Spectrum of SU(2) gauge theory with two fermions in the adjoint representation

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We present preliminary results of lattice simulations of SU(2) gauge theory with two Wilson fermions in the adjoint representation. This theory has recently attracted considerable attention because it might possess an infrared fixed point (or an almost-fixed-point), and hence be a candidate for a walking technicolor theory. In this work we study the particle spectrum of the theory, and compare it with more familiar spectrum of the theory with SU(2) gauge fields and two flavors of fundamental representation fermions.

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1. Introduction

Gauge theories with fermions in other than fundamental representation may have qualitatively different features from QCD. One of these features is so called "walking" behaviour, which is required by a class of technicolor theories [1]. Technicolor is an extension to the standard model, where the Higgs boson is replaced with a composite particle, essentially a scalar "meson" which consists of techniquarks interacting through a gauge interaction, technicolor.

In this project we study the properties of SU(2) gauge theory with two Dirac fermions in the adjoint (2-index symmetric) representation. This is a candidate theory for a "minimal" (i.e. simplest) walking technicolor [2]. For this to be viable, the theory should either have an IR fixed point (where it shows conformal behaviour) or an almost fixed point, where the coupling evolves extremely slowly with the energy scale. This theory has been studied previously in [3, 4, 5]. (See also [6, 7, 8, 9] for recent studies of related theories.)

In this first stage we shall investigate the excitation spectrum of the theory and estimate its lattice phase diagram. We use considerably larger volumes than the earlier published work. Full results will be published in [10].

2. Lattice action

The lattice action of SU(2)+adjoint quark theory is $S = S_g + S_f$, where S_g is the standard plaquette gauge action and S_f is the Wilson fermion action for spinors in the adjoint representation:

$$S_f = a^4 \sum_x \bar{\psi}(x) D \psi(x) \tag{2.1}$$

$$= \sum_x \bar{\psi}\psi(x) - \kappa \sum_\mu \left[\bar{\psi}(r - \gamma_\mu) V_\mu(x) \psi(x) + \bar{\psi}(r + \gamma_\mu) V_\mu^\dagger(x - \mu) \psi(x - \mu) \right]. \tag{2.2}$$

Here the adjoint link variables V are related to the fundamental representation ones as

$$V_\mu^{ab}(x) = 2\mathrm{Tr}(S^a U_\mu(x) S^b U_\mu^\dagger(x)), \tag{2.3}$$

where $S^a = \frac{1}{2}\sigma_a$, $a = 1, 2, 3$ are the generators of the fundamental representation.

As usual, the lattice action is parametrized with

$$\beta = \frac{2N_c}{g^2} = \frac{4}{g^2} \quad \text{and} \quad \kappa = \frac{1}{8 + 2m_{Q,\text{bare}}}, \tag{2.4}$$

where $m_{Q,\text{bare}}$ is the bare mass parameter.

3. Lattice simulations

The simulations were carried out with five different values of $\beta = 1.3$, 1.7, 1.9, 2.2 and 2.5. For each value of β we used 5 to 11 different values of κ. The volumes used were 24^4 and 32^4. The updates were performed using standard hybrid Monte Carlo algorithm and the number of trajectories was 100-700 for a single run. The timestep $\Delta\tau$ used was 0.02 for larger values of mass and was decreased to 0.004 closer to the zero mass limit. The number of integration steps N_s was chosen so that $N_s \times \Delta\tau \sim \mathcal{O}(1)$.

Particle	quark content	QCD equivalent
Pseudoscalar meson	$U\gamma_5\bar{U}$	π
Vector meson	$U\gamma_\mu\bar{U}$	ρ
Axial vector meson	$U\gamma_\mu\gamma_5 U$	b_1
"Higgs"	$U\bar{U}+D\bar{D}$	f_0 or σ
Spin 1/2 baryon	UUD	proton
Spin 3/2 baryon	UUU	Δ
Quark Gluon	UG	NA

Table 1: List of some particles in adjoint representation

We also performed some simulations in fundamental representation in order to validate the algorithms and to confirm that we observe qualitatively different behaviour and not e.g. lattice artifacts.

3.1 Phase diagram

First, in order to obtain the relevant parameter range we probe the phase diagram of the theory. We are especially interested in the critical line $\kappa_c(\beta)$, along which the quark mass vanishes. The quark mass is measured using the axial Ward identity ("PCAC mass"):

$$m_Q = \lim_{t\to\infty} \frac{1}{2}\frac{\partial_t V_{PS}}{V_{PP}}, \tag{3.1}$$

where the currents are

$$V_{PS}(x_0) = a^3 \sum_{x_1,x_2,x_3} \langle \bar{u}(x)\gamma_5 d(x)\bar{u}(0)\gamma_5 d(0)\rangle \tag{3.2}$$

$$V_{PP}(x_0) = a^3 \sum_{x_1,x_2,x_3} \langle \bar{u}(x)\gamma_0 d\gamma_5(x)\bar{u}(0)\gamma_0\gamma_5 d(0)\rangle. \tag{3.3}$$

On the left panel of Fig. 1 we have plotted the measured quark masses against $1/\kappa$, and on the right panel we have extrapolated the results to the zero quark mass limit. We include here also the results from Del Debbio et al. [3] and Catterall et al. [5]. All of the results agree well with each other.

Authors of [5] find a phase transition around $\beta = 1.9$ at zero mass. We also observe a change in the behaviour of the system at $\beta \approx 1.9$: when $\beta \lesssim 1.9$ and we decrease the quark mass (κ is increased), the system has a sharp phase transition around $m_Q \approx 0$, making the simulations in practice impossible at large volumes. This is especially noticeable at $\beta = 1.9$ simulations. This new phase is an artifact of Wilson fermions, corresponding to the "Aoki phase" of lattice QCD. On the other hand, when $\beta \gtrsim 1.9$ there is no sharp transition and, if the volume is not too large, we can decrease m_Q below zero without problems.

3.2 Mass spectrum

The "hadron" spectrum with adjoint representation quarks has more states than with the fundamental representation quarks. In Table 1 we have listed some of the low energy states. Here we
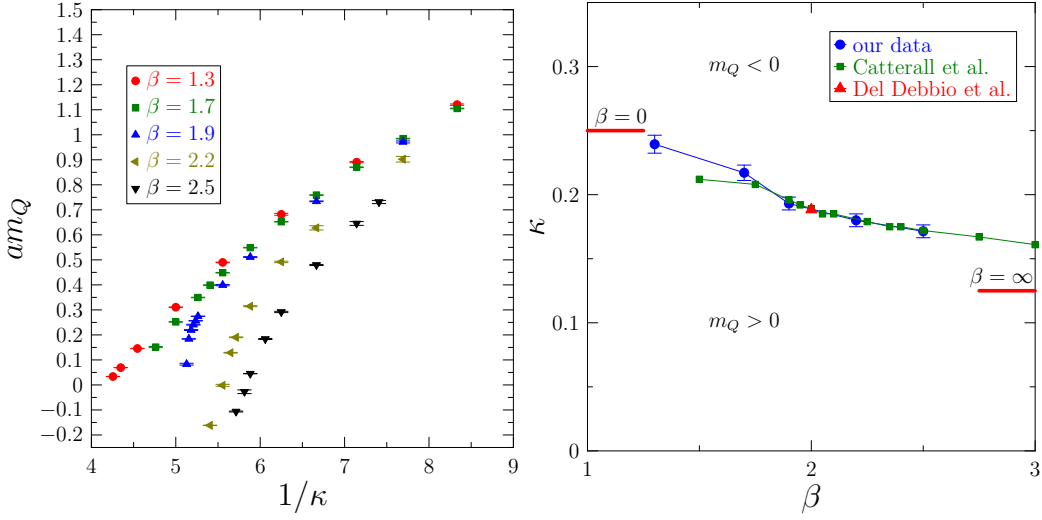
Figure 1: Left: Quark masses as functions of κ. Right: The phase diagram with $m_Q = 0$ line. Our results agree well with other groups.

will present the measurements of the pseudoscalar and vector 2-quark states, "mesons", and the spin 1/2 and spin 3/2 3-quark states, "baryons".

The measurement of the axial vector would be interesting, because it has been speculated that ratio M_{Axial}/M_ρ could be smaller than one for a conformal theory [11]. Unfortunately, we do not have yet good enough statistics at large volumes to obtain reliable measurement. The "Higgs" particle includes a quark disconnected part and we have not attempted to measure it.

The masses of the excitations are estimated by fits to the time sliced averaged correlation functions. We use wall sources at timeslice $t = 0$ (with Coulomb gauge fixing) and point sinks. For example, the correlation function for mesons reads

$$G_{\mathscr{O}}(t) \propto \sum_{x,y_1,y_2} \langle \bar{\psi}(x,t)\Gamma_{\mathscr{O}}\psi(x,t)\bar{\psi}(y_1,0)\Gamma_{\mathscr{O}}\psi(y_2,0)\rangle, \tag{3.4}$$

where $\Gamma_{\mathscr{O}} = \gamma_5$ for the pseudoscalar and $\Gamma_{\mathscr{O}} = \gamma_\mu$, $\mu = 1,2,3$ for the vector meson. The baryon correlation functions are measured analogously.

The masses of the pseudoscalar and vector mesons are plotted in Figs. 2 and 3. For a conformal theory one expects that all particle masses approach zero as $m_Q \to 0$, with the same exponent. However, at small β we observe a more or less standard pattern of chiral symmetry breaking: as $m_Q \to 0$ the pseudoscalar meson (Goldstone bosons for chiral symmetry breaking) mass behaves approximately as $\propto \sqrt{m_Q}$ at small m_Q, whereas vector meson remains massive. On the other hand, at large $\beta \gtrsim 2$ the mesons are practically degenerate, and their masses almost vanish as $m_Q \to 0$. The baryon masses are shown in Fig. 4 and show a similar pattern: at small β they extrapolate to a finite value as $m_Q \to 0$, but at large β the masses decrease linearly, but with a small intercept.

Thus, one can argue that at large β the results are compatible with a conformal behaviour (if we ignore the small intercepts in particle masses as $m_Q \to 0$). However, we emphasize that this is qualitatively also compatible with standard QCD-like running coupling: we have also measured the meson spectrum in theory with SU(2) gauge + 2 fundamental representation fermions, and the
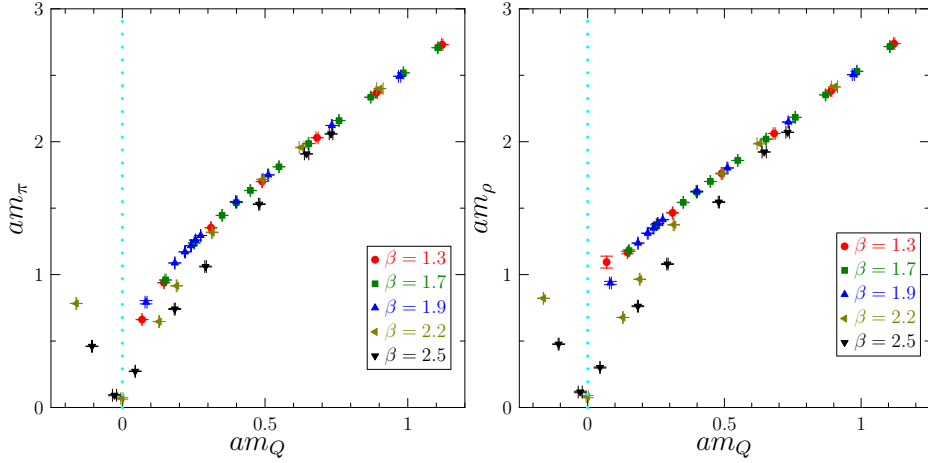
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Figure 2: Mass of π, (pseudoscalar) and mass of ρ (vector meson) with different β as a function of PCAC quark mass.
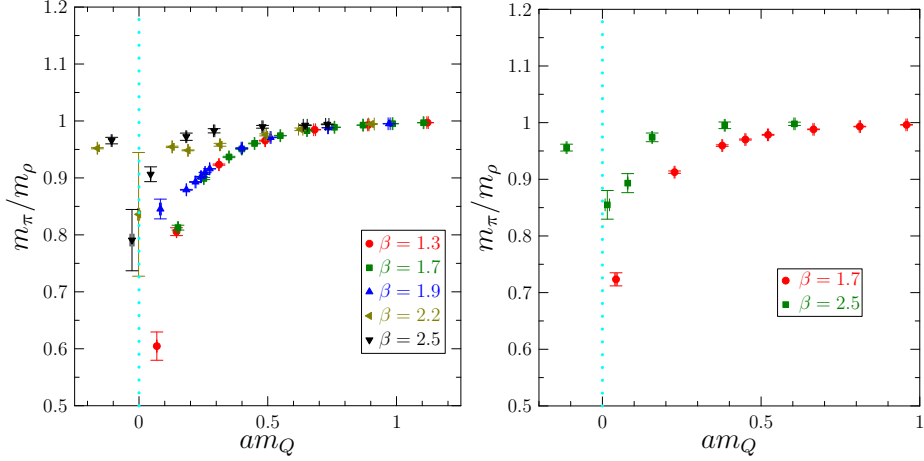

Figure 3: Ratio between π (pseudoscalar) and ρ (vector meson) mass with different β as a function of PCAC quark mass. In the left panel the results are for adjoint representation and in the right panel for the fundamental representation.

mass pattern is comparable to the one shown in Fig. 2. On Fig. 3 we show the mass ratios of pseudoscalar and vector mesons for adjoint and fundamental fermions. For fundamental fermions the reason for this behaviour is easy to understand: at small β we observe chiral symmetry breaking, as we should, but at large β the lattice spacing becomes so small that the linear size of the system becomes much smaller than the hadron size. Thus, the quarks become effectively deconfined, and we observe near-conformal behaviour also with fundamental quarks at $m_Q \approx 0$.

Thus, whether or not the adjoint quark theory has conformal or near-conformal behaviour (and hence an IR fixed point or "walking" coupling), or QCD-like running coupling, is very difficult to distinguish from the mass spectrum. However, we note that if there is a genuine IR fixed point where the theory becomes conformal, there must be a phase transition somewhere along the crit-
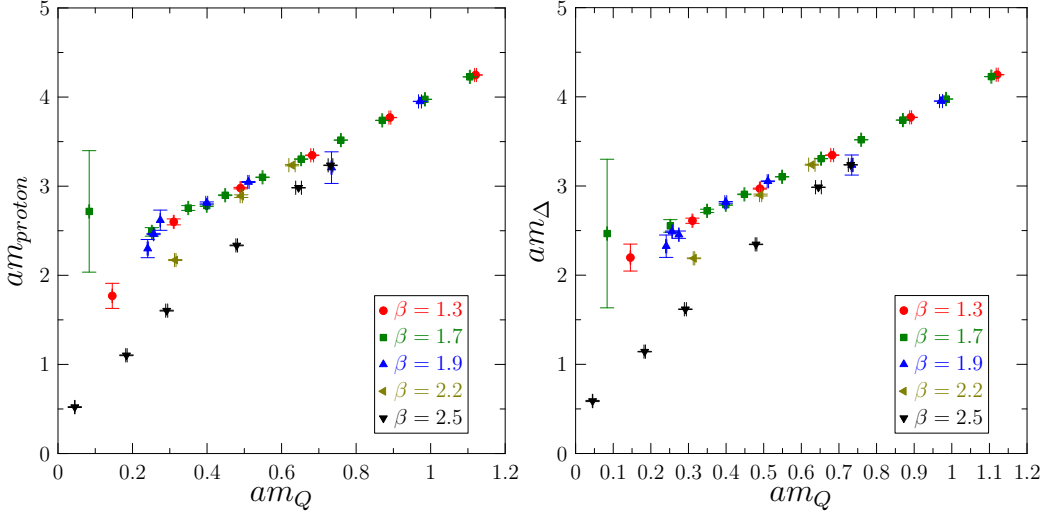
Figure 4: Mass of proton (spin 1/2 baryon) and Δ (spin 3/2 baryon) as a function of PCAC quark mass.

ical $m_Q = 0$-line ($\kappa_c(\beta)$) where the theory goes from the chirally broken phase (at small β) into the phase controlled by the IR fixed point (at large β), where IR physics is conformal. In our simulations we do observe behaviour compatible with this: there is a clear change in the $m_Q \rightarrow 0$ limit around $\beta \approx 2$. Indeed, around this point it is very difficult to even reach small m_Q values with lattice Monte Carlo. If this scenario is the correct one, then the chiral symmetry breaking is a lattice artifact (or rather, UV cutoff artifact) not present in the continuum theory. The existence of a critical point was also suggested in [5].

4. Conclusions

We have presented preliminary results of the lattice measurement of the mass spectrum in SU(2) gauge theory with two fermions in the adjoint representation. This theory has been proposed to have either a walking (i.e. very slowly evolving) coupling or even an IR fixed point, where the theory becomes conformal (in the massless fermion limit). In this case the physical states of the theory are all massless. We indeed observe an almost-massless behaviour at large inverse lattice coupling β. However, resolving the reason for this behaviour on the lattice is complicated by the fact that in practice the spectrum for a theory with a QCD-like running coupling also appears conformal at large β, due to the fact that the lattice volume becomes so small that the system is essentially deconfined. In order to resolve the issue more direct evaluation of the evolution of the coupling is required, using e.g. Schrödinger functional methods.

5. Acknowledgements

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References

[1] S. Weinberg, *Implications Of Dynamical Symmetry Breaking: An Addendum,* Phys. Rev. D **19**, 1277 (1979).

L. Susskind, *Dynamics Of Spontaneous Symmetry Breaking In The Weinberg-Salam Theory,* Phys. Rev. D **20**, 2619 (1979).

[2] F. Sannino and K. Tuominen, *Techniorientifold,* Phys. Rev. D **71**, 051901 (2005) [arXiv:hep-ph/0405209].

D. D. Dietrich, F. Sannino and K. Tuominen, *Light composite Higgs from higher representations versus electroweak precision measurements: Predictions for LHC,* Phys. Rev. D **72**, 055001 (2005) [arXiv:hep-ph/0505059].

[3] L. Del Debbio, A. Patella and C. Pica, *Higher representations on the lattice: numerical simulations. SU(2) with adjoint fermions,* arXiv:0805.2058 [hep-lat].

[4] S. Catterall and F. Sannino, *Minimal walking on the lattice,* Phys. Rev. D **76** (2007) 034504 [arXiv:0705.1664 [hep-lat]].

[5] S. Catterall, J. Giedt, F. Sannino and J. Schneible, *Phase diagram of SU(2) with 2 flavors of dynamical adjoint quarks,* arXiv:0807.0792 [hep-lat].

[6] Y. Shamir, B. Svetitsky and T. DeGrand, *Zero of the discrete beta function in SU(3) lattice gauge theory with color sextet fermions,* Phys. Rev. D **78** (2008) 031502 [arXiv:0803.1707 [hep-lat]].

[7] B. Svetitsky, Y. Shamir and T. DeGrand, *Nonperturbative infrared fixed point in sextet QCD,* arXiv:0809.2885 [hep-lat].

[8] T. DeGrand, Y. Shamir and B. Svetitsky, *Exploring the phase diagram of sextet QCD,* arXiv:0809.2953 [hep-lat].

[9] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, *"Probing technicolor theories with staggered fermions,* arXiv:0809.4890 [hep-lat]; *Nearly conformal electroweak sector with chiral fermions,* arXiv:0809.4888 [hep-lat].

[10] A. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, *Minimal walking technicolor on the lattice*, in preparation

[11] T. Appelquist and F. Sannino, *The physical spectrum of conformal SU(N) gauge theories,* Phys. Rev. D **59** (1999) 067702 [arXiv:hep-ph/9806409].