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Nearly conformal electroweak sector with chiral fermions

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$SU(3)$ gauge theory with dynamical overlap fermions in the 2-index symmetric (sextet) representation is considered. This model may be a viable model of the electroweak symmetry breaking sector along the lines of the walking technicolor paradigm. The number of fermion species is chosen such that the theory is expected to be below the conformal window. We will discuss how the ε-regime and random matrix theory can be used to test whether at any given set of parameters (N_c, N_f, representation) the theory is in the conformal phase or indeed just below it. Quenched Monte Carlo results are included in the fundamental representation and also preliminary dynamical ones in the 2-index symmetric representation.

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1. Introduction

Model building of a strongly interacting electroweak sector, with or without a Higgs resonance, requires the knowledge of the phase diagram of non-abelian gauge theories for varying number of colors N_c, number of fermion flavors N_f, and representation R. For fixed N_c and R the theory is generically in the chiral symmetry broken phase for low N_f and the conformal phase for high N_f as long as asymptotic freedom is maintained, i.e. N_f is not too high. Certain models require N_f to be just below the conformal window along the lines of the walking technicolor paradigm [1] and the knowledge of the critical N_f separating the two phases is essential.

Mapping out the phase diagram in the space of N_c, N_f and R is an interesting problem on its own and can be useful for model builders with different motivations such as unparticles. We are first and foremost concerned with the Higgs mechanism though.

In this context the parameters N_c, N_f and R are not only restricted by the phase diagram but also by electroweak precision data and the symmetry breaking pattern necessary for generating masses for the W and Z bosons. Consistency with electroweak precision data requires a small S-parameter while the simplest symmetry breaking pattern is the one which generates exactly 3 Goldstone bosons with no (techni)pions left over after the massive gauge bosons acquired their masses. The simplest model fulfilling these requirements is $SU(3)$ gauge group with $N_f = 2$ fermions in the 2-index symmetric ($2S$) representation which is the topic of our study.

In a numerical simulation at finite volume, finite lattice spacing and (usually) finite quark masses it is a non-trivial task to determine whether the theory is conformal in the continuum, massless quark and infinite volume limits, or chiral symmetry is broken just as in QCD. In section 3 a method is introduced that is capable of distinguishing the two phases based on the behavior of the low-lying Dirac eigenvalues. If the theory is QCD-like chiral symmetry is spontaneously broken and random matrix theory (RMT) will predict the distribution of low-lying eigenvalues in the ε-regime [2], whereas in the conformal phase chiral symmetry is unbroken and the spectral density of the Dirac operator goes to zero around $\lambda = 0$. One particular advantage of RMT is that it works for finite (but small of course) quark mass. This method of distinguishing phases with the help of the Dirac spectrum has been applied for dynamical staggered fermions in [3] which complements Schrodinger functional and finite temperature based investigations of similar dynamical staggered models [4, 5].

2. Perturbative expectations

As is well-known the 2-loop perturbative β-function [6] can be used to estimate the critical N_f value above which the theory is conformal and below which chiral symmetry is spontaneously broken [7]. If the first two coefficients are β_1 and β_2, asymptotic freedom requires $\beta_1 < 0$ otherwise the theory is free in the continuum. If $\beta_1 < 0$ and $\beta_2 > 0$ a non-trivial zero of the β-function exists hinting at a non-trivial IR fixed point. However if the fixed point value of the coupling is too large chiral symmetry is spontaneously broken before the flow in the IR can reach the would-be fixed point. It is nevertheless expected that a critical N_f^{crit} value exists above which the theory is really conformal even non-perturbatively.

The value N_f^{crit} can be estimated in the ladder approximation by the requirement that the anomalous dimension of $\bar{\psi}\psi$ reaches $\gamma = 1$ [8]. Using this bound, the conformal window for $SU(2)$ and representations $j = 1/2, 1$ and $3/2$ is expected to be $8 < N_f < 11$, none (for integer N_f) and $N_f = 1$ respectively, and no window for $j > 3/2$. For $SU(3)$ and fundamental, adjoint = $2A$ and $2S$ representations the conformal window is expected to be $12 < N_f < 16$, none (for integer N_f) and $N_f = 3$ respectively.

3. Dirac spectrum

To what extent the perturbative expectations of the previous section are justified is an open question in general. Non-perturbative tests of these expectations have been performed for various gauge groups, flavor number and representations [4, 5, 9, 10, 11, 12, 13] using various methods.

The low-lying spectrum of the Dirac operator is sensitive to the IR dynamics of the theory and shows characteristically different behavior in the conformal and QCD-like phases. Its measurement is in principle straightforward in a lattice simulation hence it is a good candidate to distinguish the two phases.

3.1 Chirally broken phase, ε-regime, random matrix theory

If chiral symmetry is spontaneously broken, the Banks-Casher relation connects the spectral density $\rho(\lambda)$ of the Dirac operator around zero to the chiral condensate [14],

$$\Sigma = \lim_{m\to 0}\lim_{V\to\infty} \frac{\pi\rho(0)}{V} \,. \tag{3.1}$$

It also implies that the low-lying eigenvalues are dense in the sense that the average spacing is inversely proportional to the volume,

$$\Delta\lambda = \frac{\pi}{\Sigma V} \,. \tag{3.2}$$

It has been suggested long ago that if the bare parameters β, m are tuned to the ε-regime, i.e. such that $m_\pi < L^{-1} < f_\pi$ the low-lying Dirac spectrum follows the predictions of a random matrix theory [15, 16]. The corresponding random matrix model is only sensitive to the pattern of chiral symmetry breaking, the topological charge and the rescaled fermion mass once the eigenvalues are also rescaled by the same factor ΣV.

More precisely, random matrix theory provides analytic formulae for the microscopic spectral density

$$\rho_S(\zeta) = \frac{1}{\Sigma V}\rho\left(\frac{\zeta}{\Sigma V}\right) = \sum_{k=0}^{\infty} p_k(\zeta) \,, \tag{3.3}$$

and the individual eigenvalue distributions $p_k(\zeta)$ where $\zeta = \lambda\Sigma V$. The distributions $p_k(\zeta)$ only depend on $\mu = m\Sigma V$, N_f and the topological charge ν. The value of Σ can be obtained by using μ as the fitting parameter to have

$$\frac{\langle\zeta_k\rangle}{\mu} = \frac{\langle\lambda_k\rangle}{m} \,, \tag{3.4}$$

where the left hand side is calculated in random matrix theory at a fixed charge ν while the right hand side is measured in the simulation in the given sector ν. Which eigenvalue λ_k and which sector ν is used is arbitrary in principle (as long as k is not too large, say, $k = 1,2,3$) and the quality of the whole procedure may be characterized by the (in)consistency of the obtained $\Sigma = \mu/(mV)$ values for various k and/or ν.

A more stringent test is the comparison of $p_k(\zeta)$ between the random matrix theory predictions and the simulation once a consistent Σ and corresponding μ have been obtained from the above fitting procedure. The agreement is only expected for the first few eigenvalues.

In order to see the effects of dynamical quarks the lowest eigenvalue should be larger than the fermion mass m. Otherwise the simulation is effectively quenched and random matrix theory will only agree at $N_f = 0$.

Out of the two requirements of the ε-regime, $m_\pi L < 1$ can be satisfied by tuning the fermion mass to a small value at any L. However the second requirement, $f_\pi L > 1$, is largely independent of m provided it is small enough and puts a lower bound on L. As the lower edge of the conformal window is approached from below, f_π is expected to decrease and eventually will vanish as the theory becomes conformal. Hence the $f_\pi L > 1$ condition will be more and more difficult to satisfy and larger and larger lattices will be needed the closer the theory is to the conformal window. As a result the study of nearly conformal (or walking) technicolor models is very challenging in the ε-regime.

It should be noted that the requirement $f_\pi L > 1$ is valid up to numerical constants only. From the behavior of the rotator and Goldstone spectrum of the chiral Lagrangian it can be made more precise as $f_\pi L > 1/\sqrt{2\pi}$ which is the requirement of these two spectra to separate from each other. In fact we will see that in some cases RMT gives a good description even if $f_\pi L < 1$ which is probably due to the above numerical constant $1/\sqrt{2\pi} = 0.3989...$ being smaller than 1.

3.2 Conformal phase

In the conformal phase no scale is generated and $\Sigma = 0$. The spectral density of the Dirac operator around $\lambda \sim 0$ behaves as

$$\rho(\lambda) \sim \lambda^{3+\gamma} \tag{3.5}$$

for massless quarks in the continuum and at infinite volume. Here γ is the anomalous dimension of $\bar{\psi}\psi$.

The exact dependence of γ on the conformal fixed point coupling g_* is in principle calculable in perturbation theory since g_* is presumably not large (otherwise chiral symmetry would be spontaneously broken). Certainly, $\gamma \sim g_*^2$. Of course only a non-perturbative treatment can decide whether there is room for a fixed point coupling which is large enough to be significantly different from perturbation theory and small enough so that chiral symmetry is not broken.

How the (3.5) behavior is modified by finite volume and finite quark mass is an open question that we hope to address in the future. Certainly, in the free $g_* = 0$ case the average eigenvalue spacing is inversely proportional to the linear size L of the box. The characteristic feature that is expected to hold even for a finite $g_* > 0$ is that the average eigenvalue spacing for small eigenvalues is much less dense than in the chirally broken case where it is inversely proportional to the 4-volume V; see (3.2).

If the volume is too small, the chiral condensate is squeezed out of the box and the theory behaves perturbatively even in the case when chiral symmetry is broken in an infinite volume. Hence great care is needed not to confuse a small volume chiral symmetry breaking and a (small or large volume) conformal theory which is also behaving more-or-less perturbatively.

4. Our model, $SU(3)$ with $N_f = 2$ in $2S$ representation

The simplest example of a model that – according to the perturbative expectations – is just below the conformal window, has a relatively low N_f value so that the S-parameter is relatively small, and has precisely 3 Goldstone bosons is $N_c = 3$, $N_f = 2$ and $R = 2S$. This model has been studied in [11, 12, 13] using Wilson fermions on rather small lattices and it was found to be already in the conformal window although it was indicated that more complicated possibilites are also allowed by the data.

Since exact chiral symmetry is important both for QCD-like and conformal theories we chose to use overlap fermions [17]. The simulation has to be carried out at a fixed topological charge. There are two methods available for simulating dynamical overlap fermions at fixed topology. One is the reflection/refraction algorithm [18] but always reflecting on the topological boundary. The other is employing a pair of extra Wilson fermions to suppress exact zero modes thereby suppressing tunnelling between sectors [19]. We used the second method, which is much faster, in this study.

5. Preliminary results

5.1 Quenched simulations

In order to see how well actual simulations agree or disagree with the predictions of random matrix theory (RMT) we have tested the RMT predictions for $N_f = 0$ and overlap valence quarks in the fundamental representation since in the quenched approximation chiral symmetry is guaranteed to be broken. This setup is identical to [20] but actual eigenvalue distributions were not presented there. Since valuable information can be gained from these we decided to redo this analysis on 12^4 lattices at $\beta = 5.8458$ which corresponds to a lattice size of $L = 1.49$ fm. All our parameters were the same as in [20]. In particular we used the Wilson gauge action and an unsmeared overlap operator. Our ensemble consists of 1500 configurations. The results of [20] for expectation value ratios $\langle\lambda_i\rangle/\langle\lambda_j\rangle$ have been reproduced within 1-sigma precision with occasional 1.2-sigma deviations.

The distribution of the kth eigenvalue in various topological sectors $|Q|$ are shown in figure 1 together with the RMT predictions once Σ has been fitted from $k = 1$ and $Q = 0$ as described by (3.4). Clearly, not only the expectation values $\langle\lambda_i\rangle$ follow the RMT predictions but also their width. This level of agreement with RMT was not expected since $f_\pi L < 1$ for this ensemble, however as indicated in section 3.1 a more accurate requirement of the ε-regime is $f_\pi L > 1/\sqrt{2\pi}$ which this ensemble does fulfill.

A similar comparison for the 2-index symmetric representation in the quenched approximation is ongoing. Since RMT is only sensitive to the topological charge, N_f, μ and the pattern of chiral symmetry breaking, the same random matrix model is expected to describe this representation as

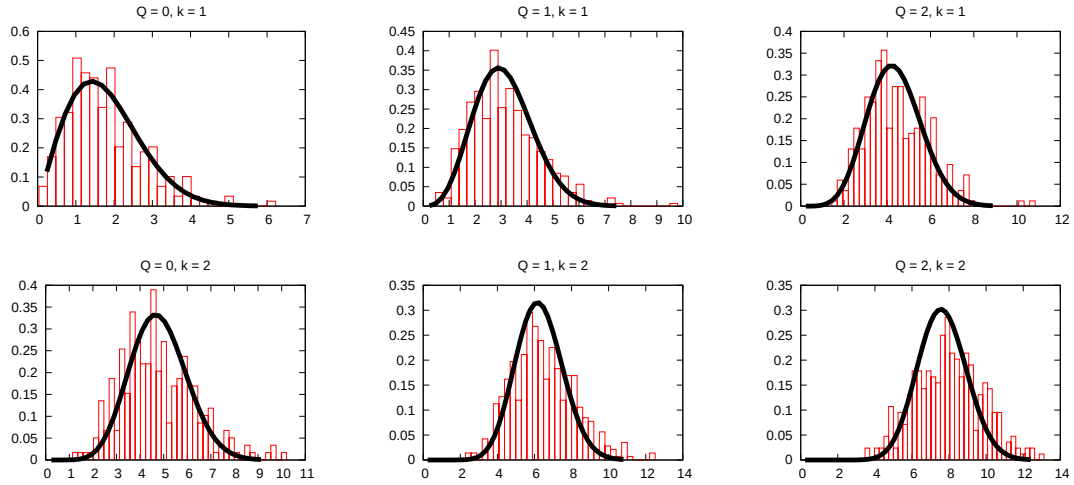

Figure 1: Rescaled quenched eigenvalue distributions $p_k(\zeta)$ in the fundamental representation and $N_f = 0$ RMT predictions for $|Q| = 0, 1, 2$ and $k = 1, 2$

the one used for the fundamental. Agreement with the same RMT using a different representation than fundamental will be a non-trivial check of its universality.

5.2 Dynamical simulations

In the 2-index symmetric representation three dynamical ensembles were generated on 6^4 lattices using the tree-level improved Symanzik gauge action at $\beta = 4.850$, 4.975 and 5.100 and $N_f = 2$ flavors of massive quarks with $m = 0.05$. The negative Wilson mass in the overlap operator was $m_W = -1.3$ and 2 levels of stout smearing with smearing parameter $\rho = 0.15$ have been applied. The topology change suppressing action of [19] was used with mass $M = 0.2$ for the ghost Wilson fermions and only the topological sector $Q = 0$ was sampled. Since chiral symmetry is preserved by overlap fermions at finite lattice spacing $N_f = 2$ RMT is applicable. Fitting Σ from the average first eigenvalue as in (3.4) one obtains $0.083(4)$, $0.084(4)$ and $0.080(4)$ in lattice units for the three β values respectively.

The eigenvalue distributions look qualitatively the same for the three β values and the first 3 eigenvalues are plotted in figure 2 for $\beta = 4.850$ together with the $N_f = 2$ RMT predictions after rescaling both the eigenvalues and the quark mass.

Clearly, the RMT predictions are very far from the simulation results, neither the averages nor the widths follow the RMT curves. This may be due to several reasons the most likely of which is small volume. We have not measured either m_π or f_π so it is not clear if the simulation was in the ε-regime at all. Simulations on larger volumes as well as measurements of m_π and f_π are ongoing.

If the larger volume simulations agree with the above conclusion the 2-index symmetric representation for gauge group $SU(3)$ and $N_f = 2$ is already in the conformal window.

6. Conclusions and outlook

Needless to say that the results on the 2-index symmetric representation are preliminary. The

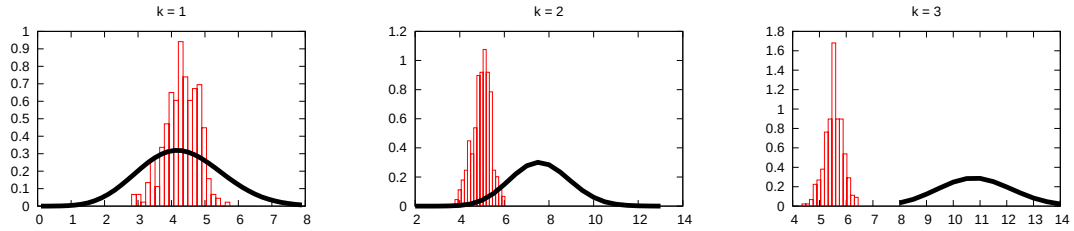

Figure 2: Rescaled dynamical eigenvalue distributions $p_k(\zeta)$ in the 2-index symmetric representation and $N_f = 2$ RMT predictions for $Q = 0$ and $k = 1, 2, 3$

quenched fundamental representation simulations shows that the RMT predictions are very precise for the first few eigenvalues once the volume is large enough. A similar conclusion is expected for the quenched 2*S* representation which will in addition test the universality of RMT. Presently, the result of the 6^4 dynamical 2*S* representation simulation are preliminary and the deviation from RMT is thought to be due to small volume. Larger volume simulations are ongoing for both the quenched and fully dynamical cases.

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