Stochastic quantization of a finite temperature lattice field theory in the real time formula

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We apply Parisi-Wu type Stochastic Quantization Method to a finite temperature lattice field theory of the real-time formula. In the theory, the time axis is extended to a complex contour proposed by Matsumoto et al. and Niemi and Semenoff. The finite temperature property is guaranteed by (anti-) periodicity of the time contour in the imaginary direction and a part of the time contour along the real axis describes the real evolution of the system. Taking correlations on the real-time part, we can directly obtain the relaxation of the system.

We apply numerically this method to a scalar field on the lattice. In the stationary limit of the stochastic process expectation values of physical quantities converge. Taking field correlation on the real-time part, relaxation like behavior of the system appears.

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1. Introduction

Describing and understanding the time evolution and relaxation of the system is one of the goal of theoretical physics, but it is difficult, in general, especially for many-body system. Numerical simulation of the lattice field theory based on the Monte Carlo method works as a quite powerful tool to investigate the many-body system. However, usually, the time axis there is converted to the inverse temperature through Wick rotation and most of the discussions are limited to the stationary properties such as thermodynamic quantities. Even in the calculation of the transport coefficients at fixed temperature, some sophisticated analytic continuation is inevitable.

Ordinarily Monte Carlo method is based on the probability distribution e^{-S_E}, therefore Euclidean time through Wick rotation is indispensable. Recently, Berges and Stamatescu succeed to make a numerical simulation of the real-time evolution based on a Stochastic Quantization Method[1]. They apply Parisi-Wu type Stochastic Quantization Method to the system of the Schwinger-Keldish closed-time-path formula and investigate time evolution of the field on the lattice. However, because of the non-equilibrium property of the system, they must specify the boundary condition which may strongly restrict the evolution of the system.

Describing non-equilibrium evolution of the system is a great challenge of the physics but how to specify the boundary condition seems to be too much for us. Hence, we focus our attention to the finite temperature equilibrium system. Even in equilibrium, a real-time correlation of currents provides us a relaxation of the fluctuation which is related to a transport coefficient through linear response theory.

A finite temperature field theory with real-time was formulated and established energetically around 80's. Takahashi and Umezawa proposed operator formula named Thermo Filed Dynamics[2]. An equivalent formula in perturbative sense is formulated in path integral with a complex time contour by Niemi and Semenoff[3]. The aim of this paper is stochastically quantize the finite temperature system of Niemi-Semenoff type real-time formula.

2. Stochastic Quantization Method

Parisi and Wu proposed a quantization method based on a stochastic process with an additional time axis so called "fictitious time"τ [4, 5]. The chronological evolution of the system along the fictitious time is subject to a Langevin equation,

$$\frac{d\phi(x,\tau)}{d\tau} = -\frac{\delta S_E}{\delta\phi(x,\tau)} + \eta(x,\tau), \tag{2.1}$$

with S_E being Euclidean action, x being Euclidean space-time and $\eta(x,\tau)$ being a white Gaussian noise. Statistical properties of $\eta(x,\tau)$ are given as,

$$\langle\eta(x,\tau)\rangle|_\eta = 0, \tag{2.2}$$

$$\langle\eta(x,\tau)\eta(x',\tau')\rangle|_\eta = \delta^4(x-x')\delta(\tau-\tau'). \tag{2.3}$$

Taking stationary limit at $\tau\to\infty$, we can obtain equivalent expectation values to ones of the Euclidean path integral formula,

$$\lim_{\tau\to\infty}\langle\phi(x,\tau)\phi(x',\tau)\rangle|_\eta = \langle\phi(x)\phi(x')\rangle|_{PI}. \tag{2.4}$$

The starting point of the Stochastic Quantization Method is a Euclidean field theory. Our basic idea is to extent Euclidean time in above x to the Niemi-Semenoff type complex contour which includes Minkowski time.

3. Finite Temperature Field Theory with Real Time

Niemi and Semenoff proposed a path integral with a complex contour shown in fig. 1 with β being a inverse temperature. The theory is equivalent to Thermo Field Dynamics in perturbative sense[6]. A correlation of the fields on the path (OA) corresponds to a thermal expectation of a real-time green function, $\mathrm{tr}\{e^{-\beta H}\phi(\vec{x},t)\phi(\vec{x}',t')\}$ and a field on the path (BC) corresponds to the "tilde field" in Thermo Field Dynamics. Putting (anti-) periodic condition between O and D, we can obtain Kubo-Martin-Schwinger condition which guarantees thermal equilibrium state[7].

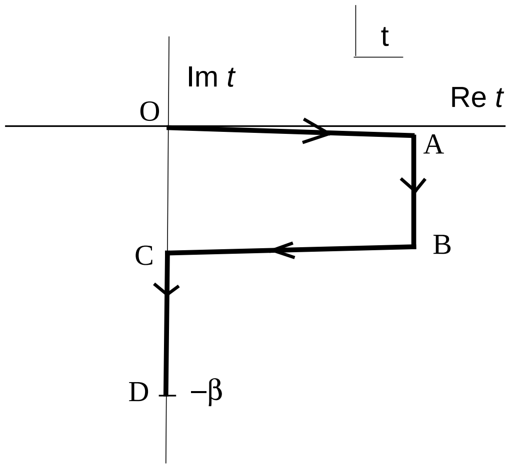

Figure 1: The time contour on the complex plane

In the complex contour in fig. 1, Niemi and Semenoff put AB = CD = $\beta/2$. Schwinger-Keldish formula is obtained if we take AB $\to 0$[8]. When we extend a time axis to a complex contour, there exists a restriction that the path must tilt in forward direction[9]. This restriction comes from semi-definite properties of Hamiltonian and fixes the direction of Wick rotation. Even in the ordinary Minkowski theory, there exists originally a small negative imaginary part in the time path.

4. Method

We apply Stochastic Quantization Method to the system with complex time contour shown in fig. 1. The kinetic term in the Langevin equation (2.1) is changed as,

$$\frac{\delta S_E}{\delta\phi(t_c,x,\tau)} = [\partial_{t_c}\partial_{t_c}\phi(t_c,x,\tau)+\cdots.$$

∂_{t_c} is the derivative along complex contour and causes a path dependent phase. The phase makes Langevin equation complex. Stochastic Quantization Method with complex Langevin equation has long history and it is also known that sometimes numerical simulations lead to wrong results[5]. However, Nakazato and Yamanaka have discussed Stochastic Quantization with Minkowski time and they conclude that, by virtue of a small $i\varepsilon$ in Lagrangian which corresponds to Feynman causality, the propagator obtained in Minkowski Stochastic Quantization becomes the same to the one in ordinary quantum field theory[10]. We also expect the phase which comes from the tilt of the complex time contour make Langevin equation converge and provides us a non-trivial expectation value.

5. Concluding remarks

We are now carrying out numerical simulation of scalar field with complex time path. Though our simulation is still preliminary stage, obtained results show that even with the Minkowski time region in the complex path, correlation function seems to converge in $\tau \to \infty$ limit. Correlation functions of the physical quantities on real-time part(OA in fig. 1) show appropriate relaxation-like behavior. Detailed results will be published elsewhere.

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