

Scaling behavior and sea quark dependence of pion spectrum with HYP-smearred staggered fermions

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We study the pion spectrum (and in particular taste-symmetry breaking within it) using HYP-smearred valence staggered fermions on the coarse and fine MILC lattices (which have asqtad staggered sea quarks). We focus on the dependence on lattice spacing and sea-quark mass. We also update our results on source dependence. Our main conclusion is that on the MILC fine lattices the appropriate power-counting for SU(3) staggered chiral perturbation theory may have discretization errors entering at next-to-leading order rather than at leading-order.

*The XXVI International Symposium on Lattice Field Theory
July 14 - 19, 2008
Williamsburg, Virginia, USA*

*Speaker.

1. Introduction

In this note we give an update on our results (most recently presented in Ref. [1]) for the pion spectrum using valence HYP-smearred staggered fermions. These results are needed as input into our parallel calculation of B_K (whose status is described in Ref. [2]), and more generally provide information on the systematic errors that are likely to be found with improved staggered actions (including the “highly improved staggered quark” [HISQ] action [3]).

Smearing the gauge links in the fermion action is a widely used method to reduce discretization and perturbative errors. We use the unimproved staggered action with HYP-smearred [4] links. One conclusion of Ref. [1] was that this smearing reduced the dominant discretization error—taste-symmetry breaking in the pion spectrum—by a factor 2.5–3 compared to quarks improved with the asqtad action [5]. The resulting taste-breaking was thus reduced to the same level as with the HISQ action. Thus, for light quarks, where the additional improvements in the HISQ action are less important, HYP-smearred quarks represent a simpler alternative to the HISQ action.

The reduction in taste-breaking was studied in Ref. [1] on the coarse MILC lattices (with $a \approx 0.125$ fm). For these lattices it was found that, even after HYP-smearing, the size of $O(a^2)$ errors was comparable, for the lightest quark masses, to the $O(p^2)$ terms in chiral perturbation theory (χ PT). Thus the standard $a^2 \sim p^2$ power-counting of staggered chiral perturbation theory ($S\chi$ PT) must still be used. It was suggested, however, that on the fine MILC lattices (with $a \approx 0.09$ fm) taste-breaking would be reduced to the extent that the appropriate power-counting would be $a^2 \sim p^4$, thus simplifying the required fitting. Our new results allow us to study this point.

Another observation of Ref. [1] was that $O(a^2 p^2)$ effects were significantly smaller than expected. With improved statistics we can now quantify this statement.

Our final new result concerns the dependence of the pion spectrum on the sea-quark mass.

2. Scaling behavior of pion spectrum

In Fig. 1, we show m_π^2 as a function of quark mass for both coarse and fine MILC lattices with sea quark mass-ratio of $m_l/m_s = 1/5$. Parameters for the numerical study are summarized in Table

gauge action	1-loop tadpole-improved Symanzik gluon action	
sea quarks	$N_f = 2 + 1$ Asqtad staggered fermions	
valence quarks	HYP staggered fermions	
parameters	MILC fine lattices	MILC coarse lattices
a	0.09fm	0.12fm
geometry	$28^3 \times 96$	$20^3 \times 64$
# of confs	995	671
sea quark masses	$am_l = 0.0062,$ $am_s = 0.031$	$am_l = 0.01,$ $am_s = 0.05$
valence quark masses	0.003, 0.006, ..., 0.030	0.005, 0.01, ..., 0.05

Table 1: Parameters for the numerical study on scaling violations.

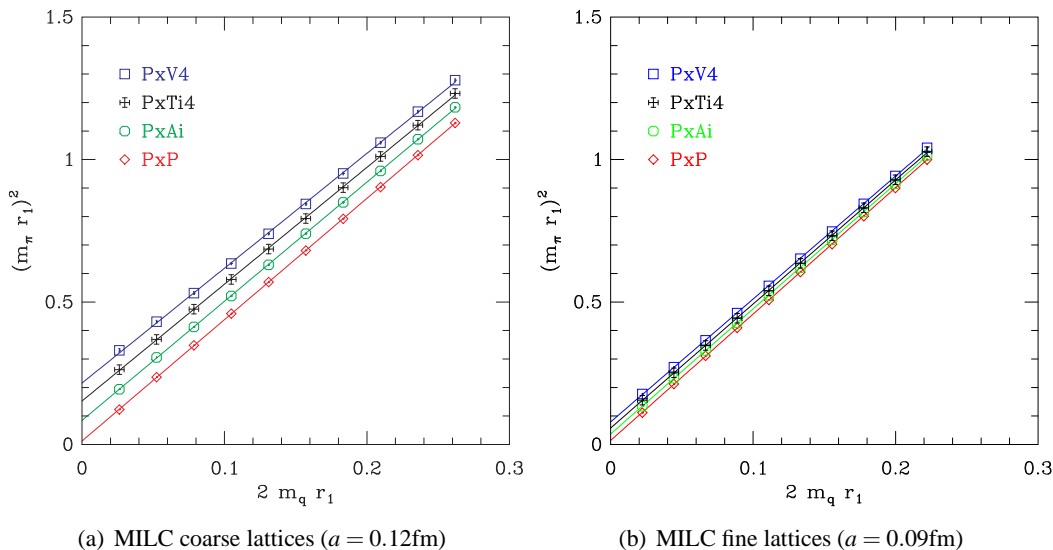


Figure 1: m_π^2 vs. $2m_q$: the data is obtained using HYP-smearred staggered fermions with cubic wall sources and Golterman sink operators [1]. The legend indicates the tastes.

1—we note that the lightest valence quarks have a mass of $\approx m_s^{\text{phys}}/10$. We show only results with degenerate valence quarks, and display a linear fit. For clarity, we show only the four tastes with the smallest errors (those created by pseudoscalar operators which are local in time [1]).

Taste-breaking is substantially reduced on the fine lattices. To make this quantitative, we tabulate results for the splitting in the chiral limit

$$\Delta(T) = \lim_{m_q \rightarrow 0} [m_\pi^2(T) - m_\pi^2(\xi_5)], \quad (2.1)$$

with T the taste, in Table 2. Our results are consistent with the $SO(4)$ symmetry expected at $O(a^2)$ [6, 7], and we quote the result for the taste with the smallest error. The ratio $\Delta(T, \text{fine})/\Delta(T, \text{coarse})$ is about 0.3 for all tastes. For HYP-smearred staggered fermions, taste-symmetry breaking is of order $\mathcal{O}(a^2\alpha_s^2)$.¹ Our results are consistent with this expectation, as shown in Fig. 2.

In light of these results, what is the appropriate power-counting for $S\chi\text{PT}$ on the fine lattices? When considering kaon properties (e.g. B_K), there are loops containing valence $d\bar{d}$ pions of all

Taste (T)	$\Delta(T)$ [(GeV) ²]		$\Delta(\text{Fine})/\Delta(\text{Coarse})$
	$a = 0.125\text{fm}$	$a = 0.09\text{fm}$	
$\xi_{5\mu}$	0.0278(6)	0.0087(3)	0.314(12)
$\xi_{\mu\nu}$	0.0540(13)	0.0168(4)	0.310(11)
ξ_μ	0.0783(17)	0.0250(6)	0.319(11)
I	0.1005(84)	0.0300(31)	0.299(40)

Table 2: $\Delta(T)$ in physical units on coarse and fine MILC lattices.

¹Smearing removes $\mathcal{O}(a^2\alpha_s)$ taste symmetry breaking terms.

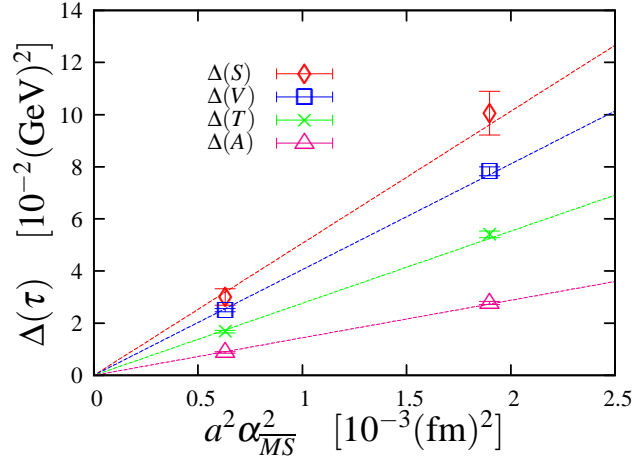


Figure 2: $\Delta(T)$ vs. $a^2\alpha_{\overline{MS}}^2$: $\alpha_{\overline{MS}}^2$ is obtained at $\mu = 1/a$ using four-loop running.

Taste (T)	slope(T)	
	$a = 0.125\text{fm}$	$a = 0.09\text{fm}$
ξ_5	4.258(11)	4.436(12)
ξ_{i5}	4.182(14)	4.391(12)
ξ_{i4}	4.100(20)	4.352(13)
ξ_4	4.031(23)	4.314(15)
ξ_{45}	4.180(33)	4.414(33)
ξ_{ij}	4.115(42)	4.373(35)
ξ_i	4.052(53)	4.340(37)
I	3.952(73)	4.312(40)

Table 3: Slopes, c_2 , of the linear fits $(r_1 m_\pi(T))^2 = c_1 + c_2(2r_1 m_q)$.

tastes. For our lightest valence quark, the ratio of $O(a^2)$ to $O(p^2)$ contributions is characterized by

$$(m_\pi^2(\xi_{\mu\nu}) - m_\pi^2(\xi_5))/m_\pi^2(\xi_{\mu\nu}) \equiv \Delta(\xi_{\mu\nu})/m_\pi^2(\xi_{\mu\nu}) \approx 0.25. \quad (2.2)$$

(We use tensor taste as it is the most numerous and its mass lies near the average of the multiplet.) Since χ PT for kaons is characterized by an expansion parameter of $(m_K^{\text{phys}}/1 \text{ GeV})^2 \approx 25\%$, it may be appropriate to use the power-counting $O(a^2) \sim O(p^4)$, i.e. to treat discretization errors as of next-to-leading order (NLO). This will not, however, be appropriate for calculations of f_π using $SU(2)$ chiral perturbation theory, where the expansion parameter is smaller.

Further information on the power counting can be obtained by comparing the slopes of the linear fits, differences between which are of $O(a^2 p^2)$. Our results for these slopes for both lattices and all tastes are given in Table 3. We observe that there are significant splittings between slopes for different tastes (the differences are more significant than the errors suggest because of correlations), but that these splittings are small. For example, the splitting between slopes for tastes ξ_5 and ξ_{i4}

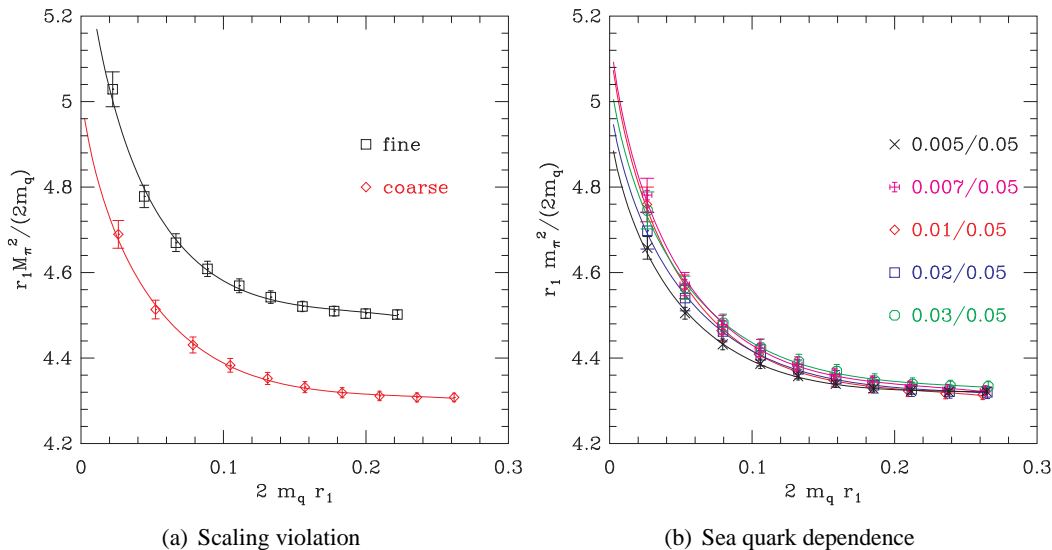


Figure 3: $r_1 m_\pi^2 / (2m_q)$ vs. $2r_1 m_q$ for the Goldstone pion.

are $\approx 4\%$ and 2% on coarse and fine lattices, respectively. On the fine lattices, this is consistent with the power-counting $O(a^2) \sim O(p^4)$, in which case the splittings between slopes are a NNLO effect.

We also note that $SO(4)$ breaking in the slopes is much smaller than the splittings allowed by $SO(4)$, although both are effects of the same order, $O(a^2 p^2)$ [8]. This remains to be understood.

Finally, we note that the difference between the slopes for the two lattice spacings, which is roughly 5%, is of the expected size for a discretization effect (i.e. $(a\Lambda_{\text{QCD}})^2$). This is also seen in Fig. 3(a). We note that this difference is comparable to the taste splittings on the coarse lattices, indicating that the latter are not significantly enhanced over taste-symmetric discretization errors.

3. Sea quark dependence of pion masses

In Table 4, we list the MILC coarse-lattice ensembles we use to study the dependence of the pion spectrum on sea quark masses. We find no significant dependence within our errors. This is

Geometry	$am_l(\text{sea})$	$am_s(\text{sea})$	$m_{\pi, \text{sea}} L$
$24^3 \times 64$	0.005	0.05	3.83
$20^3 \times 64$	0.007	0.05	3.78
$20^3 \times 64$	0.010	0.05	4.49
$20^3 \times 64$	0.020	0.05	6.23
$20^3 \times 64$	0.030	0.05	7.56

Table 4: Parameters for numerical study of sea quark mass dependence on coarse lattices.

shown for the Goldstone pion in Fig. 3(b), and for all tastes in Fig. 4. Note that in Fig. 4, different colors represent different tastes while different symbols correspond to different light sea-quark masses.

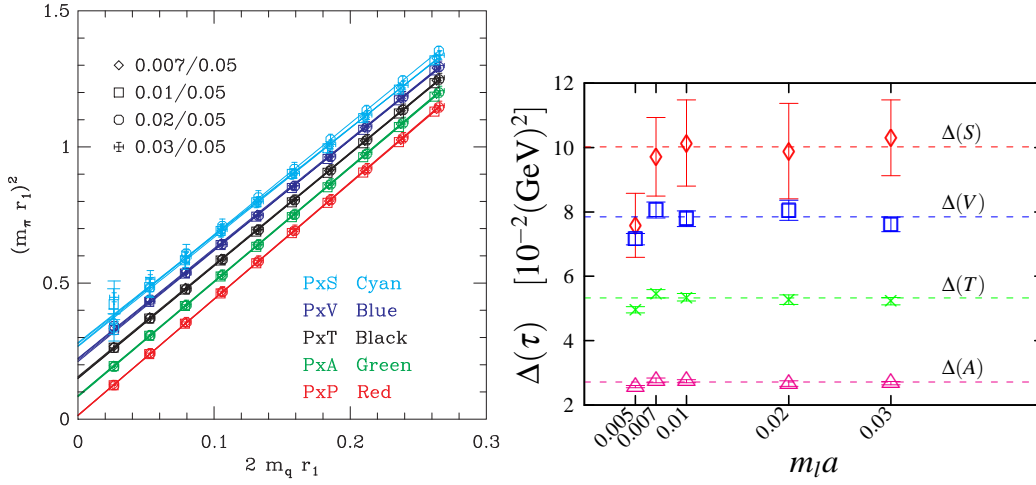


Figure 4: $(m_\pi r_1)^2$ vs. $2m_q r_1$ (left) and $\Delta(T)$ vs. $am_l(\text{sea})$ (right)

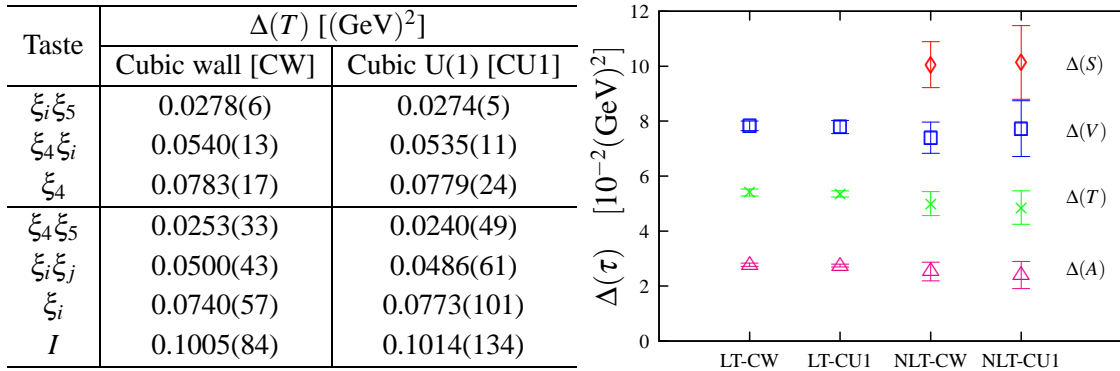


Figure 5: $\Delta(T)$ for cubic wall and cubic U(1) sources.

The righthand plot in Fig. 4 shows $\Delta(T)$ as a function of the light sea quark mass. There is a 2σ drop for $am_{l,\text{sea}} = 0.005$, but only improved statistics will allow us to determine if this is significant.

The independence of the pion spectrum on the sea-quark mass is consistent with the results of the MILC collaboration with asqtad quarks [9]. We note, however, that other quantities, e.g. f_π and f_K , do show some dependence on m_{sea} .

4. Efficacy of different sources

Finally, we have updated our results on the relative efficacy of our two sources: “cubic wall” and “cubic U(1)” sources. (For their definitions see Ref. [1].) Both sources project onto specific irreps of the cubic group, but we do not know *a-priori* what the resulting signal/noise ratio will be. Here we extend the study of Ref. [1] by comparing the sources on the MILC coarse lattice ensemble with $am_l = 0.01$ and $am_s = 0.05$.

In Fig. 5, we present $\Delta(T)$ for cubic wall (CW) and U(1) sources (CU1). In this plot, LT represents “local-in-time” tastes ($\{\xi_i \xi_5, \xi_4 \xi_i, \xi_4\}$) and NLT represents “non-local-in-time” tastes ($\{\xi_4 \xi_5, \xi_i \xi_j, \xi_i, I\}$). Thus, for example, $\xi_i \xi_5$ and $\xi_4 \xi_5$ form two different irreps of SW_4 but belong to

the same irrep of $SO(4)$. $S\chi$ PT predicts that the pion spectrum should respect the $SO(4)$ symmetry in the chiral limit, both at LO [6, 7] and at NLO [8]. In Fig. 5, we notice that $SO(4)$ symmetry is well respected in $\Delta(T)$ since the data for LT tastes and NLT tastes are consistent with each other within statistical uncertainty.

In the case of LT tastes, the results from both sources have comparable errors. However, for the NLT tastes, CW sources are clearly preferred, with errors being smaller by about a factor of 2. This is a more significant difference than we observed in Ref. [1] on quenched lattices.

5. Conclusions

We have observed the expected reduction in taste-symmetry breaking by a factor of ≈ 3 when passing from the coarse to the fine MILC lattices. This implies that, for $SU(3)$ staggered chiral perturbation theory, using HYP-smearred valence quarks probably allows one to treat discretization errors as a NLO effect. Whether this is really the case can only be determined by detailed fitting, such as that we are undertaking for B_K . It will also be interesting to do NLO fits of our pion-spectrum data itself, but this requires NLO expressions for pions of all tastes, which are in progress but not yet completed.

Our finding that the pion spectrum is almost independent of the sea-quark mass, which is consistent with the MILC asqtad results, simplifies our calculation of B_K , since we need not recalculate the entire spectrum for each MILC fine lattice (although we will check this by calculating the Goldstone taste).

6. Acknowledgment

C. Jung is supported by the US DOE under contract DE-AC02-98CH10886. The research of W. Lee is supported in part by the KICOS grant K20711000014-07A0100-01410, by the KRF grants (KRF-2007-313-C00146 and KRF-2008-314-C00062), by the BK21 program, and by the US DOE SciDAC-2 program. The work of S. Sharpe is supported in part by the US DOE grant no. DE-FG02-96ER40956, and by the US DOE SciDAC-2 program.

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