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QCD on the light cone and heavy ion collisions: the CGC, the Glasma and multi-gluon correlations

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High energy heavy ion collisions are efficiently described as the collision of two sheets of Color Glass Condensate. The dynamics of the collision can be studied *ab initio* in a light cone effective field theory approach. Factorization theorems allow one to separate the initial state evolution of the wave functions with energy from the final state interactions that produce matter with high energy densities called the Glasma. We discuss how this matter is formed, its remarkable properties and its relevance for understanding thermalization of the Quark Gluon Plasma. We discuss multi-gluon correlations in the Glasma. In particular, long range rapidity correlations, reflected in a Glasma flux tube picture of the collision, are not washed out by subsequent final state interactions and may provide a sensitive probe of early time dynamics. A near side "ridge"–like structure observed in two particle correlation studies of central heavy ion collisions can be explained as resulting from radially flowing flux tubes.

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1. Introduction

Multi-particle production in QCD at high energies is generated by small x partons in hadron and nuclear wavefunctions. These partons have properties best described as a Color Glass Condensate (CGC) [1]. When two sheets of CGC collide in a high energy heavy ion collision, the partons are released in bulk and create energy densities an order of magnitude above the energy density required for the crossover from hadronic to partonic degrees of freedom. This matter, at early times after a heavy ion collision, is a coherent classical field, which expands, decays into nearly on shell partons and may eventually thermalize to form a Quark Gluon Plasma (QGP). Because it is formed by melting the frozen CGC degees of freedom, and because it is the non-equilibrium matter preceding the QGP, this matter is called the Glasma [2].

The Glasma is of intrinsic interest because it corresponds to chromo-electric and chromo-magnetic field configurations that, in absolute magnitude, are some of the strongest such fields in nature; they are greater (by several orders of magnitude) than the magnetic fields on the surface of magnetars. The Glasma is also important for quantifying the initial conditions for the QGP. In particular, two measures of QGP formation, the flow of bulk matter and the energy loss of jets, can be significantly influenced by the properties of the Glasma. In the former case, the initial matter distribution and non-equilibrium flow are important for a quantitative determination of the properties of the “perfect" fluid QGP. In case of the latter, jets can experience significant energy loss in their interactions with the Glasma.

There is a strong analogy between the physics of the little bang in a heavy ion collision and the big bang that created our universe. In the big bang, the inflaton field with large occupation number $O(\frac{1}{g^2})$ decays rapidly with the expansion of the universe. Likewise, in the little bang, the Glasma field with occupation number $O(\frac{1}{g^2})$ decays rapidly after the collision. In the big bang, low momentum quantum fluctuations are explosively amplified in a process known as pre-heating [3]. In the little bang, the explosive amplification of low momentum quantum fluctuations may be related to a Weibel [4] or a Nielsen-Olesen [5] type instability. The interaction of quantum fluctuations with the decaying inflaton field can lead to rapid “turbulent" thermalization [6]. A similar phenomenon may be responsible for rapid thermalization in heavy ion collisions [7]. Understanding thermalization from first principles is an outstanding problem in heavy ion collisions. There is also likely a concrete analogy between super horizon fluctuations observed in the COBE and WMAP measurements [8] and the near side Ridge measured at RHIC [9]. Another strong analogy is between sphaleron driven topological transitions that may induce P and CP odd metastable states in heavy ion collisions [10] and the matter–anti-matter asymmetry generated by C and CP violating topological transitions during electroweak baryogenesis. What both long range rapidity correlations and topological transitions in strong fields have in common is the likelihood that they both survive the later stage interactions that thermalize the system causing it to lose memory of how it was formed.

I will first briefly describe the properties of hadron and nuclear wavefunctions at high energies in the CGC framework. This is understood within the framework of an effective field theory approach on the light cone and relies heavily on the ideas and methods of light cone quantization in quantum field theory. I will next describe the quantum field theory framework of particle production in strong time dependent fields. Multi-particle production in the Glasma can be computed systematically in this framework. An important part of this systematic computation is a proof of high energy factorization. This was first proved in the leading logarithmic approximation for multi-gluon production in a restricted kinematic window in rapidity but has recently been extended to describe inclusive particle production in a wide window in rapidity. I will describe briefly how plasma instabilities arise and are accounted for in this framework. These plasma instabilities may play an important role in isotropizing matter distributions leading to early thermalization of the matter formed. Finally, I will discuss how Glasma flux tubes form the near side ridge seen in heavy ion collisions.

2. Before the little bang in heavy ion collisions

At high energies, the competition between QCD bremsstrahlung, which enhances the parton density, and screening/recombination processes, which deplete it, leads to a saturation of parton densities when the field strengths squared become maximal: $E^2 \sim B^2 \sim \frac{1}{\alpha_S}$. This non-linear strong field regime of QCD is characterized by a saturation scale [11] $Q_S(x,A)$; modes in the nuclear wavefunction with momenta $k_\perp \leq Q_S$ have high occupation numbers typical of classical fields. As one goes to higher energies, higher and higher momentum modes in nuclear and hadron wave functions become saturated. The kinematics of the problem lends itself to an effective field theory formulation. Large x modes correspond to long lived modes on the light cone and small x modes (of comparable transverse momenta) correspond to short lived modes. The former can therefore be described as static light cone sources, while the latter are dynamical degrees of freedom. In the CGC effective field theory (EFT), classical gauge fields are the dynamical degrees of freedom that couple stochastically to static light cone color sources $\rho^a(\mathbf{x}_\perp)$ at large x [12]; their source distribution is given by a gauge invariant weight functional $W_Y[\rho]$, where $Y = \ln(1/x)$ is the rapidity.

While this separation of degrees of freedom is arbitrary at some initial scale, its evolution with energy is described by the renormalization group (RG) equation $\frac{\partial W_Y[\tilde{\rho}]}{\partial Y} = H_{\rm JIMWLK}[\tilde{\rho}]W_Y[\tilde{\rho}]$, where $H_{\rm JIMWLK}[\tilde{\rho}]$ is the JIMWLK Hamiltonian [13]. (Here $\tilde{\rho}$ is the color charge density gauge rotated from light cone gauge to Lorentz gauge.) The Balitsky-Kovchegov (BK) equation [14] is a useful mean field (large N_C, large A) simplification of the JIMWLK equation describing, in closed form, the energy evolution of the "dipole" operator corresponding to the forward scattering amplitude in deep inelastic scattering. The JIMWLK and BK equations are derived in the leading logarithmic approximation in x, where running coupling effects are neglected. There is currently an intense on-going theoretical effort to compute next-to-leading order corrections to the kernels of the leading order RG equations [15]. We shall return to a discussion of the JIMWLK Hamiltonian in our discussion of factorization in A+A collisions.

Phenomenological "dipole" models that incorporate the physics of saturation have been very successful-see Ref. [16] for a recent review of comparisons of these models to the HERA, fixed target e+A, D+A and A+A RHIC data. A detailed dipole model study [17] shows that the saturation scale $Q_S^2(x,A) = A^{1/3} Q_S^2(x,A=1)$ when one compares their values in nuclei and nucleons respectively for median impact parameters. Because of the nuclear "oomph", saturation (CGC) effects should become increasingly visible in deuteron+gold collisions at RHIC, p+A collisions at the LHC, and especially at a future electron ion collider (EIC) [18]. We emphasize that the

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CGC effective theory is more general than the dipole picture and is used to study nucleus–nucleus collisions; it is not possible to do this in the dipole framework.

3. Multiparticle production in the Glasma at leading order

When two CGC sheets collide in an A+A collision, the static light cone color sources in the nuclear wavefunctions become time dependent. An *ab initio* computation of the Glasma therefore requires a formalism to compute particle production in quantum field theories coupled to external time dependent sources [19, 20]. Multi-particle production in the Glasma is always non–perturbative; the relevant question is whether the physics is one of strong coupling or weak coupling. Only the latter will be considered here.[1]

For the inclusive gluon distribution, the leading order (LO) contribution is of order $O(\frac{1}{g^2})$ but all orders in $g\rho_{1,2}$. It can be expressed as

$$E_{\mathbf{p}}\frac{dN}{d^3\mathbf{p}} = \frac{1}{16\pi^3}\lim_{x_0,y_0\to+\infty}\int d^3\mathbf{x}\,d^3\mathbf{y}\,e^{ip\cdot(x-y)}\,(\partial_x^0-iE_{\mathbf{p}})(\partial_y^0+iE_{\mathbf{p}}) \times\sum_{\lambda}\varepsilon_\lambda^\mu(\mathbf{p})\varepsilon_\lambda^\nu(\mathbf{p})\,\langle A_\mu(x)A_\nu(y)\rangle\,. \tag{3.1}$$

The gauge fields on the right hand side can be computed numerically for proper times $\tau\geq 0$ [22] by solving the classical Yang-Mills (CYM) equations in the presence of the light cone current $J^{\mu,a}=\delta^{\mu+}\delta(x^-)\rho_1(x_\perp)+\delta^{\mu-}\delta(x^+)\rho_2(x_\perp)$ corresponding to the local color charge densities of the two nuclei with initial conditions determined by matching the (known) solutions to the CYM equations in the backward light cone. The energy densities of produced gluons can be computed in terms of Q_S and one obtains $\varepsilon\sim 20-40$ GeV/fm^3 for the previously mentioned values of Q_S obtained by extrapolating from fits to the HERA and fixed target e+A data.

This LO formalism was applied to successfully predict the RHIC multiplicity at $y\sim 0$ [22] as well as the rapidity and centrality distribution of the multiplicities [23]. CGC model comparisons to the RHIC data on limiting fragmentation [24] or solutions of CYM equations [21], extrapolated to the LHC, give $dN/d\eta|_{\eta\sim 0}\approx 1000-1400$ charged particles[2]. At LO, the initial transverse energy is $E_T\sim Q_S$, which is about 3 times larger than the final measured E_T, while (assuming parton hadron duality) $N_{\rm CGC}\sim N_{\rm had.}$. The two conditions are consistent if one assumes nearly isentropic flow which reduces E_T due to PdV work while conserving entropy. This assumption has been implemented directly in hydrodynamic simulations [26].

A powerful measure of flow is the second moment of the inclusive distribution as a function of the azimuthal angle; this quantity is commonly referred to as elliptic flow, and it is sensitive to the initial spatial geometry of the collision. For semi-peripheral collisions, the "almond" shaped reaction plane has a large spatial "eccentricity" which is a measure of the difference in the pressure gradient along the short axis of the almond relative to the long axis. (For strictly central collisions with zero impact parameter, the eccentricity is zero and there is no elliptic flow.) How much flow is generated in the Glasma before thermalization? The primordial Glasma has occupation numbers

[1] The magnitude of the saturation scale in a gold nucleus is $Q_S^2\sim 1-1.4$ GeV^2 at RHIC; estimates for lead nuclei at LHC energies are $Q_S^2\sim 2.6-3.9$ GeV^2 [21].

[2] See Ref. [25] for other model predictions.

$f \sim \frac{1}{\alpha_S}$ and can be described as a classical field. As the Glasma expands, higher momentum modes increasingly become particle like and eventually the modes have occupation numbers $f < 1$, which may be described by a thermal spectrum. A first computation of elliptic flow of the Glasma found only about half the observed elliptic flow [27] albeit the computation did not properly treat the interaction between hard and soft modes in the Glasma. For more recent discussion on the issue of the initial eccentricity, see Ref. [28]. Formulating a kinetic theory that describes this evolution is a challenging problem in heavy ion collisions–for a preliminary discussion, see [29]. It is important to know the amount of flow in the Glasma because that can help quantify the viscosity of the subsequent hydrodynamic flow of a thermalized Quark Gluon Plasma.

The LO Glasma result, from the solutions of Yang–Mills equations, has very interesting properties. Firstly, the solution is boost invariant in the strong sense–the fields are independent of the space-time rapidity; the dynamics of the produced gluon fields is purely transverse as a function of proper time. Another interesting feature is that the chromo E and B fields are purely longitudinal after the collision [2]. This result suggests that one can generate topological Chern–Simons charge in heavy ion collisions [30]. Because the range of color correlations in the transverse plane is of order $1/Q_S$, the LO picture that emerges is of color flux tubes with finite topological charge stretching between the valence color sources after the collision. As we shall see later, this picture provides a plausible explanation of a near side ridge structure in central heavy ion collisions.

The LO field configurations are very unstable and lead to very anisotropic momentum distributions at later times. Such distributions can trigger an instability analogous to the Weibel instability in QED plasmas [4]. Romatschke and I showed (in 3+1-D numerical solutions of CYM equations) [31] that small rapidity dependent quantum fluctuations in the initial conditions generate transverse E and B fields that grow rapidly as $\exp(\sqrt{Q_S\tau})$. They are the same size as the rapidly diluting longitudinal E and B fields on time scales of order $\frac{1}{Q_S}\ln^2(\frac{C}{\alpha_S})$. The transverse E and B fields may cause large angle deflections of colored particles leading to p_T broadening and energy loss of jets–numerical simulations by the Frankfurt group appear to confirm this picture [32]. These interactions of colored high momentum particle like modes with the soft coherent classical field modes may also generate a small “anomalous viscosity" whose effects on transport in the Glasma may mask a larger kinetic viscosity [33]. The same underlying physics may cause “turbulent isotropization" by rapidly transferring momentum from soft “infrared" longitudinal modes to ultraviolet modes [7]. Finally, albeit the LO result demonstrated that one could have non-trivial Chern-Simons charge in heavy ion collisions, the boost invariance of CYM equations disallows sphaleron transitions that permit large changes in the Chern-Simons number [30]. With rapidity dependent quantum fluctuations, sphaleron transitions can go. These may have important consequences–in particular P and CP odd metastable transitions that cause a novel “Chiral Magnetic Effect" [10] in heavy ion collisions.

4. QCD Factorization and the Glasma instability

The discussion at the end of the last section strongly suggests that next-to-leading order (NLO) quantum fluctuations in the Glasma, while (superficially) parametrically suppressed, may alter our understanding of heavy ion collisions in a fundamental way. To cosmologists, this will not come as a surprise–quantum fluctuations play a central role there as well. In recent papers, it was shown for a scalar theory that moments of the multiplicity distribution at NLO in A+A collisions could be computed as an initial value problem with retarded boundary conditions; this framework has now been extended to QCD [19]. In QCD, the problem is subtle because some quantum fluctuations occur in the nuclear wavefunctions and are responsible for how the wavefunctions evolve with energy; others contribute to particle production at NLO.

A factorization theorem organizing these quantum fluctuations shows that all order leading logarithmic contributions to an inclusive gluon operator $\mathcal{O}$ in the Glasma gives [20]

$$\langle \mathcal{O} \rangle_{\text{LLog}} = \int [D\widetilde{\rho}_1][D\widetilde{\rho}_2] \, W_{Y_{\text{beam}}-Y}[\widetilde{\rho}_1] \, W_{Y_{\text{beam}}+Y}[\widetilde{\rho}_2] \; \mathcal{O}_{\text{LO}} \left[\widetilde{\rho}_1, \widetilde{\rho}_2\right] \,, \tag{4.1}$$

where $\mathcal{O}_{\text{LO}}$ is the same operator evaluated at LO by solving classical Yang–Mills equations and $W_{Y_{\text{beam}}\mp Y}[\widetilde{\rho}_{1,2}]$ are Y dependent weight functionals (introduced in section 2) that obey the JIMWLK Hamiltonians describing the rapidity evolution of the projectile and target wavefunctions respectively. The functions $\tilde{\rho}_{1,2} \equiv \tilde{\rho}_{1,2}(x_\perp, Y)$ are here functions of Y, having acquired the Y dependence by successive integration of slices in rapidity that convert the fields in each slice into sources. This theorem is valid for an arbitrary rapidity interval in which the operator $\mathcal{O}$ is measured.

This factorization theorem is a necessary first step before a full NLO computation of gluon production in the Glasma. Eq. (4.1) includes only the NLO terms that are enhanced by a large logarithm of $1/x_{1,2}$, while the complete NLO calculation would also include non enhanced terms. These would be of the same order in α_S as the production of quark-antiquark pairs [34] from the classical field. To be really useful, this complete NLO calculation likely has to be promoted to a Next-to-Leading Log (NLL) result by resumming all the terms in $\alpha_S(\alpha_S \ln(1/x_{1,2})^n$. Now that evolution equations in the dense regime are becoming available at NLO, work in this direction is a promising prospect [15].

In addressing the role of instabilities at NLO, note that small field fluctuations fall into three classes: i) Zero modes ($p_\eta = 0$) that generate the leading logs resummed in eq. 4.1; the coefficients of the leading logs do not depend on $x^\pm$. ii) Zero modes that do not contribute at leading log because they are less singular than the leading log contributions. These become relevant in resumming NLL corrections to the factorization result [15]. Because they are zero modes, they do not trigger plasma instabilities. iii) Non zero modes ($p_\eta \neq 0$) that do not contribute large logarithms of $1/x_{1,2}$, but grow exponentially as $\exp(\sqrt{Q_S \tau})$. While these boost non-invariant terms are suppressed by α_S, they are enhanced by exponentials of the proper time after the collision. These leading temporal divergences can be resummed–we conjecture that the expression for inclusive gluon operators in the Glasma can be revised to read

$$\begin{aligned} \langle \mathcal{O} \rangle_{\text{LLog+LInst}} = & \int [D\widetilde{\rho}_1][D\widetilde{\rho}_2] \, W_{Y_{\text{beam}}-Y}[\widetilde{\rho}_1] \, W_{Y_{\text{beam}}+Y}[\widetilde{\rho}_2] \\ & \times \int \left[Da(\vec{\mathbf{u}})\right] \widetilde{Z}[a(\vec{\mathbf{u}})] \; \mathcal{O}_{\text{LO}}[\widetilde{\mathcal{A}_1^+} + a, \widetilde{\mathcal{A}_2^-} + a] \end{aligned} \tag{4.2}$$

where $\widetilde{\mathcal{A}_1^+}(x) = -\frac{1}{\partial_\perp^2} \widetilde{\rho}_1(x_\perp, x^-)$ and $\widetilde{\mathcal{A}_2^-}(x) = -\frac{1}{\partial_\perp^2} \widetilde{\rho}_2(x_\perp, x^+)$. The effect of the resummation of instabilities would therefore be to add fluctuations to the initial conditions of the classical field, with a distribution that depends on the outcome of the resummation. This spectrum $\widetilde{Z}[a(\vec{\mathbf{u}})]$ is the final incomplete step in determining all the leading singular contributions to particle production in the Glasma–for a first attempt, see Ref.[35]. The stress-energy tensor $T^{\mu\nu}$ can then be determined *ab initio* and matched smoothly to kinetic theory or hydrodynamics at late times.

5. Two particle correlations in the Glasma and the Ridge

Striking "ridge" events were revealed in studies of the near side spectrum of correlated pairs of hadrons at RHIC [9]. The spectrum of correlated pairs of hadrons on the near side of the STAR detector extends across the entire detector acceptance in pseudo-rapidity of order $\Delta\eta \sim 2$ units but is strongly collimated for the relative azimuthal angle $\Delta\phi$ between the measured pairs. Preliminary analyses of measurements by the PHENIX and PHOBOS collaborations corroborate the STAR results. In the latter case, the ridge is observed to span the PHOBOS acceptance in pseudo-rapidity of $\Delta\eta \sim 3-4$ units.

Causality dictates (in strong analogy to CMB superhorizon fluctuations) that long range rapidity correlations causing the ridge must have occured at proper times $\tau \leq \tau_{\rm freezeout}\, e^{-\frac{1}{2}|y_A-y_B|}$, where y_A and y_B are the rapidities of the correlated particles. If the ridge span in psuedo-rapidity is large, these correlations must have originated at very early times–in the Glasma. For inclusive 2-gluon correlations, the factorization formula in eq. (4.1) takes the form

$$\left\langle \frac{d^2N_2}{d^3\mathbf{p}d^3\mathbf{q}} \right\rangle_{\rm LLog} = \int [D\widetilde{\rho}_1^p][D\widetilde{\rho}_2^p][D\widetilde{\rho}_1^q][D\widetilde{\rho}_2^q]\; Z_{Y_{\rm beam}-Y_p}[\widetilde{\rho}_1^p]\, Z_{Y_{\rm beam}+Y_q}[\widetilde{\rho}_2^q]$$
$$\times\; G_{Y_p,Y_q}(\widetilde{\rho}_1^p,\widetilde{\rho}_1^q) G_{Y_q,Y_p}(\widetilde{\rho}_2^q,\widetilde{\rho}_2^p)\; \left.\frac{dN}{d^3\mathbf{p}}\right|_{\rm LO} (\widetilde{\rho}_1^p,\widetilde{\rho}_2^p)\; \left.\frac{dN}{d^3\mathbf{q}}\right|_{\rm LO} (\widetilde{\rho}_1^q,\widetilde{\rho}_2^q)\,, \qquad (5.1)$$

to leading log accuracy. In this formula, $Z_{Y_{\rm beam}-Y_p}[\widetilde{\rho}_1^p] = \int [d\widetilde{\rho}_1(Y,x_\perp)]\delta(\widetilde{\rho}_1(Y,x_\perp)-\widetilde{\rho}_1^p)W[\widetilde{\rho}_1(Y,x_\perp)]$ and $G_{Y_p,Y_q}(\widetilde{\rho}_1^p,\widetilde{\rho}_1^q)$ is a Green's function which satisfies the JIMWLK equation $\partial_{Y_p} G_{Y_p,Y_q}(\widetilde{\rho}_1^p,\widetilde{\rho}_1^q) = H_{\rm JIMWLK}(\widetilde{\rho}_1^p) G_{Y_p,Y_q}\widetilde{\rho}_1^p,\widetilde{\rho}_1^q)$ with the initial condition $\lim_{Y_p\to Y_q} G_{Y_p,Y_q}(\widetilde{\rho}_1^p,\widetilde{\rho}_1^q) = \delta(\widetilde{\rho}_1^p-\widetilde{\rho}_1^q)$.

This formula holds, as long as one resums only leading logs, for arbitrarily large $\Delta Y = Y_p - Y_q$ (as long as $\Delta Y \ll 2Y_{\rm beam}$). At RHIC energies, where the effects of the quantum evolution of sources are not too large, and the rapidity separation between the produced gluons is restricted as well, the classical picture is a good first approximation. The leading order two gluon correlation is independent of the separation of the gluons in rapidity because the leading order single inclusive spectrum is boost invariant [36]. Therefore, at the classical level, one obtains a geometrical picture of "Glasma flux tubes" that are of transverse size $1/Q_S$ (the typical scale of color correlations in the nuclei) stretching between the two nuclei. These flux tubes have parallel chromo E and B fields and therefore contain topological charge. (They differ from conventional string pictures in these two features.) It follows immediately from this flux tube picture (and from detailed computation) that the two particle correlation has a strength $1/Q_S^2/S_\perp$ given by the geometrical ratio of the area of a flux tube over the transverse overlap area ($S_\perp$) of the two nuclei. The long range rapidity correlations contained in eq. (5.1) can be tested thoroughly in the wide kinematic window in rapidity allowed at the LHC.

An important result is that particles produced in a flux tube are isotropic locally in the rest frame but are collimated in azimuthal angle when boosted by final state transverse "Hubble" flow [37] of the hot and dense matter. Thermalization and flow are local in rapidity, so they do not destroy the initial long range rapidity correlation but have a significant effect on the angular correlation. Combining our dynamical calculation of two particle correlations with a simple "blast wave" model of transverse flow, one obtains reasonable agreement with 200 GeV STAR data on the amplitude of the correlated two particle spectrum relative to the number of binary collisions per participant pair. An important caveat to the work of ref. [36] was that the dependence of the angular width of the ridge on centrality was much wider than seen in the data. Recently however, it has been shown in ref. [38] that a proper treatment of final state flow effects and hadronization can give quantitative agreement of the Glasma flux tube picture for both the amplitude and the angular width as a function of centrality for two different RHIC energies. The Glasma flux tube model has several additional attractive features consistent with observations-a particularly interesting test would be the relative amplitude of the three gluon near side correlation to the two gluon correlation.

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