector, of Fock sectors containing fermion-antifermion pairs completely removes extra ω-dependent contributions in $\Gamma_2(s = m^2)$. Work is in progress to extend this calculation to our nonperturbative approach.

3. Perspectives

The general framework we have developed so far - with an explicitly covariant formulation of light-front dynamics and a systematic nonperturbative renormalization scheme - enables us to calculate physical observables of physical systems in truncated Fock space.

We have presented a preliminary study of the fermion anomalous magnetic moment in the Yukawa model, for a nontrivial three-body Fock space truncation. This calculation embedded all features (general structure of the state vector in terms of spin components, new nonperturbative renormalization scheme) of a more general study and, in principle, can be extended to the case of Fock space truncations of arbitrary order rather easily. Applications to gauge theories and to effective field theories are under consideration.

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