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Hadronic interactions and nuclear physics

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I give an overview of efforts in the last year to calculate interactions among hadrons using lattice QCD. Results discussed include the extraction of low-energy phase shifts and three-body interactions, and the study of pion and kaon condensation. A critical appraisal is offered of recent attempts to calculate nucleon-nucleon and nucleon-hyperon potentials on the lattice.

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1. Motivation

Lattice QCD promises to revolutionize our understanding of multi-hadron systems and nuclear physics. Being one level of difficulty removed from the physics of single hadrons, the study of nuclei has traditionally been confined to phenomenological descriptions which are, for the most part, disconnected from the Standard Model of particle physics. Over the past decade, a great deal of progress has been made in formulating various low-energy effective field theories (EFT) of QCD which describe few-nucleon systems [1, 2], and even finite nuclei [3]. The study of many-hadron systems using lattice QCD has only begun very recently, with accurate results now available for some meson systems, as we will see in this review.

The underlying motivation for studying complicated hadronic systems (like nuclei) varies. While many lattice theorists are interested in calculating hadronic quantities that are relevant for an understanding of physics beyond the standard model, there exists an entirely independent motivation: there are many intrinsically interesting hadronic quantities for which there is essentially no experimental information, and for these quantities lattice QCD calculations will have substantial impact. For instance, while there were several experiments decades ago which measured the low-energy hyperon-nucleon (YN) interaction, very little is known about such basic scattering quantities as the scattering lengths and effective ranges[1] And yet, for instance, the $\Sigma^- n$ interaction is an important ingredient in the nuclear Equation of State that determines the fate of dense astrophysical objects like neutron stars [5]. Yet another quantity of interest is $h_{\pi NN}$, the parity-violating, flavor conserving, pion-nucleon coupling constant. After many decades of intense experimental effort this quantity remains mysterious, and while a lattice determination poses serious challenges [6], the required computer-time resources constitute a minute fraction of what is required for an experiment in nuclear or particle physics. As a final example, the $K\pi$ scattering lengths have recently been determined using lattice QCD [5], providing a prediction for the global experimental effort led by the DIRAC collaboration [7] to measure these quantities by observing the decays of mesonic atoms.

As is well known, lattice correlation functions involving baryons face a significant signal to noise problem; i.e. signal to noise degrades exponentially with time [8]. Since this issue is so fundamental to lattice QCD studies of nuclear physics, I will first review this basic result. I will then discuss progress over the last year in calculating hadron-hadron and multi-hadron interactions using lattice QCD. Before calculating a scattering process that is unknown or poorly known experimentally, it is essential to "benchmark" against quantities that are well known either from experiment and/or from independent theoretical considerations. The s-wave $\pi\pi$ scattering lengths offer a powerful means of benchmarking lattice QCD methods as these quantities are known with remarkable accuracy from the Roy equation method [9]. I will discuss a recent lattice QCD calculation of the $I = 2$ $\pi\pi$ scattering length which has achieved accuracy at the 1% level. Other recent results in the meson sector will also be mentioned, including K^+K^+ scattering and the interactions of up to twelve pions and kaons, which allow determinations of three-body interactions as well as chemical potentials relevant to a description of pion and kaon condensation.

[1] There is some movement to remedy this deficiency; for instance, a recent YN scattering experiment at relatively high energies is described in Ref. [4].

In meson-baryon scattering, benchmarking is significantly more difficult because the channels which have been calculated to date do not have disconnected diagrams and these are not well known experimentally. I will review the status of these calculations.

In nucleon-nucleon scattering, benchmarking the s-wave scattering length poses an extreme challenge to lattice QCD theorists. On the one hand, there is a severe signal to noise problem which requires vast computer resources to overcome. Then there is the problem of extrapolation: since the physical scattering lengths are fine tuned, the chiral EFT description is necessarily non-perturbative. Furthermore, the radius of convergence of the EFT is smaller than in the sector with mesons or a single baryon, and therefore smaller quark masses are required in order to extrapolate. In spite of these difficulties, a great deal of progress is being made in providing a lattice postdiction of the threshold s-wave NN scattering parameters. While a fully-dynamical lattice QCD calculation of low-energy YN phase shifts has been carried out, presently one can only compare to model calculations whose reliability is not clear. I will review existing results for NN and YN scattering.

Recent calculations of NN and YN potentials on the lattice have generated a great deal of interest. Unfortunately, the flaws in these calculations have not engendered as much attention. Therefore, I will give a detailed critique of recent attempts to calculate NN and YN potentials using lattice QCD.

2. Signal to Noise Estimates

As is well known [8], very general field-theoretic arguments allow a robust estimate of the noise to signal ratio of hadronic correlation functions calculated on the lattice. With an eye towards lattice QCD attempts to describe nuclei, it is worth briefly noting the fundamental difference between lattice-measured correlation functions involving mesons and baryons. As an example, consider the noise to signal ratio of a correlation function involving n pion fields, where the small interaction is neglected,

$$\frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{\sqrt{\left(A_2-A_0^2\right)}\, e^{-nm_\pi t}}{\sqrt{N}A_0\, e^{-nm_\pi t}} \sim \frac{1}{\sqrt{N}} \quad . \tag{2.1}$$

Here $\langle\theta(t)\rangle$ is the correlation function, $\sigma(t)$ is the variance and the A_i are amplitudes. It is noteworthy that in this ratio, the time dependence of the variance mirrors the time dependence of the correlator itself. One therefore concludes that correlators involving arbitrary numbers of pions have time-independent errors, as is indeed observed in lattice calculations. This of course renders the study of mesonic correlators quite pleasurable (from the statistical perspective).

The baryons provide a more disturbing story; consider the noise to signal ratio for a proton correlation function:

$$\frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{\sqrt{A_2}\, e^{-\frac{3}{2}m_\pi t}}{\sqrt{N}A_0 e^{-m_p t}} \sim \frac{1}{\sqrt{N}}\, e^{\left(m_p-\frac{3}{2}m_\pi\right)t} \quad . \tag{2.2}$$

Here the variance is dominated by the three-pion state rather than by the proton, and therefore the noise to signal ratio of the proton correlator grows exponentially with time. More generally, for a system of A nucleons, the noise to signal ratio behaves as

$$\frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{1}{\sqrt{N}}\, e^{\,A\left(m_p-\frac{3}{2}m_\pi\right)t} \quad . \tag{2.3}$$

Therefore the situation worsens as one adds nucleons. Fig. 1 compares the theoretical expectation from eq. 2.3 with $A = 2$ to data calculated by the NPLQCD collaboration [5].

These estimates, which follow from very general field theoretic arguments, indicate that nucleon and nuclear physics require *exponentially more resources* than meson physics to achieve the same level of accuracy. There has been an attempt to get around this problem [10] by eliminating the pion zero modes through a clever choice of boundary conditions. However, this method does not yet have a practical implementation. Clearly more effort should be dedicated to this very important problem.

3. n bosons in a box

The ground-state energy of an n-boson system [11] in a finite volume is calculated with an interaction of the form

$$V(\mathbf{r}_1,\ldots,\mathbf{r_n}) = \eta \sum_{i<j}^{n} \delta^{(3)}(\mathbf{r}_i - \mathbf{r}_j) + \eta_3 \sum_{i<j<k}^{n} \delta^{(3)}(\mathbf{r}_i - \mathbf{r}_k)\delta^{(3)}(\mathbf{r}_j - \mathbf{r}_k) + .., \tag{3.1}$$

where η and η_3 are the two- and three-body pseudo-potentials, respectively, and the ellipsis denote higher-body interactions. In general, m-body interactions will enter at $\mathcal{O}(L^{3(1-m)})$ in the large volume expansion. For an s-wave scattering phase shift, $\delta(p)$, the two-body contribution to the

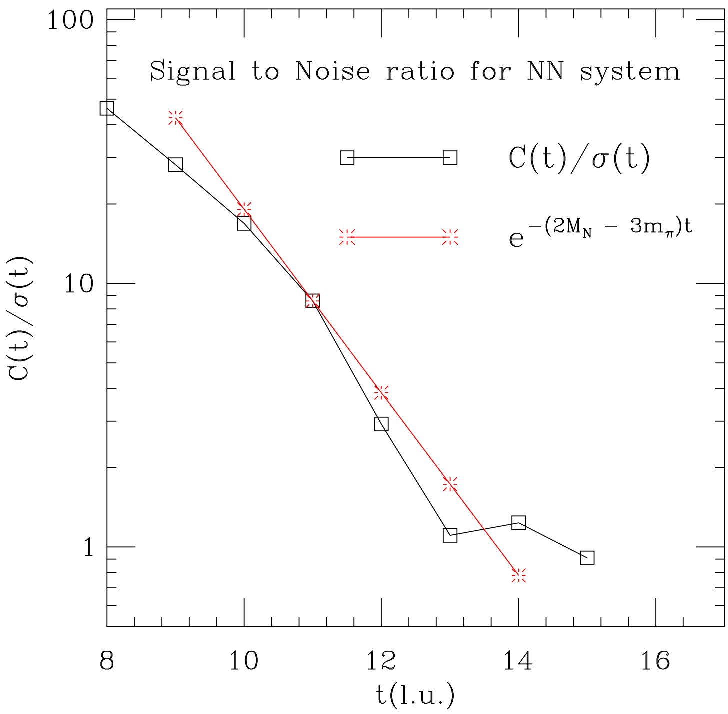

Figure 1: Signal to noise ratio for the NN system in the $^1S_0$ channel. The lattice data (black line) is generated from the MILC coarse ensemble with pion mass ~ 350 MeV, as discussed in Ref. [5]. The theoretical prediction (red line) is from eq. 2.3.

pseudo-potential is given by $\eta = -\frac{4\pi}{M} p^{-1} \tan\delta(p) = \frac{4\pi}{M} a + \frac{2\pi}{M} a^2 r p^2 + \ldots$, keeping only the contributions from the scattering length and effective range, a and r, respectively. To $\mathcal{O}(L^{-6})$ the coefficient of the three-body pseudo-potential, η_3, is momentum independent.

As an example, consider the 2-boson energy. The volume dependence of the energy may be built up using Rayleigh-Schrödinger time-independent perturbation theory. The leading contribution to the perturbative expansion of the energy is given by

$$\Delta E_2^{(1)} = \langle -\mathbf{k}, \mathbf{k} | V(\mathbf{r}_1, \mathbf{r}_2) | -\mathbf{p}, \mathbf{p} \rangle \; , \tag{3.2}$$

where $|-\mathbf{p}, \mathbf{p}\rangle$ are the two-body momentum eigenstates in the center-of-mass system. The single-particle wavefunctions in the finite volume are given by: $\langle \mathbf{r} | \mathbf{p} \rangle = \exp(i\mathbf{k} \cdot \mathbf{r})/L^{3/2}$. Inserting two complete sets of position eigenstates in eq. (3.2), one finds

$$\Delta E_2^{(1)} = \frac{\eta}{L^3} = \frac{4\pi a}{M L^3} \; . \tag{3.3}$$

One sees that in the large volume limit, the two-particle ground-state energy, say, calculated on the lattice, is related to the scattering length. (Excited levels then allow reconstruction of the entire phase shift.) It is straightforward to calculate higher-order $1/L$ corrections in this manner. Similarly, for n bosons in a finite volume, one finds

$$\Delta E_n^{(1)} = \binom{n}{2} \frac{4\pi a}{M L^3} \; . \tag{3.4}$$

The volume dependence of the energy of the n-boson ground state in a periodic cubic spatial volume of periodicity L has now been calculated [12, 13, 14, 15, 16, 11, 17, 18] up to $\mathcal{O}\left(1/L^7\right)$. While the calculational framework described here is non-relativistic, the results remain valid relativistically. In the two-body case, this has been shown by Lüscher [16]. With $n \geq 3$ the interaction of three particles due to the two-body interaction first enters at L^{-5}, and relativistic effects in such interactions are suppressed by $(ML)^{-2}$. Hence, the first relativistic effects occur at $\mathcal{O}(L^{-7})$ [11] and have been calculated perturbatively in Ref. [18]. The finite-volume energy formulas for fermions are considered in Ref. [19].

4. Meson-meson interactions

As the simplest application of eq. (3.3), consider recent results for the $\pi\pi$ interaction [20]. Due to the chiral symmetry of QCD, pion-pion ($\pi\pi$) scattering at low energies is the simplest and best-understood hadron-hadron scattering process. The scattering lengths for $\pi\pi$ scattering in the s-wave are uniquely predicted at leading order in chiral perturbation theory (χ-PT) [21]:

$$m_\pi a_{\pi\pi}^{I=0} = 0.1588 \; ; \; m_\pi a_{\pi\pi}^{I=2} = -0.04537 \quad , \tag{4.1}$$

at the charged pion mass. While experiments do not provide stringent constraints on the scattering lengths, a determination of s-wave $\pi\pi$ scattering lengths using the Roy equations has reached a remarkable level of precision [9, 22]:

$$m_\pi a_{\pi\pi}^{I=0} = 0.220 \pm 0.005 \; ; \; m_\pi a_{\pi\pi}^{I=2} = -0.0444 \pm 0.0010 \quad . \tag{4.2}$$

The Roy equations [23, 24, 25], use dispersion theory to relate scattering data at high energies to the scattering amplitude near threshold. At present lattice QCD can compute $\pi\pi$ scattering only in the $I = 2$ channel as the $I = 0$ channel contains disconnected diagrams. It is of course of great interest to compare the precise Roy equation predictions with lattice QCD calculations. Fig. 2 summarizes the theoretical (left panel) and experimental (right panel) constraints on the s-wave $\pi\pi$ scattering lengths [22]. It is clearly a strong-interaction process where theory has outpaced the very-challenging experimental measurements.

The only existing fully-dynamical lattice QCD prediction of the $I = 2$ $\pi\pi$ scattering length involves a mixed-action lattice QCD scheme of domain-wall valence quarks on a rooted staggered sea. Details of the lattice calculation can be found in Refs. [20, 26]. The energy difference ΔE_2 (and via eq. (3.3) the scattering length) was computed at pion masses, $m_\pi \sim 290$ MeV, 350 MeV, 490 MeV and 590 MeV, and at a single lattice spacing, $b \sim 0.125$ fm and lattice size $L \sim 2.5$ fm [20]. The physical value of the scattering length was obtained using two-flavor mixed-action χ-PT (MAχ-PT) which includes the effect of finite lattice-spacing artifacts to $\mathcal{O}(m_\pi^2 b^2)$ and $\mathcal{O}(b^4)$ [27]. Figure 3 (left panel) is a plot of $m_\pi\, a_{\pi\pi}^{I=2}$ vs. m_π / f_π with the lattice results and the fit curves from MAχ-PT. The final result is:

$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042 \quad , \tag{4.3}$$

where the statistical and systematic uncertainties have been combined in quadrature. Notice that 1% precision is claimed in this result. This result is consistent with all previous determinations within uncertainties (see Figure 3 (right panel)). In particular the agreement between this result and the Roy equation determination is a striking confirmation of the lattice methodology, and a powerful demonstration of the constraining power of chiral symmetry in the meson sector.

It would be of great interest to see other (fully-dynamical) lattice QCD calculations of the

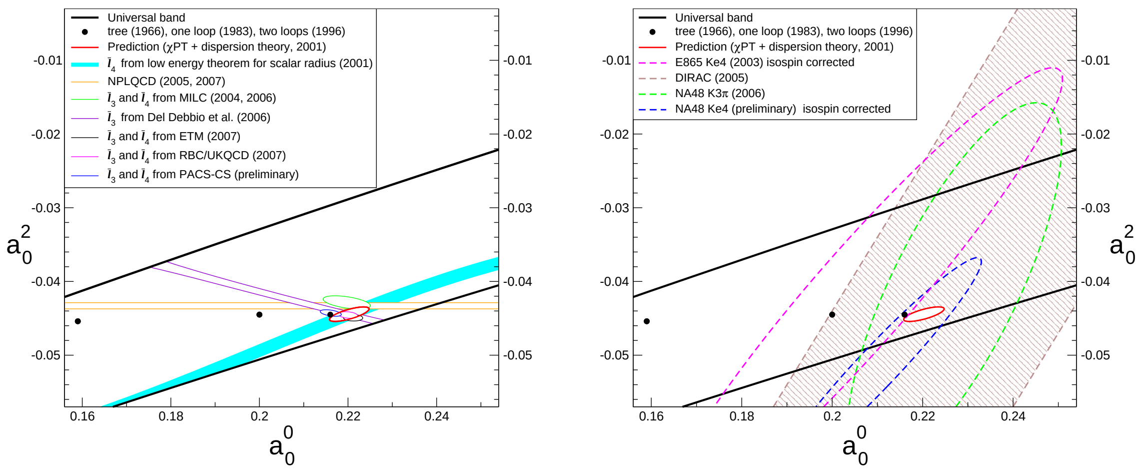

Figure 2: The state of threshold s-wave $\pi\pi$ scattering. Left panel: theoretical results. Noteworthy are the red ellipse from the Roy equation analysis and the orange band from the lattice QCD calculation of the $I = 2$ scattering length, as discussed in the text. Right panel: experimental results. For detailed information about all of the curves on these plot, see Ref. [22]

s-wave $\pi\pi$ scattering lengths using different types of fermions. Recently, χ-PT for Wilson-type quarks has been developed for $\pi\pi$ scattering [30, 31] with an eye towards lattice calculations. One may also wonder about new methodologies for computing scattering which compete with the finite volume method. In this respect, there has been promising recent work on calculating the phase shift from the two-pion wavefunction [32].

The $I = 3/2$, K^+K^+ scattering length has also been computed by the NPLQCD collaboration [33, 26]. At the physical value of m_{K^+}/f_{K^+},

$$m_{K^+}\, a_{K^+K^+} \;=\; -0.352 \pm 0.016 \,, \tag{4.4}$$

where statistical and systematic errors have been added in quadrature. This scattering parameter, which is not measured experimentally, may be useful for the study of kaon interferometry in heavy-ion collisions.

5. Multi-Meson Interactions

Perhaps surprisingly, lattice QCD calculations with up to twelve pions and kaons have recently been carried out. Using the large-volume expansion of the ground state energies of these systems, it has proved possible to extract a signature of a three-pion force [34, 35, 36, 37]. The difficulty with this result is that, unlike the relation between two-body interactions and scattering, it is not

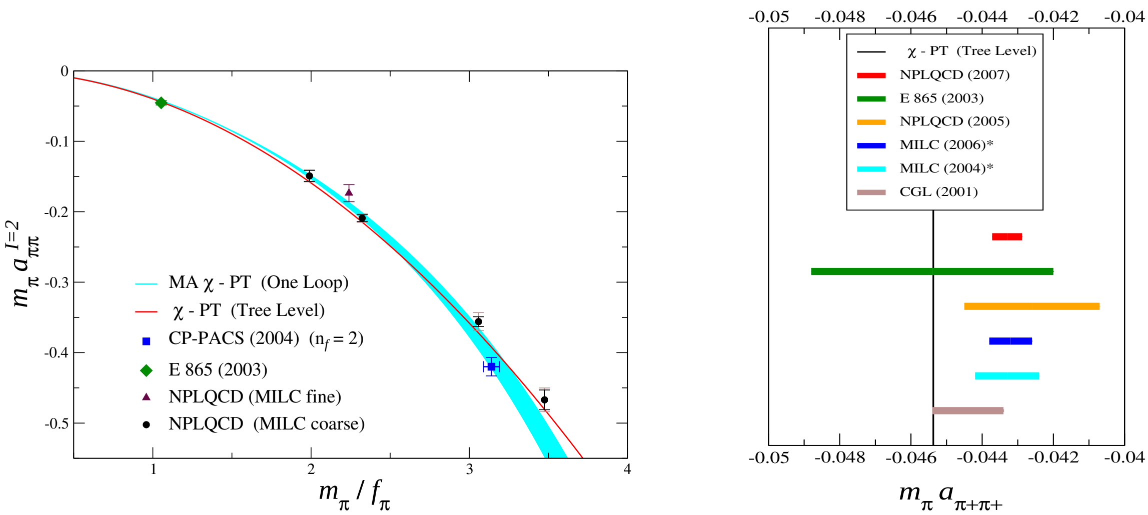

Figure 3: Left panel: $m_\pi\, a_{\pi\pi}^{I=2}$ vs. m_π/f_π (ovals) with statistical (dark bars) and systematic (light bars) uncertainties. Also shown are the experimental value from Ref. [28] (diamond) and the lowest quark mass result of the $n_f = 2$ dynamical calculation of CP-PACS [29] (square). The blue band corresponds to a weighted fit to the lightest three data points using the one-loop MAχ-PT formula (the shaded region corresponds only to the statistical error). The red line is the tree-level χ-PT result. Right panel: A compilation of the various measurements and predictions for the $I = 2$ $\pi\pi$ scattering length. The prediction described in these proceedings is labeled NPLQCD (2007), and the Roy equation determination of Ref. [9] is labeled CGL (2001).

obvious how to relate the three-body interaction (which is proportional to the coefficient of a three-body operator in a non-relativistic Lagrangian) to an observable quantity. Nevertheless, Ref. [35] has noted that a comparison can be made between χ-PT predictions for pion condensation [38] and the lattice QCD results. And indeed the lattice-extracted three-body force appears to be essential for agreement with the χ-PT result, which in principle contains all n-pion forces. Similar results have been found for kaons in Ref. [37]; however the kaon three-body is consistent with zero (See Fig. 4.) This agreement is quite remarkable given that the lattice calculation is clearly not in the thermodynamic limit. This result demonstrates that lattice QCD calculations with a finite number of particles are useful for the study of many-body physics, like pion and kaon condensation.

6. Meson-Baryon Interactions

Pion-nucleon scattering has long been considered a paradigmatic process for the comparison of χ-PT and experiment. To this day, controversy surrounds determinations of the pion-nucleon coupling constant and the pion-nucleon sigma term. While it would be of great interest to calculate scattering parameters for this process on the lattice, pion-nucleon correlation functions necessarily involve disconnected diagrams. Indeed, considering the meson and baryon octets, there are six processes that are free of annihilation: $\pi^+\Sigma^+$, $\pi^+\Xi^0$, K^+p, K^+n, $\bar{K}^0\Sigma^+$, and $\bar{K}^0\Xi^0$. Preliminary lattice QCD results by the NPLQCD collaboration now exist which use domain-wall valence quarks on a rooted

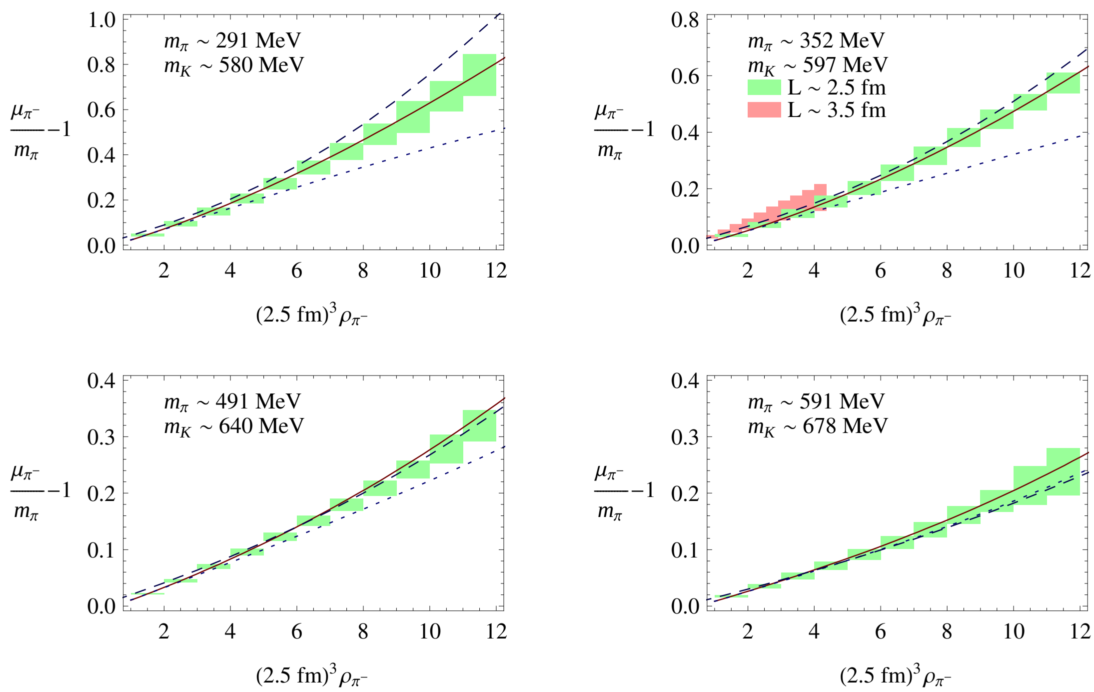

Figure 4: K^- chemical potential as a function of K^- density on the coarse MILC lattices. Finite differences obtained from lattice data appear as boxes. The curves are: leading-order χ-PT (dashed), fitted scattering length with three-body interaction (solid) and same with no three-body interaction (dotted)

staggered sea [39]. An example of effective scattering lengths is shown in Fig. 5. An interesting aspect of this system of six processes is that the χ-PT description at next-to-leading order contains two free parameters [40]. Hence the overconstrained nature of the system provides an interesting test of chiral symmetry and the lattice methodology. It is also worth noting that an understanding of meson-baryon energy levels is an essential ingredient in any attempt to extract excited-baryon masses from lattice calculations [41].

7. Baryon-Baryon Interactions

7.1 Potentials or Phase Shifts? A No-Go Theorem

Modern nucleon-nucleon (NN) potentials fit the NN phase shift "data" at low energies with a chi-squared of order one. One might then envisage calculating an NN potential directly from lattice QCD which could be input into the Schrödinger equation to generate first principles predictions

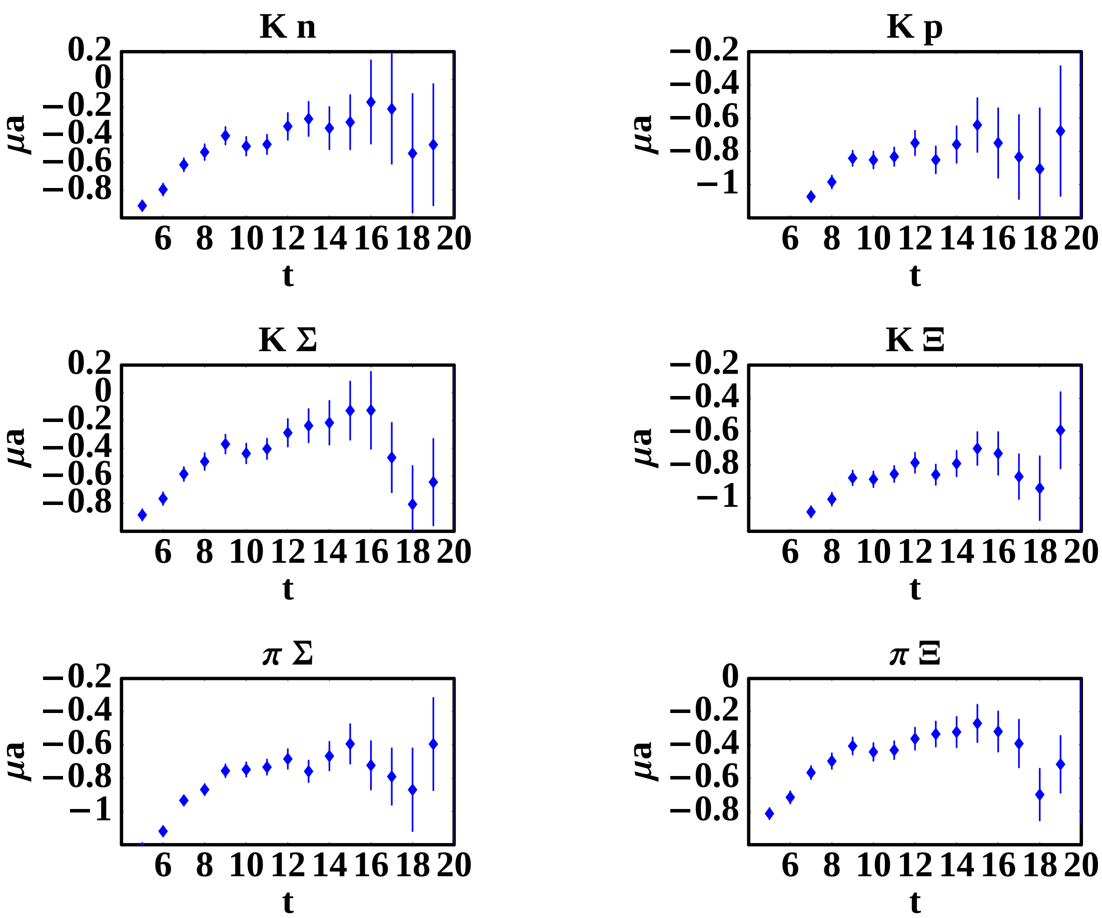

Figure 5: Effective scattering length (times reduced mass) plots for the six annihilation-free meson-baryon processes. For this MILC ensemble the pion mass is roughly 600 MeV, $b \sim 0.125$ fm and $L \sim 2.5$ fm.

for phase shifts. A priori, this seems problematic; after all, unless the scattering particles are infinitely heavy, the potential is not an observable in quantum mechanics, as a unitary transformation will change the potential and the wavefunction in such a way as to leave observables invariant. Therefore at best one can calculate a potential that has been defined as a particular lattice QCD correlation function with the understanding that this choice is not unique.

With this defined NN potential in hand, could one compare it to modern NN potentials? Modern NN potentials tend to treat the short-range part of the NN force in completely different ways, indeed sometimes using arbitrary parameterizations, while maintaining more or less the same almost-perfect agreement with data at low energies. This is no surprise if one thinks in the language of the renormalization group. Ref. [42] has considered momentum-space matrix elements of various modern NN potentials, $V_{NN}(k,k)$, that fit the low-energy data and yet look quite different at short distances. See Fig. 6. If one integrates out physics above a cutoff Λ, one sees that as Λ is reduced, the various potentials, $V_{\mathrm{low\,k}}(k,k)$, approach a universal curve, thus indicating that the details of the short-distance physics are irrelevant to low-energy scattering data. The bottom line is that if one is able to compute the NN potential from QCD, then there is no meaningful way in which the short-distance part of the potential may be compared to phenomenological NN potentials. The only utility of the potential would be to calculate phase shifts.

Recently, it has been claimed that the NN and YN potentials can be extracted from the lattice wavefunctions of two nucleons [43, 44], extending the technique that CP-PACS has successfully used to determine $I = 2$ $\pi\pi$ scattering parameters [45]. Given the widespread attention that this work has received, it is worth repeating here why this method is flawed [46].

The NN correlation function measured on the lattice in Ref. [43] (IAH) is

$$G_{NN}(\mathbf{x},\mathbf{y},t) \;=\; \langle 0|\hat{\mathscr{O}}_1(\mathbf{x},t)^i_\alpha \hat{\mathscr{O}}_1(\mathbf{y},t)^j_\beta \bar{J}(0)|0\rangle$$

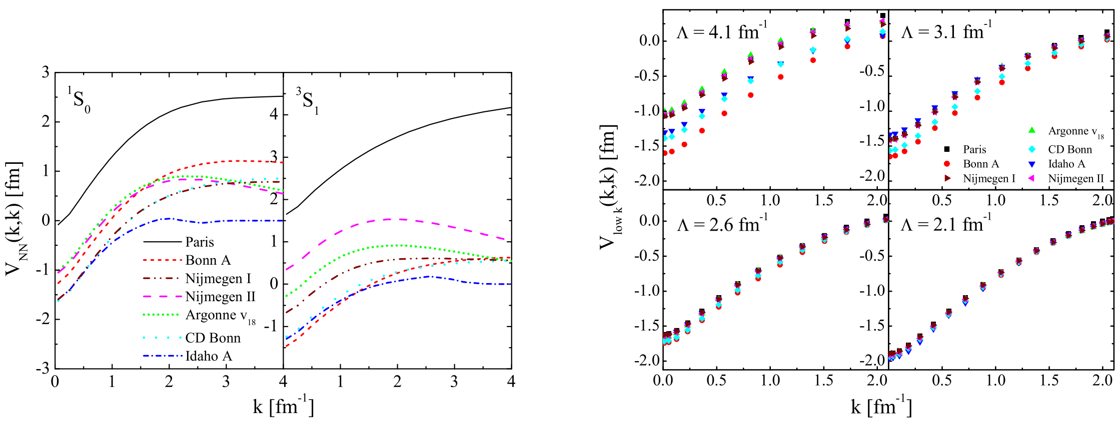

Figure 6: Left panel: momentum-space matrix elements for an assortment of bare NN potentials in the $^1S_0$ and $^3S_1$ channels. Right panel: momentum-space matrix elements for NN potentials with short distance physics excluded beyond a cutoff Λ.

$$= \sum_n \langle 0|\hat{\mathscr{O}}_1(\mathbf{x},0)^i_\alpha \hat{\mathscr{O}}_1(\mathbf{y},0)^j_\beta|\psi_n\rangle\langle\psi_n|\bar{J}(0)|0\rangle\ \frac{e^{-E_n t}}{2E_n} \quad , \tag{7.1}$$

where $\hat{\mathscr{O}}_1(\mathbf{x},t)^i_\alpha$ is a nucleon interpolating field with Dirac-index i, and isospin index α. $\bar{J}$ is a wall-source on the initial time-slice $t_0 = 0$, and $|\psi_n\rangle$ are the eigenstates of the Hamiltonian in the finite-volume. In particular, $|\psi_n\rangle$ are states of definite baryon number and isospin, and transform non-trivially under the hyper-cubic group. Setting $\langle\psi_n|\bar{J}(t_0)|0\rangle = A_n(t_0)$, at long times the correlation function becomes

$$G_{NN}(\mathbf{x},\mathbf{y},t)\ \rightarrow\ A_0(0)\ \langle 0|\hat{\mathscr{O}}_1(\mathbf{x},0)^i_\alpha \hat{\mathscr{O}}_1(\mathbf{y},0)^j_\beta|\psi_0\rangle\ \frac{e^{-E_0 t}}{2E_0} \quad , \tag{7.2}$$

where E_0 is the ground-state energy shifted from $2M$ by boundary effects (ΔE_2 in the notation given above in eq. 3.3). From this object, IAH generate the potential:

$$U_{E_0}(r)\ =\ E_0\ +\ \frac{1}{2\mu}\frac{\nabla^2 G_{NN}}{G_{NN}} \quad , \tag{7.3}$$

where μ is the reduced mass of the NN system. Here the energy dependence of the potential $U_{E_0}(r)$ has been made explicit and $\Psi = G_{NN}$ trivially satisfies the Schrödinger equation for this potential. IAH then assert that $\langle 0|\hat{\mathscr{O}}_1(\mathbf{x},t_0)^i_\alpha \hat{\mathscr{O}}_1(\mathbf{y},t_0)^j_\beta|\psi_0\rangle$ is proportional to the non-relativistic, equal-time, Bethe-Salpeter (BS) wavefunction $\Phi^{ij}_{\alpha\beta} \equiv \langle 0|N(\mathbf{x},t_0)^i_\alpha N(\mathbf{y},t_0)^j_\beta|\psi_0\rangle$, where $N(\mathbf{x},t)$ is a free-field nucleon annihilation operator.

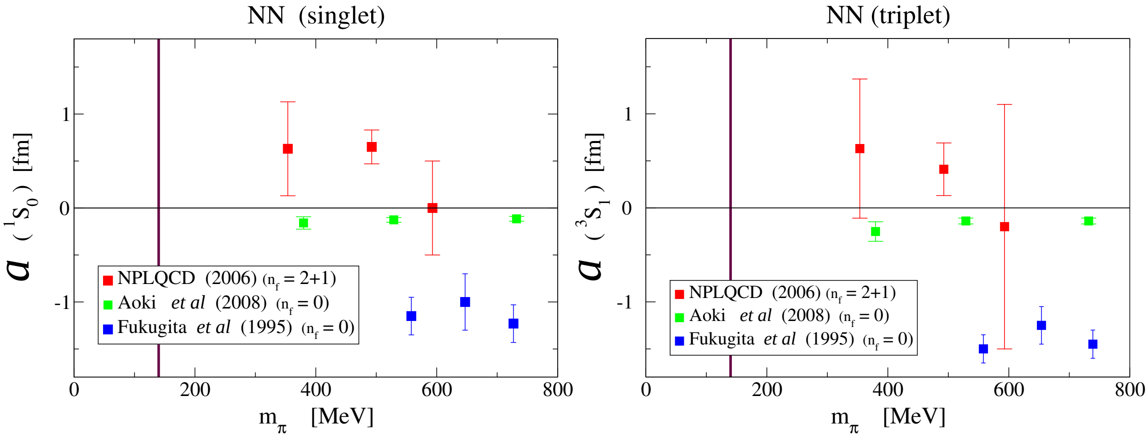

Figure 7: The current status of lattice QCD calculations of the s-wave NN scattering lengths; left panel: $^1S_0$; right panel: $^3S_1$. The solid vertical line indicates the physical pion mass.

However, this identification of the Bethe-Salpeter wavefunction is not correct as the most general form for the matrix element is

$$\langle 0|\hat{\mathscr{O}}_1(\mathbf{x},t_0)^i_\alpha \hat{\mathscr{O}}_1(\mathbf{y},t_0)^j_\beta|\psi_0\rangle\ =\ Z^{(S,I)}_{NN}(|\mathbf{r}|)\ \langle 0|N(\mathbf{x},t_0)^i_\alpha N(\mathbf{y},t_0)^j_\beta|\psi_0\rangle + \ldots , \tag{7.4}$$

where $Z^{(S,I)}_{NN}(|\mathbf{r}|)$ is an unknown function that depends on details of the composite sink, $\hat{\mathscr{O}}^i_{1,\alpha}\hat{\mathscr{O}}^j_{1,\beta}$ and on the separation $\mathbf{r} = \mathbf{x} - \mathbf{y}$. The ellipses denote additional contributions from the tower of states of the same global quantum numbers.

In the limit $|\mathbf{r}| \to \infty$, $Z_{NN}^{(S,I)}(|\mathbf{r}|) \to (\sqrt{Z_N})^2$ (where $Z_N = |\langle 0|\hat{\mathcal{O}}_1|N\rangle|^2$) and the additional terms in eq. (7.4) containing $p > 2$ particles are suppressed. Consequently the scattering parameters can be rigorously extracted from G_{NN}. However, inside the range of the NN interaction ($|\mathbf{r}| < m_\pi^{-1}$), G_{NN} depends explicitly on the interpolating fields that are used. Hence the "potential" defined in eq. 7.3 contains only a single piece of useful physics: the phase shift $\delta(E_0)$, evaluated at the specific energy E_0, which is precisely what one extracts using the finite-volume method described above. The energy dependence of the BS equation has been considered in toy models in Ref. [47], however the implications for the realistic problem are not clear.

Perhaps not surprisingly, this no-go theorem implies that the only meaningful information about hadron-hadron scattering that can be rigorously computed in lattice QCD consists of S-matrix elements.

7.2 Current status of Baryon-Baryon Scattering

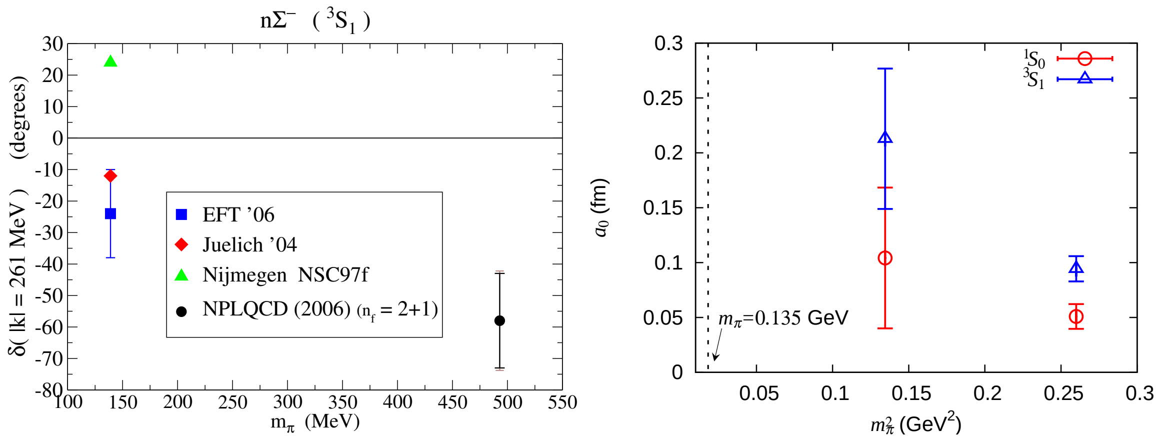

Figure 8: Lattice QCD calculations of the s-wave YN scattering lengths; left panel: the $n\Sigma^-$ $^3S_1$ phase shift evaluated at center-of-mass momentum $|k| = 261$ MeV, compared to various potential models; right panel: the s-wave $p\Xi^0$ scattering lengths at various pion masses in quenched QCD.

Recently, the NPLQCD collaboration has performed the first full-QCD calculation of the s-wave NN scattering lengths [48]. At the pion masses used in these calculations, the NN scattering lengths were found to be of natural size in both channels, and much smaller than the $L \sim 2.5$ fm lattice spatial extent. (See Fig. 7, which also includes the quenched calculations of Refs. [49, 50].) The lowest pion mass calculated (~ 350 MeV) is at the upper limit of where one expects the EFT describing NN interactions to be valid. While the NN system is clearly plagued by the signal to noise problem discussed above, as the NN signals improve with increased statistics, a lattice QCD prediction of the low-energy scattering parameters will become possible. However, it may well be the case that accurate benchmarking for the NN system will initially be done with higher partial waves, whose effective range parameters are of natural size and dominated by pion physics.

Study of the interactions of hyperons with nucleons and nuclei is an exciting area of nuclear physics, as mentioned in the introduction. YN interactions influence the structure and energy-levels of hypernuclei and are expected to be a basic input in studies of the Equation of State of dense stellar matter. Initial investigations of these interactions using lattice QCD have been carried through. Here scattering lengths of natural size are expected as there are probably no YN bound states near threshold. As an example, Fig. 8 (left panel) displays the spin-triplet $n\Sigma^-$ phase shift at (center of mass) momentum $|k| = 493$ MeV and $m_\pi \sim 350$ MeV calculated by the NPLQCD collaboration in fully-dynamical lattice QCD [51], compared to various potential models. Fig. 8 (right panel) displays the s-wave $p\Xi^0$ scattering lengths in a recent quenched calculation, for various pion masses [44].

8. Conclusion

Lattice QCD calculations of two- and three-body interactions of pions and kaons are now a precision science (for those channels that do not involve disconnected diagrams). The study of multi-pion systems has led to the first lattice QCD evidence of many-body forces. While these results provide an important test of the basic methodology for extracting many-body physics from lattice QCD, they are also useful for the study of many-body physics like meson condensation. It will be of great interest to see results of competing calculations with different fermion discretizations in the meson sector.

A milestone for this area of research is to see a definitive signal for nuclear physics. Here one is plagued by a severe signal to noise problem and, for the case of the NN interaction, a fine-tuned system that requires a non-perturbative effective field theory description. However a great deal of progress has been made in a short period; initial results for NN and YN scattering parameters now exist in fully-dynamical lattice calculations and the advent of petascale computing aligns nicely with the need for very high-statistics calculations, which promise to herald a golden age of exploration for nuclear physics.

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