

A lattice study of light scalar tetraquarks with isospins 0, 1/2 and 1

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The observed mass pattern of scalar resonances below 1 GeV gives preference to the tetraquark assignment over the conventional $\bar{q}q$ assignment for these states. We present a search for tetraquarks with isospins 0, 1/2, 1 in lattice QCD, where isospin channels 1/2 and 1 have not been studied before. Our simulation uses Chirally Improved fermions on quenched gauge configurations. We determine three energy levels for each isospin using the variational method. The ground state is consistent with the scattering state, while the two excited states have energy above 2 GeV. Therefore we find no indication for light tetraquarks at our range of pion masses 344 – 576 MeV.

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1. Introduction

The observed mass pattern of scalar mesons below 1 GeV, illustrated in Fig. 1, does not agree with the expectations for the conventional $\bar{q}q$ nonet. The observed ordering $m_\kappa < m_{a_0(980)}$ can not be reconciled with the conventional $\bar{u}s$ and $\bar{u}d$ states since $m_{\bar{u}s} > m_{\bar{u}d}$ is expected due to $m_s > m_d$. This is the key observation which points to the tetraquark interpretation, where light scalar tetraquark resonances may be formed by combining a “good” scalar diquark

$$[qQ]_a \equiv \varepsilon_{abc} [q_b^T C \gamma_5 Q_c - Q_b^T C \gamma_5 q_c] \quad (\text{color and flavor anti-triplet}) \quad (1.1)$$

with a “good” scalar anti-diquark $[\bar{q}\bar{Q}]_a$ [1]. The states $[qq]_{\bar{3}_f, \bar{3}_c} [\bar{q}\bar{q}]_{3_f, 3_c}$ form a flavor nonet of color-singlet scalar states, which are expected to be light. In this case, the $I = 1$ state $[us][\bar{d}\bar{s}]$ with additional valence pair $\bar{s}s$ is naturally heavier than the $I = 1/2$ state $[ud][\bar{d}\bar{s}]$ and the resemblance with the observed spectrum speaks for itself.

Light scalar tetraquarks have been extensively studied in phenomenological models [1], but there have been only few lattice simulations [2, 3, 4]. The main obstacle for identifying possible tetraquarks on the lattice is the presence of the scattering contributions in the correlators. All previous simulations considered only $I = 0$ and a single correlator, which makes it difficult to disentangle tetraquarks from the scattering. The strongest claim for σ as tetraquark was obtained for $m_\pi \simeq 180 - 300$ MeV by analyzing a single correlator using the sequential empirical Bayes method [3]. This result needs confirmation using a different method (for example the variational method used here) before one can claim the existence of light tetraquarks on the lattice with confidence.

We study the whole flavor pattern with $I = 0, 1/2, 1$ and our goal is to find out whether there are any tetraquark states on the lattice, which could be identified with observed resonances $\sigma(600)$, $\kappa(800)$ and $a_0(980)$. Our methodology and results are explained in more detail in [5].

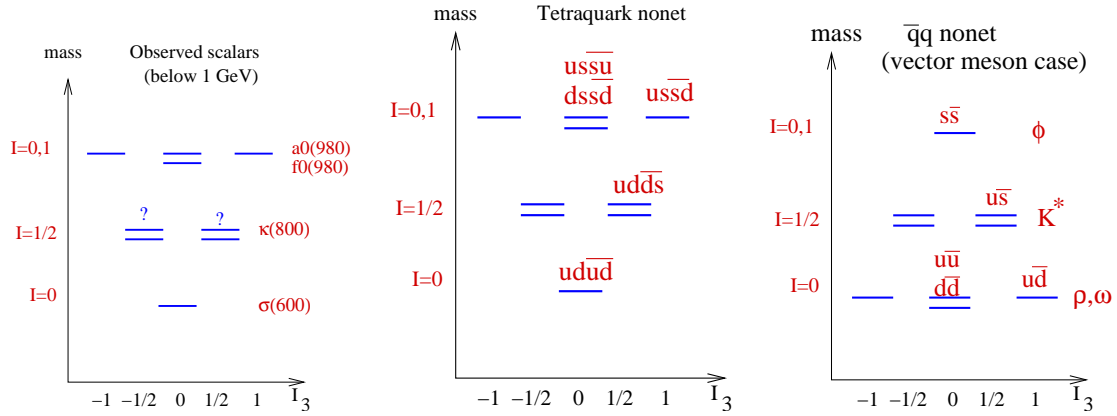


Figure 1: Observed spectrum of scalar mesons below 1 GeV (left), together with the expected spectrum for the nonet of scalar tetraquarks (middle) and compared with a typical $\bar{q}q$ spectrum (right).

2. Lattice simulation

In our simulation, tetraquarks are created and annihilated by diquark anti-diquark interpolators

$$\mathcal{O}^{I=0} = [ud][\bar{u}\bar{d}], \quad \mathcal{O}^{I=1/2} = [ud][\bar{d}\bar{s}], \quad \mathcal{O}^{I=1} = [us][\bar{d}\bar{s}]. \quad (2.1)$$

In each flavor channel we use three different shapes of interpolators at the source and the sink

$$\mathcal{O}_1^I = [q_n Q_n][\bar{q}'_n \bar{Q}'_n], \quad \mathcal{O}_2^I = [q_w Q_w][\bar{q}'_w \bar{Q}'_w], \quad \mathcal{O}_3^I = [q_n Q_w][\bar{q}'_w \bar{Q}'_n]. \quad (2.2)$$

Here q_n and q_w denote “narrow” and “wide” Jacobi-smearred quarks. We use exactly the same two smearings as applied in [6], which have approximately Gaussian shape and a width of a few lattice spacings. In order to extract energies E_n of the tetraquark system, we compute the 3×3 correlation matrix for each isospin

$$C_{ij}^I(t) = \sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle 0 | \mathcal{O}_i^I(\vec{x}, t) \mathcal{O}_j^{I\dagger}(\vec{0}, 0) | 0 \rangle_{\vec{p}=\vec{0}} = \sum_n \langle 0 | \mathcal{O}_i^I | n \rangle \langle n | \mathcal{O}_j^{I\dagger} | 0 \rangle e^{-E_n t} = \sum_n w_n^{ij} e^{-E_n t}. \quad (2.3)$$

Like all previous tetraquark simulations, we use the quenched approximation and discard the disconnected quark contractions. These two approximations allow a definite quark assignment to the states and discard $[\bar{q}\bar{q}][qq] \leftrightarrow \bar{q}q \leftrightarrow \text{vac}$ mixing, so there is even a good excuse to use them in these pioneering studies. We work on two¹ volumes $V = L^3 \times T = 16^3 \times 32$ and $12^3 \times 24$ at the same lattice spacing $a = 0.148$ fm [6]. The quark propagators are computed from the Chirally Improved Dirac operator [7] with periodic boundary conditions in space and anti-periodic boundary conditions in time. We use $m_l a = m_{u,d} a = 0.02, 0.04$ and 0.06 corresponding to $m_\pi = 344, 475$ and 576 MeV, respectively. The strange quark mass $m_s a = 0.08$ is fixed from m_ϕ . The analysis requires the knowledge of the kaon masses, which are 528, 576, 620 MeV for $m_l a = 0.02, 0.04, 0.06$.

The extraction of the energies from the correlation functions using a multi-exponential fit $C_{ij} = \sum_n w_n^{ij} e^{-E_n t}$ is unstable. A powerful method to extract excited state energies is the variational method, so we determine the eigenvalues and eigenvectors from the hermitian 3×3 matrix $C(t)$ ²

$$C(t)\vec{v}_n(t) = \lambda_n(t)\vec{v}_n(t). \quad (2.4)$$

The resulting large-time dependence of the eigenvalues $\lambda_n(t) = w_n e^{-E_n t} [1 + \mathcal{O}(e^{-\Delta_n t})]$ allows a determination of energies $E_{0,1,2}$ and spectral weights $w_{0,1,2}$. The eigenvectors $\vec{v}_n(t)$ are orthogonal and represent the components of physical states in terms of variational basis (2.2).

3. Results

Our interpolators couple to the tetraquarks, if these exist, but they also unavoidably couple to the scattering states $\pi\pi$ ($I = 0$), $K\pi$ ($I = 1/2$) and $K\bar{K}$, $\pi\eta_{ss}$ ($I = 1$) as well as to the heavier states with the same quantum numbers. The lowest few energy levels of the scattering states $P_1(\vec{k})P_2(-\vec{k})$

$$E^{P_1(\vec{j})P_2(-\vec{j})} \simeq m_{P_1} + m_{P_2}, \dots, \sqrt{m_{P_1}^2 + \left(\frac{2\pi\vec{j}}{L}\right)^2} + \sqrt{m_{P_2}^2 + \left(\frac{2\pi\vec{j}}{L}\right)^2}, \dots \quad (3.1)$$

are well separated for our L and we have to identify them before attributing any energy levels $E \simeq m_{\sigma, \kappa, a_0}$ to the tetraquarks.

The three energy levels for $I = 0$ are represented by the effective masses of the three eigenvalues $\lambda_n(t)$ in Fig. 2. Similar effective masses are obtained for all isospins, quark masses and

¹Equal smearings are used on both volumes.

²We use the standard eigenvalue problem in order to study $w(L)$.

volumes. The energies $E_{0,1,2}$ extracted from $\lambda_{0,1,2}(t)$ are summarized in Fig. 3 for all isospin channels. The excited energies were obtained from a conventional two-parameter fit $\lambda_{1,2}(t) = w_{1,2}[e^{-E_{1,2}t} + e^{-E_{1,2}(T-t)}]$. The fit of $\lambda_0(t)$ (A.1) takes into account a non-standard time dependence at finite T (noticed from decreasing m_{eff} near $t \simeq T/2$) and is described in Appendix.

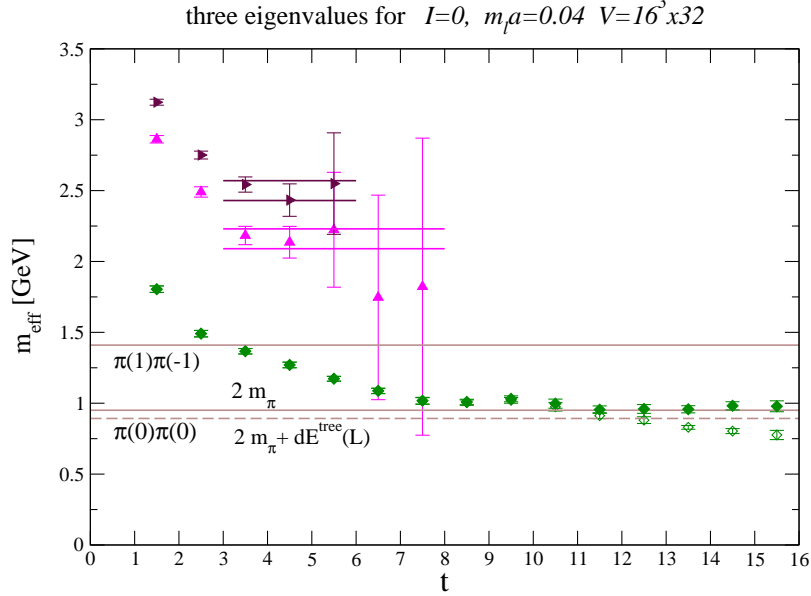


Figure 2: Effective masses for the three eigenvalues $\lambda_{0,1,2}(t)$ at $I = 0$, $m_l a = 0.04$ and $V = 16^3 \times 32$. They are obtained from conventional cosh-type definition, except for the full diamonds which are obtained from $\lambda_0(t) - const = w_0[e^{-m_{eff}t} + e^{-m_{eff}(T-t)}]$ (see Appendix). The lines give energy levels for the scattering states: full lines present non-interacting energies, while dashed lines take into account tree-level energy shifts.

The *ground state* energies in $I = 0, 1/2$ and 1 channels are close to $2m_\pi$, $m_\pi + m_K$ and $2m_K$, $m_\pi + m_{\eta_{ss}}$, respectively, which indicates that all ground states correspond to the scattering states $P_1(\vec{0})P_2(\vec{0})$ (see Fig. 3). Another indication in favor of this interpretation comes from $w_0(L = 12)/w_0(L = 16) \simeq 16^3/12^3$ (see Fig. 4). This agrees with the expected dependence $w_0 \propto 1/L^3$ for scattering states [3], which follows from the integral over the loop momenta $\int \frac{d\vec{k}}{(2\pi)^3} f(\vec{k}, t) \rightarrow \frac{1}{L^3} \sum_{\vec{k}} f(\vec{k}, t)$ with $dk_i = 2\pi/L$. The third indication comes from a non-standard time dependence of $\lambda_0(t)$, which agrees with the expected time-dependence of $P_1(\vec{0})P_2(\vec{0})$ at finite T (see Appendix).

The most important feature of the spectrum in Fig. 3 is a large gap above the ground state: *the first and the second excited states* appear only at energies above 2 GeV. Whatever the nature of these two excited states are, they are much too heavy to correspond to $\sigma(600)$, $\kappa(800)$ or $a_0(980)$, which are the light tetraquark candidates we are after. The two excited states may correspond to $P_1(\vec{k})P_2(-\vec{k})$ with higher \vec{k} or to some other energetic state. We refrain from identifying the excited states with certain physical objects as such massive states are not a focus of our present study.

Our conclusion is that we find no evidence for light tetraquarks at our range of pion masses 344 – 576 MeV. This is not in conflict with the simulation of the Kentucky group [3], which finds

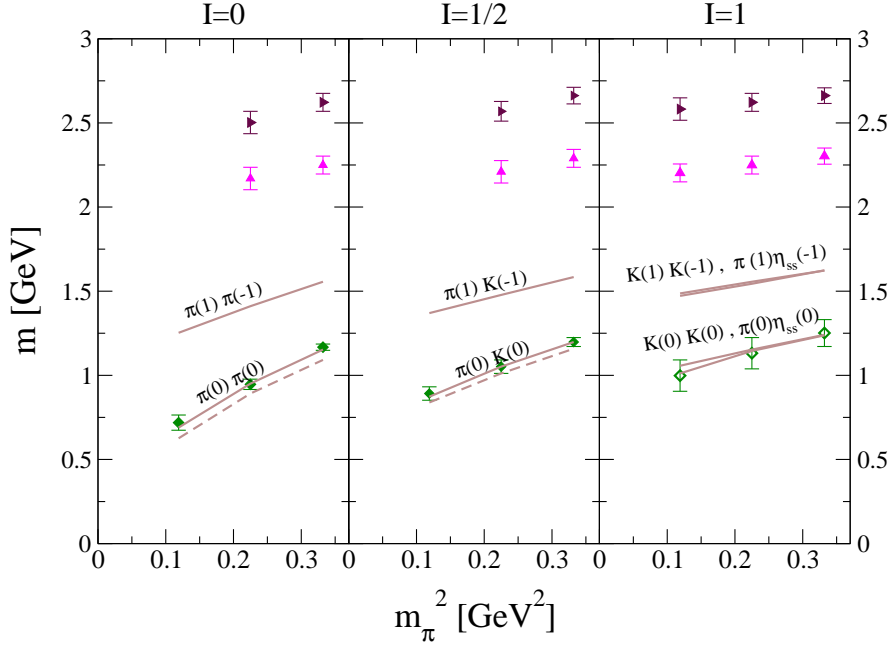


Figure 3: Three lowest energy levels from tetraquark correlators in $I = 0, 1/2, 1$ channels at lattice volume $16^3 \times 32$. The lines give analytic energy levels for scattering states: full lines present non-interacting energies, while dashed lines take into account tree-level energy shifts.

indication for an $I = 0$ tetraquark for pion masses 180 – 300 MeV (but not above that) since all our pion masses are just above 300 MeV.

4. The absence of scattering states with $|\vec{k}| = 2\pi/L$ in the spectrum

Why are there no states close to the energies of $P_1(1)P_2(-1)$ with $|\vec{k}| = 2\pi/L$ in the spectrum of Figs. 2 and 3? We believe this is due to the fact that all our sources (2.2) have a small spatial extent and behave close to point like. The point source couples to all the scattering states equally $\sum_{\vec{k}} g_{\vec{k}} |P_1(\vec{k})P_2(-\vec{k})\rangle$ up to a factor $g_{\vec{k}}$, which gives a Lorentz structure of the coupling. In this approximation each source $\mathcal{O}_{1,2,3}$ (2.2) couples to the few lowest scattering states equally (within our error bars)

$$\mathcal{O}_i = c_i \sum_{\vec{k}} g_{\vec{k}} |P_1(\vec{k})P_2(-\vec{k})\rangle + a_i |a\rangle + b_i |b\rangle \quad i = 1, 2, 3 \quad (4.1)$$

and only the overall strengths $c_{1,2,3}$ are different. We assume also that the sources couple to two other physical states a and b . Given these linear combinations, one can construct the corresponding 3×3 correlation matrix and it can be easily shown that its three eigenvalues are

$$\lambda_0(t) = w_0 \sum_{\vec{k}} g_{\vec{k}} \frac{e^{-(E^{P_1(\vec{k})} + E^{P_2(\vec{k})})t}}{E^{P_1(\vec{k})} E^{P_2(\vec{k})}}, \quad \lambda_{a,b}(t) = w_{a,b} e^{-E_{a,b}t}. \quad (4.2)$$

The physical states a and b get their own exponentially falling eigenvalues, while a tower of few lowest scattering states contributes to a single eigenvalue in this approximation. The agreement

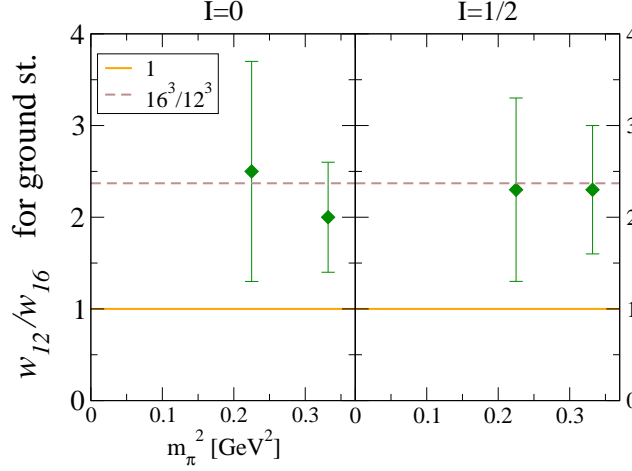


Figure 4: The ratio of spectral weights $w_0(L=12)/w_0(L=16)$ for $I=0, 1/2$ as computed from ground state eigenvalues for two volumes L^3 .

of $\lambda_0(t)$ and the analytic prediction (4.2) with $g_k = 1$ [5] gives us confidence about the hypothesis (4.1). So our basis does not disentangle the few lowest scattering states into separate eigenvalues. They would contribute to separate eigenvalues only by using a different or larger basis.

5. Conclusions and outlook

Our lattice simulation gives no indication that the observed resonances $\sigma(600)$, $\kappa(800)$ and $a_0(980)$ are tetraquarks. However, one should not give up hopes for finding these interesting objects on the lattice. Indeed, our simulation with pion masses 344 – 576 MeV does not exclude the possibility of finding tetraquarks for lighter m_π or for a different interpolator basis. A stimulating lattice indication for σ as a tetraquark at $m_\pi = 182 - 300$ MeV has already been presented in [3].

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A. Appendix: Effect of finite T on scattering states

We find that the cosh-type effective mass for the ground state $P_1(\vec{0})P_2(\vec{0})$ is decreasing near $t \simeq T/2$ (empty symbols in Figs. 2 and 5), which means that $\lambda_0(t)$ does not behave as $e^{-E_0 t} + e^{-E_0(T-t)}$ at large t . The time dependence of $P_1(\vec{0})P_2(\vec{0})$ with anti-periodic propagators in time is

$$\lambda_0^{P_1(0)P_2(0)}(t) = w_0[e^{-E_0 t} + e^{-E_0(T-t)}] + A[e^{-m_{P_1} t} e^{-m_{P_2}(T-t)} + e^{-m_{P_2} t} e^{-m_{P_1}(T-t)}], \quad E_0 \simeq m_{P_1} + m_{P_2}. \quad (\text{A.1})$$

In the last term one pseudoscalar propagates forward and the other backward in time (see Appendix A of [8]). The ground state energies for isospins 0 and 1/2 in Fig. 3 were obtained by fitting $\lambda_0(t)$ to (A.1) with three unknown parameters E_0 , w_0 and A ³. The effective mass obtained from $\lambda_0(t) - A[e^{-m_{p1}t}e^{-m_{p2}(T-t)} + e^{-m_{p2}t}e^{-m_{p1}(T-t)}]$ is flat near $t \simeq T/2$ (full symbols in Figs. 2 and 5).

The $I = 1$ channel contains $K\bar{K}$ and $\pi\eta_{ss}$ scattering states and is therefore more challenging. The ground state energies were obtained from a naive one-state (cosh) fit since a two-state fit is very unstable.

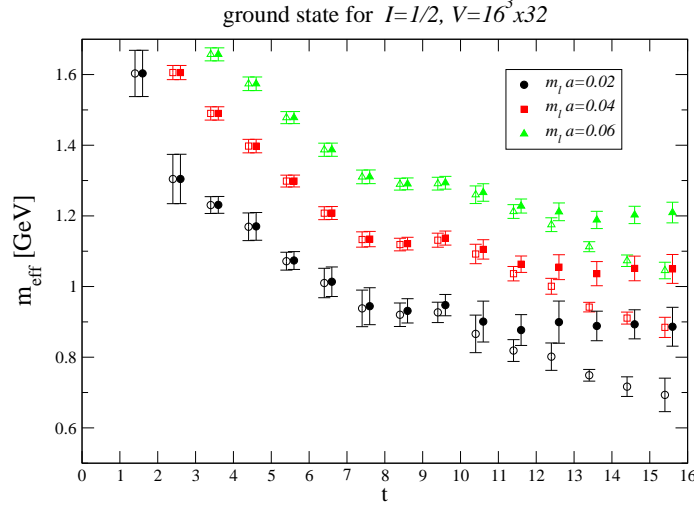


Figure 5: The effective masses for the $I = 1/2$ ground states at various $m_l a$ and $V = 16^3 \times 32$. Empty symbols are obtained using the conventional cosh-type effective mass $\lambda_0(t) = w_0[e^{-m_{eff}t} + e^{-m_{eff}(T-t)}]$, while the full symbols take into account the correct form $\lambda_0(t) - A[e^{-m_{\pi}t}e^{-m_K(T-t)} + e^{-m_Kt}e^{-m_{\pi}(T-t)}] = w_0[e^{-m_{eff}t} + e^{-m_{eff}(T-t)}]$ for $K(\vec{0})\pi(\vec{0})$ scattering (see Appendix).

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³At the time of our presentation at Lattice 2008, we were not aware of the last term in (A.1), so the numerical results on our slides are slightly different. The general physical conclusions concerning the tetraquarks are however the same.