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Continuum emission from HII regions and dusty molecular clouds

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The classical description of the formation and evolution of ionized (HII) regions associated with early-type stars is reviewed, with illustrative examples taking into account constant and power-law density profiles. The expression for the free-free continuum flux density from such HII regions is then calculated and the main HII region parameters are derived as a function of measurable quantities. The thermal emission from dust grains in molecular clouds is also briefly described and template spectral energy distributions in the simple case of homogeneous, isothermal, spherical clouds are discussed.

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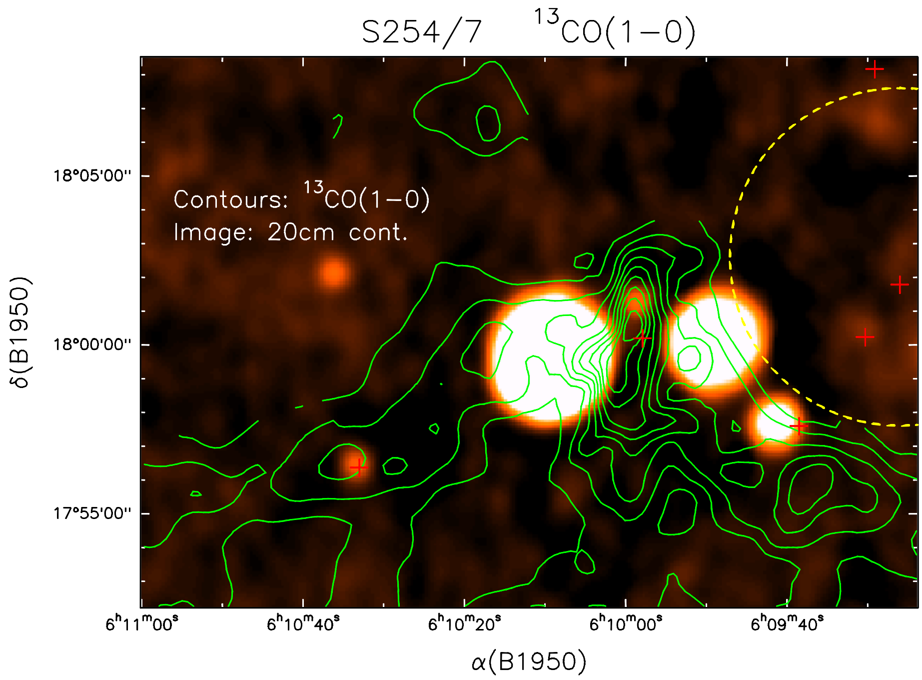

Figure 1: Continuum image (colour scale) at 20 cm of the high-mass star forming region S254/7 obtained with the Very Large Array interferometer of the National Radio Astronomy Observatory. The yellow dashed circle denotes a large HII region which was resolved out by the interferometer. The overlaied contours are a map of the emission integrated under the $J = 1 \rightarrow 0$ rotational transition of the ^{13}CO molecule obtained with the SEST telescope. Clearly, the molecular gas is enshrouding the ionized regions, as expected if the ionizing stars formed inside a molecular cloud.

1. Size of HII regions

It is well known that early-type main-sequence stars are powerful emitters of Lyman continuum radiation. Such stars are born deeply embedded in the densest parts of molecular clouds, whose main constituent is molecular hydrogen (H_2). Stellar photons with wavelengths < 912 Å can first dissociate the H_2 molecules and then ionize the H atoms, thus creating a so-called "HII region" around the star. Figure 1 shows a number of these objects in the S254/7 star forming region, whose radio continuum emission has been imaged with the Very Large Array interferometer. The overlaied map of the ^{13}CO(1–0) line emission demonstrates that the HII regions are still enshrouded by the dense molecular cloud where the O-B stars were born.

While the shape and density of an HII region is strongly dependent on the initial distribution of the circumstellar neutral gas, the size of it is determined in all cases by the balance between ionization and recombinations to the ground state of the H atom. In fact, recombinations to levels $n > 1$ are bound to produce photons with $\lambda > 912$ Å, which are lost to the ionization process. This concept is expressed by the equation

$$N_{\rm Ly} = \int {n_{\rm e}}^2 \alpha_2 {\rm d}V \qquad (1.1)$$

where the integral extends over the whole volume of the HII region and $N_{\rm Ly}$ is the number of ionizing photons emitted by the star per unit time, $n_{\rm e}$ the electron density inside the HII region, and ${n_e}^2\alpha_2$ the number of recombinations per unit time and volume to levels $n \geq 2$. The underlying assumptions are that the HII region is fully ionized and all photons due to recombinations to the ground state, $n = 1$, are immediately re-absorbed by the H atoms ("on-the-spot" approximation).

It is instructive to discuss the simple case of a spherical, isothermal HII region and obtain the radius at which ionization equilibrium is attained, the so-called Strömgren radius, R_S. In particular, in the following we consider a constant density HII region and one with a power-law density profile.

1.1 Constant density HII region

In spherical symmetry, Eq. (1.1) can be rewritten as

$$N_{\rm Ly} = \int_{R_\star}^{R_S} \alpha_2 {n_e}^2 \, 4\pi R^2 \, dR \tag{1.2}$$

with R distance from the star, $R_\star$ stellar radius, and $\alpha_2(\mathrm{cm^3\,s^{-1}}) \simeq 4.1\times10^{-10}\,[T_e(\mathrm{K})]^{-0.8}$. Here T_e is the electron temperature, which is quite constant (typically several 1000 K) inside the HII region due to the nature of the balance between cooling and heating (see [6]).

If the star lies inside a homogeneous molecular cloud with H_2 volume density n_{H_2}, one has also n_e=constant. Taking into account that, in practice, $R_S \gg R_\star$, one obtains

$$R_S \simeq \left(\frac{3}{4\pi}\frac{N_{\rm Ly}}{\alpha_2 {n_e}^2}\right)^{\frac{1}{3}} \tag{1.3}$$

with $n_e = 2n_{H_2}$. The factor 2 is due to the fact that each H_2 molecule produces two electrons (and two protons), because molecular hydrogen is first dissociated into two H atoms (forming a layer of atomic hydrogen around the HII region) and then each H atom is ionized into one electron and one proton.

The condition of ionization equilibrium represented by this expression does not correspond to *pressure* equilibrium, because the external pressure due to the neutral cold H_2 gas cannot balance the internal pressure of a $\sim 10^4$ K bubble of ionized gas. One has,

$$p_{\rm HII} = 2n_e kT_e \gg p_{H_2} = n_{H_2} kT_{H_2} \tag{1.4}$$

because the molecular gas temperature, T_{H_2}, is typically a few 10 K and the electron density is twice the H_2 density (see above). Due to this non-equilibrium situation, the HII region undergoes expansion. Since the ionization time scale is much shorter than the dynamical time scale, during the expansion the ionization equilibrium condition expressed by Eq. (1.3) is satisfied at all times. This implies that $n_e \propto {R_S}^{-\frac{3}{2}}$.

One can calculate the Strömgren radius as a function of time (see [1]):

$$R_S(t) = R_S(0)\left(1 + \frac{7C_{\rm II}}{4R_S}\,t\right)^{\frac{4}{7}} \tag{1.5}$$

where $R_S(0)$ is given by Eq. (1.3) and $C_{\rm II} = (2kT_e/m_H)^{1/2}$ is the isothermal sound speed in the HII region. Expansion will go on until pressure equilibrium is attained, when the final density satisfies the condition $2n_e^f kT_e = n_{H_2} kT_{H_2}$. From this expression and the relation $n_e = 2n_{H_2}$ (see above), one obtains

$$\frac{n_e^f}{n_e} = \frac{n_e^f}{2n_{H_2}} = \frac{T_{H_2}}{4T_e} \simeq 10^{-3}. \tag{1.6}$$

Taking into account that $n_\mathrm{e} \propto {R_\mathrm{S}}^{-\frac{3}{2}}$, one finally gets

$$\frac{{R_\mathrm{S}}^\mathrm{f}}{R_\mathrm{S}} = \left(\frac{n_\mathrm{e}^\mathrm{f}}{n_\mathrm{e}}\right)^{-\frac{2}{3}} = \left(\frac{T_{\mathrm{H}_2}}{4T_\mathrm{e}}\right)^{-\frac{2}{3}} \simeq 100 \tag{1.7}$$

where ${R_\mathrm{S}}^\mathrm{f}$ is the final Strömgren radius. This result shows that the final size of an HII region is much larger than the initial one.

1.2 Power-law density profile

In the real world, OB stars form in the densest parts of molecular clouds, where circumstellar material is free-falling onto the star. The free-fall velocity and conservation of mass are expressed by

$$v_\mathrm{ff} = \sqrt{\frac{2GM_\star}{R}} \tag{1.8}$$

$$\dot{M} = 4\pi R^2 m_{\mathrm{H}_2} n_{\mathrm{H}_2} v_\mathrm{ff} \tag{1.9}$$

from which, taking into account that $n_\mathrm{e} = 2n_{\mathrm{H}_2}$, one obtains the power-law density profile

$$n_\mathrm{e} = \sqrt{\frac{\dot{M}^2}{8\pi^2 GM_\star m_{\mathrm{H}_2}^2}}\,R^{-\frac{3}{2}}. \tag{1.10}$$

In this case the solution of Eq. (1.2) is

$$R_\mathrm{S} = R_\star \exp\left(\frac{N_\mathrm{Ly} 2\pi GM_\star m_{\mathrm{H}_2}^2}{\alpha_2 \dot{M}^2}\right) \tag{1.11}$$

which means that *in spherical symmetry* an HII region is squelched (i.e. $R_\mathrm{S} \simeq R_\star$) if the mass accretion rate onto the star exceeds a critical value:

$$\dot{M} > \sqrt{\frac{N_\mathrm{Ly} 2\pi GM_\star m_{\mathrm{H}_2}^2}{\alpha_2}}. \tag{1.12}$$

2. Continuum emission from HII regions

The main source of continuum emission from HII regions at radio wavelengths ($\lambda > 1$ cm) is *bremsstrahlung* (free-free) radiation. Here, we want to derive an expression for the continuum spectrum of such a region and show that important physical parameters of the HII region can be obtained from this.

In the simple case n_e=constant, the solution of the radiative transfer equation along a line of sight crossing the HII region is:

$$I_\nu = B_\nu(T_\mathrm{e})\left(1 - \mathrm{e}^{-\tau_\mathrm{ff}}\right) \tag{2.1}$$

The free-free optical depth is equal to (see [4])

$$\begin{aligned}\tau_\mathrm{ff} &= 3.014\times10^{-2} T_\mathrm{e}^{-1.5} \nu^{-2} \left[\ln\left(\frac{4.955\times10^{-2}}{\nu}\right) + 1.5\ln(T_\mathrm{e})\right] \int_\mathrm{l.o.s.} {n_\mathrm{e}}^2 \mathrm{d}z \simeq \\ &\simeq 8.235\times10^{-2} T_\mathrm{e}^{-1.35} \nu^{-2.1} \int_\mathrm{l.o.s.} {n_\mathrm{e}}^2 \mathrm{d}z\end{aligned}$$

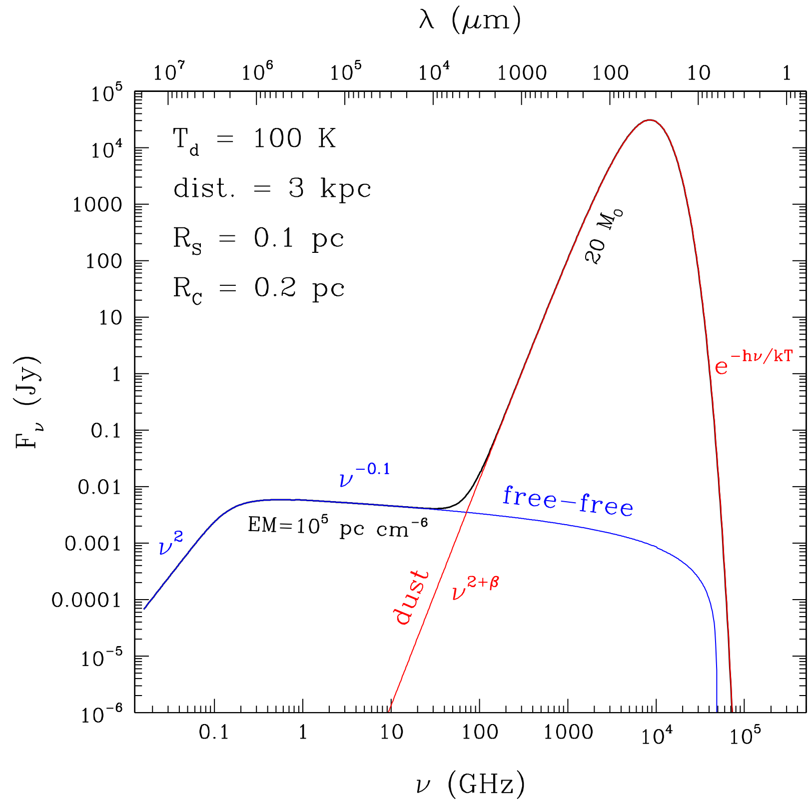

Figure 2: Template spectrum (black curve) of HII region embedded in a dusty molecular cloud. The relevant physical parameters are given in the figure, where $EM = 2R_S\, n_e{}^2$ and R_C is the radius of the cloud. Here, $\beta = 2$ is adopted for the dust opacity. The blue and red curves denote the individual contributions respectively of the free-free and thermal dust emission.

where the integral is made along the line of sight (l.o.s.), T_e is expressed in K, ν in GHz, n_e in cm^{-3}, and z in pc. The quantity $EM = \int_{\rm l.o.s.} n_e{}^2 {\rm d}z$ is called "emission measure".

Although most of the following results hold in general, for the sake of simplicity we will refer to a spherical HII region. In this case $\tau_{\rm ff}(\theta) = \tau_S \sqrt{1 - \left(\frac{\theta}{\theta_S}\right)^2}$ with $\tau_S \simeq 8.235 \times 10^{-2}\ T_e^{-1.35} \nu^{-2.1}\ 2R_S\, n_e{}^2$ and the flux density of the HII region is

$$F_\nu = \int_{\Omega_S} I_\nu {\rm d}\Omega = \int_0^{\theta_S} B_\nu(T_e)\left(1 - {\rm e}^{-\tau_{\rm ff}(\theta)}\right) 2\pi\theta {\rm d}\theta \tag{2.2}$$

$$= \pi\theta_S^2 B_\nu(T_e)\left[1 + \frac{2}{\tau_S}\left({\rm e}^{-\tau_S} + \frac{{\rm e}^{-\tau_S} - 1}{\tau_S}\right)\right] \tag{2.3}$$

In practice, $h\nu \ll kT_e$ and the Rayleigh-Jeans approximation can be used for the black-body brightness: $B_\nu(T_e) \simeq \frac{2kT_e\nu^2}{c^2}$. An example of free-free continuum spectrum from an HII region is shown by the blue curve in Fig. 2.

It is useful to derive the approximate expression for Eq. (2.3) in the optically thin and thick limits. For $\tau_S \gg 1$ one has

$$F_\nu \simeq \pi\theta_S^2 B_\nu(T_e) \propto \theta_S^2\, T_e\, \nu^2 \tag{2.4}$$

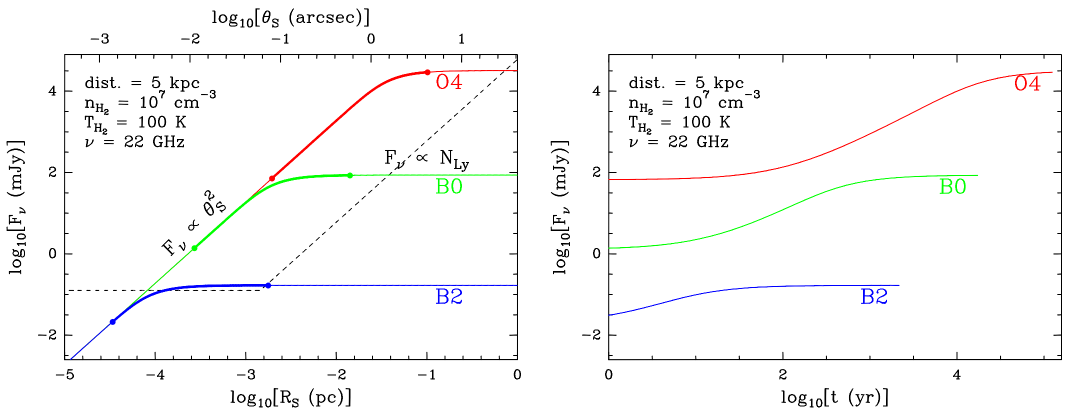

Figure 3: *Left panel*: Flux density of expanding HII regions as a function of the Strömgren radius, for the fiducial set of physical parameters listed in the top left. Different colours correspond to different stellar spectral types, as specified close to each curve. The thick part of each curve comprised between two solid dots represents the flux effectively emitted by the HII region between the initial radius (given by Eq. 1.3 with $n_e = 2\,n_{H_2}$) and the final one (given by Eq. 1.7). For the sake of comparison the dashed lines denote the sensitivity at 22 GHz of the VLA interferometer in the most extended configuration, after 1 hour integration on source. *Right panel*: Flux density as a function of time for the same curves as in the left panel.

from which one can get an estimate of the HII region angular radius $\theta_S \propto \nu^{-1}\sqrt{\frac{F_\nu}{T_e}}$.

For $\tau_S \ll 1$

$$F_\nu \simeq \pi\theta_S^2 B_\nu(T_e)\frac{2}{3}\tau_S \propto \theta_S^2\, T_e^{-0.35}\, {n_e}^2\, R_S\; \nu^{-0.1} \tag{2.5}$$

from which the electron density can be obtained: $n_e \propto \sqrt{\frac{F_\nu T_e^{0.35}\nu^{0.1}}{\theta_S^3 d}}$. Alternatively, the flux density can be expressed as

$$F_\nu \propto T_e^{-0.35}\, {R_S}^3\, {n_e}^2\, d^{-2}\; \nu^{-0.1} \tag{2.6}$$

which makes it possible to derive the Lyman continuum luminosity of the star: $N_{Ly} = \frac{4}{3}\pi {R_S}^3 {n_e}^2 \alpha_2 \propto F_\nu d^2\, T_e^{-0.45}\, \nu^{0.1}$. For convenience of the reader, we also express the electron density and Lyman continuum luminosity in terms of physical units commonly adopted in astronomy:

$$n_e(\mathrm{cm}^{-3}) \;\simeq\; 1.44\times 10^5\; \sqrt{F_\nu(\mathrm{Jy})\, \nu^{0.1}(\mathrm{GHz})\, d^{-1}(\mathrm{kpc})\, \theta_S^{-3}(\mathrm{arcsec})} \tag{2.7}$$

$$N_{Ly}(\mathrm{s}^{-1}) \;\simeq\; 7.55\times 10^{46}\; F_\nu(\mathrm{Jy})\, d^2(\mathrm{kpc})\, \nu^{0.1}(\mathrm{GHz}) \tag{2.8}$$

2.1 Flux density of expanding HII region

As previously discussed, an HII region is in general not in pressure equilibrium and undergoes expansion. Consequently, its flux density will change with time. The exact expression of F_ν as a function of t can be obtained by substituting Eq. (1.5) into Eq. (2.3). This is illustrated in Fig. 3 where F_ν is shown both as a function of R_S and t, for a given set of fiducial parameters of the HII region and molecular gas.

It is instructive, though, to discuss the solution in the optically thick and thin limits. At a given frequency, the free-free opacity is bound to decrease with time, because we have seen that $\tau_{ff} \propto R_S\, {n_e}^2$ and $n_e \propto {R_S}^{-\frac{3}{2}}$, which implies $\tau_{ff} \propto {R_S}^{-\frac{1}{2}}$. Therefore, initially the HII region is likely optically thick and its flux is $F_\nu \propto {R_S}^2$, while later on it will become thin with $F_\nu \propto {R_S}^3 {n_e}^2 \propto N_{Ly}$. This means that after undergoing a rapid increase, the HII region flux will soon saturate at a value depending only on N_{Ly}, i.e. on the spectral type of the star.

3. Dust emission in molecular clouds

Dust grains are a fundamental component of the interstellar medium, despite their mass density being only 1/100 of that of the gas. The goal of this contribution is not to review the origin and physical properties of dust (for which the reader can refer, e.g., to [2]), but to describe the characteristics of the continuum emission from a typical molecular, dusty cloud and show how knowledge of the corresponding spectral energy distribution (SED) can provide us with an estimate of the mean physical parameters of the cloud itself.

The dust emissivity beside depending on the density and temperature of the dust grains is also a function of their composition and shape. While a detailed description of the dust optical properties goes beyond our purposes, one can naïvely describe the dust opacity from the radio to the optical as a decreasing function of wavelength, mostly due to pure absorption in the (sub)millimeter regime (where most grains are larger than the relevant wavelength) and to both absorption and scattering in the IR and optical.

Both observations and theory (e.g. [3]) indicate that a suitable approximation of the dust absorption coefficient in the (sub)millimeter is $\kappa \propto \nu^\beta$, with β ranging approximately from 0 to 2 depending on the mean grain size. As already done for the free-free emission from HII regions, it is instructive to consider a toy model consisting of a homogeneous, isothermal, spherical cloud and calculate the SED of the dust continuum emission. The flux density is given by Eq. (2.3) where T_e and τ_{ff} must be replaced respectively by the dust temperature, T_d, and opacity, τ_d. This is often referred to as "grey body" emission as it may be naïvely described as a black-body spectrum becoming optically thin at long wavelengths. An example is shown by the red curve in Fig. 2.

For a practical example, we consider a cloud with radius $R = 0.5$ pc, T_d=30 K, gas density $\rho = 3 \times 10^{-19}$ g cm^{-3}, a mass gas-to-dust ratio of 100, $\kappa(\mathrm{cm}^{-1}) = 0.005\,\rho(\mathrm{g\,cm}^{-3})\,(\nu/230.6\mathrm{GHz})^\beta$, and $\beta = 2$. For these fiducial values, τ_d=1 at $\lambda = 90$ μm and the Rayleigh-Jeans approximation ($B_\nu(T_d) \simeq \frac{2kT_d\nu^2}{c^2}$) holds for $\lambda \gg 500$ μm. For $\lambda \ll 90$ μm (i.e. in the optically thick regime)

$$F_\nu \simeq \pi\theta^2 B_\nu(T_d) \tag{3.1}$$

with θ angular radius of the cloud. This shows that the measured flux depends only on the apparent source size and on the dust temperature, so that one can obtain an estimate of the latter if the former is known, e.g., from a map of the source. On the other hand, for $\lambda \gg 90$ μm (i.e. in the optically thin limit), one has

$$F_\nu \simeq \pi\theta^2 B_\nu(T_d)\frac{2}{3}\tau_d \tag{3.2}$$

which in the Rayleigh-Jeans regime ($\lambda \gg 500$ μm) takes the form

$$F_\nu \propto T_d M d^{-2} \nu^{\beta+2} \tag{3.3}$$

with M mass of the cloud. One can easily obtain β from the ratio between two values of F_ν measured at two different wavelengths (provided $\lambda \gg 500$ μm), while T_d can be estimated in the optically thick limit, as explained above. Then, Eq. (3.3) can be used to calculate the cloud mass.

While this idealized example is very useful for illustrative purposes, it must be kept in mind that real molecular clouds are much more complex, which makes the derivation of their physical parameters a challenging task. In particular, temperature gradients make questionable the assumption of constant temperature. Inhomogeneities, density gradients, and complex cloud geometries all contribute to cause significant deviations of the SED from the ideal grey-body approximation. This is especially relevant at short wavelengths, where clumpiness may cause strong variations of the measured flux depending also on the orientation of the cloud with respect to the observer. Complex numerical models have been developed to elaborate more realistical SEDs (see [5]).

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