PROCEEDINGS OF SCIENCE

RHMC simulation of two-dimensional N=(2,2) super Yang-Mills with exact supersymmetry

Issaku Kanamori*
Theoretical Physics Laboratory, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
E-mail: kanamori-i@riken.jp

We report our numerical simulation of two-dimensional $\mathcal{N}=(2,2)$ super Yang-Mills. The lattice model we use is one proposed by F. Sugino which keeps one exact supersymmetry at finite lattice spacing. We use Rational Hybrid Monte Carlo (RHMC) method to implement the dynamical fermion. We apply the simulation to measure the ground state energy which is useful to observe dynamical SUSY breaking.

The XXVI International Symposium on Lattice Field Theory
July 14 - 19, 2008
Williamsburg, Virginia, USA
Report Number: RIKEN-TH-138

*Speaker.

PoS(LATTICE 2008)232

1. Introduction

It seems impossible to put the SUSY on the lattice, because SUSY algebra contains infinitesimal translation but on the lattice we have only finite translations. However, what we have realized in the recent development is that it is possible to formulate supersymmetric models on the lattice if $\mathcal{N} \geq 2$. Lots of lattice models especially for super Yang-Mills are known along this line [1, 2, 3, 4, 5] and relations among them have become clear [6, 7, 8]. [1] Most of these formulations utilize the topological twist. After the twist, we have a scalar supercharge instead of spinors. We can put the scalar on a lattice site and keep it exactly at finite lattice spacing.[2] Some of the simulation have already done aiming the check of the formulation [13, 14, 15].[3]

In this talk we report our simulation with dynamical fermions and its application. We utilize the Rational Hybrid Monte Carlo algorithm [19]. The target model is two-dimensional $\mathcal{N}=(2,2)$ super Yang-Mills model based on a formulation with one exactly kept supersymmetry proposed by Sugino [2]. As an application of the simulation, we measure the ground state energy which is useful to observe dynamical SUSY breaking. We also sketch the method of observing dynamical SUSY breaking which we proposed in [20, 21].

2. Lattice Model

The target theory in the continuum has $\mathcal{N}=(2,2)$ twisted supersymmetry. After the twist we have four supercharges, one from a scalar Q, two from a two-dimensional vector $(Q_0,\ Q_1)$, and one from a pseudo scalar $\tilde{Q}$. The following is a part of the twisted SUSY algebra:

$$Q^2 = \delta^{(\text{gauge})}_{\phi}, \qquad Q_0^2 = \delta^{(\text{gauge})}_{\overline{\phi}}, \qquad \{Q, Q_0\} = 2i\partial_0 + 2\delta^{(\text{gauge})}_{A_0}, \tag{2.1}$$

where $\delta^{(\text{gauge})}_{\bullet}$ denotes an infinitesimal gauge transformation with the parameter $\bullet$. The supercharges are nilpotent up to gauge transformation. The action is Q-exact and because of the nilpotency, Q-invariance is manifest.

On the lattice, we keep the scalar Q exactly. The following Q transformation on the lattice keeps the nilpotency even at the finite lattice spacing [2]:

$$\begin{aligned}
&QU(x,\mu) = i\psi_\mu(x)U(x,\mu), \quad Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu - i\big(\phi(x) - U(x,\mu)\phi(x+\hat{\mu})U(x,\mu)^{-1}\big),\\
&Q\phi(x) = 0,\\
&Q\chi(x) = H(X), \qquad QH(x) = [\phi(x), \chi(x)],\\
&Q\overline{\phi}(x) = \eta(x), \qquad Q\eta(x) = [\phi(x), \overline{\phi}(x)],
\end{aligned}$$

where $U(x,\mu)$ is a gauge link variable, scalar fields $\phi, \overline{\phi}$ and auxiliary field H are defined on the sites, fermions in the twisted basis η, χ, ψ_μ are defined on the sites. The action is defined as Q-exact

[1] For the review, see [9].

[2] For recent developments in keeping whole supersymmetry exactly on the lattice, see [10, 11]; See also [12].

[3] See [16, 17, 18] for interesting attempts towards simulation in three and four dimensions.

PoS(LATTICE 2008)232

as in the continuum case:

$$
\begin{aligned}
S = Q \frac{1}{a^2 g^2} \sum_x \mathrm{tr} \Big[& \chi(x) H(x) + \frac{1}{4} \eta(x) [\phi(x), \overline{\phi}(x)] - i\chi(x) \hat{\Phi}(x) \\
& + i \sum_{\mu=0,1} \left\{ \psi_\mu(x) \left(\overline{\phi}(x) - U(x,\mu) \overline{\phi}(x + a\hat{\mu}) U(x,\mu)^{-1} \right) \right\} \Big] \\
= \frac{1}{a^2 g^2} \sum_x \mathrm{tr} \Big[& \frac{1}{4} \hat{\Phi}_{\mathrm{TL}}(x)^2 + \dots \qquad \Big],
\end{aligned}
$$

where g is the dimensionful gauge coupling, $i\hat{\Phi}(x) = \dfrac{U(x,0,1) - U(x,0,1)^{-1}}{1 - ||1 - U(x,0,1)||^2/\varepsilon^2}$, $\hat{\Phi}_{\mathrm{TL}}$ is the traceless part of $\hat{\Phi}$ and $U(x,0,1)$ is the usual plaquette variable. We impose the admissibility condition $||1 - U(x,0,1)|| < \varepsilon$ for a constant ε in order to kill artifact vacua. Because of the nilpotency and Q-exactness, the action is manifestly Q invariant at the finite lattice spacing. The other three supercharges, Q_0, Q_1 and $\tilde{Q}$, will be automatically restored in the continuum limit as long as a perturbative power counting is valid.

3. Simulation Details

Since fermions play an important role in supersymmetry the effect of the dynamical fermion is crucial. We use the Rational Hybrid Monte Carlo algorithm. The path integration of the fermion gives Pfaffian of the Dirac operator D, which contains the Yukawa interaction terms as well. We rewrite the Pfaffian using pseudo fermion integration with rational function. Symbolically, contribution from the fermionic part of the action S_{fermion} becomes

$$
\begin{aligned}
\int \mathscr{D} f \exp(-S_{\mathrm{fermion}}) = \mathrm{Pf}(D) &= \int \mathscr{D} F \exp(-F^\dagger (D^\dagger D)^{-1/4} F) \\
&= \int \mathscr{D} F \exp\left(-F^\dagger \left[a_0 + \sum_{i=1}^{n} \frac{a_i}{D^\dagger D + b_i} \right] F \right), \qquad (3.1)
\end{aligned}
$$

where f is the fermion, F the pseudo fermion, D the Dirac operator, a_i and b_i are numerical constants.[4] Here we ignore a phase factor of the Pfaffian $\mathrm{Pf}(D)$ because it is almost 1 (i.e., almost real and positive) in the current model. If it is needed we would reweight this phase factor in the measurements. We also utilize the multi time step evolution in the molecular dynamics [23]. We calculate forces from S_{fermion} every several calculations of forces from the other part of the action. We evolve pseudo fermion F as well as other bosonic fields $U(x,\mu)$, ϕ and $\overline{\phi}$ in the molecular dynamics.

The parameters we use are the following. We set the gauge group to $SU(2)$ and ε for the admissibility condition to 2.6.[5] The lattice size is 3×12–30×10 and the lattice spacing is $ag =$ 0.07071–0.2357. The degree of the rational approximation is typically 20. The length of time evolution in the molecular dynamics in each trajectory is fixed to 0.5. We store the configurations every 10 trajectories. We keep the acceptance in the Metropolis test to be greater than 0.8. Because

[4] We use a program from [22] to obtain a_i and b_i.

[5] The possible maximum value for ε is $2\sqrt{2} = 2.8284...$ in $SU(2)$ case.

PoS(LATTICE 2008)232

of the flat directions in the scalar potential which will be discussed later, and since the magnitude of the molecular dynamical force depends on the magnitude of the scalar fields, the acceptance fluctuates during the simulation.[6] The magnitude of the scalar fields tends to be larger and the acceptance tends to be smaller as the simulation runs. See also Table 1 for the parameters and number of configurations we use in the application.

4. Application: observing dynamical SUSY breaking

What can we do with this simulation? We use it to observe the dynamical SUSY breaking using a method we proposed in [20, 21]. The requirement for the lattice model in the method is that it should have nilpotent Q and exact Q-invariance, which the current model satisfies. Since the SUSY is not broken in the perturbation if it is not broken in the tree level, a way of observing SUSY breaking due to non-perturbative effects is very important. Usually, the Witten index provides such a method but in this system, two-dimensional $\mathscr{N}=(2,2)$ pure super Yang-Mills, it is not available. What we know without numerical simulations is an argument by Hori and Tong that SUSY is probably spontaneously broken in this system [24].

What we measure is the ground state energy using the Hamiltonian as the order parameter. As well known, vacuum expectation value of the Hamiltonian is zero if and only if the SUSY is not broken. The advantage of using the Hamiltonian is that it requires one-point function which is numerically much less expensive than two-point function.

Since we are interested in that it is zero or not, the correct choice of the origin of the Hamiltonian is crucial. We use the SUSY algebra to define the Hamiltonian. We regard the anti-commutator in (2.1) as follow:

$$Q\mathscr{J}_0^{(0)} = 2\mathscr{H}, \tag{4.1}$$

that is, Q transformation of the 0-th component of the Noether current corresponding Q_0, $\mathscr{J}_0^{(0)}$, gives the Hamiltonian density $\mathscr{H}$. On the lattice we have only Q transformation but no Q_0 transformation. Therefore, we discretize the continuum version of the Noether current by hand. We use the following as the 0-th component of the current for Q_0:

$$\begin{aligned}\mathscr{J}_0^{(0)}(x) = \frac{1}{a^4g^2}\operatorname{tr}\Big\{&\eta(x)[\phi(x),\overline{\phi}(x)]^2 + 2\chi(x)H(x)\\ &- 2i\psi_0(x)\big(\overline{\phi}(x) - U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\big)\\ &+ 2i\psi_1(x)\big(\overline{\phi}(x) - U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\big)\Big\}.\end{aligned} \tag{4.2}$$

Since we know the Q transformation on the lattice so it is straightforward to obtain the Hamiltonian.

Another point is the boundary condition. As usual method for observing spontaneous symmetry breaking, we apply an external field conjugate to the order parameter. The conjugate to the Hamiltonian is the temperature. That is, we impose the anti-periodic condition in the time direction for fermion. Therefore we break SUSY by boundary condition or equivalently by the temperature. Then we take zero temperature limit and see the effect of the breaking is left or not.

[6] This is for the case in which we impose anti-periodic boundary condition in time direction for fermions. In the periodic case, the flat direction is lifted so that we do not observe such fluctuations of the acceptance.

PoS(LATTICE 2008)232

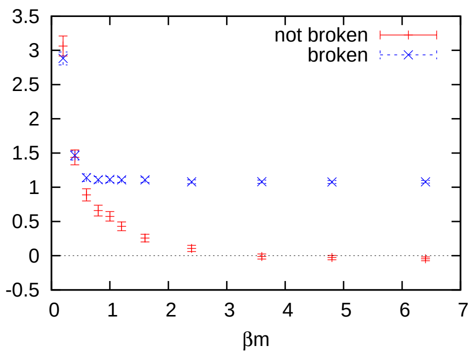

Figure 1: Expectation value of the Hamiltonian for supersymmetric quantum mechanics versus inverse temperature β. All quantities are measured in a dimensionful parameter m in the potential.

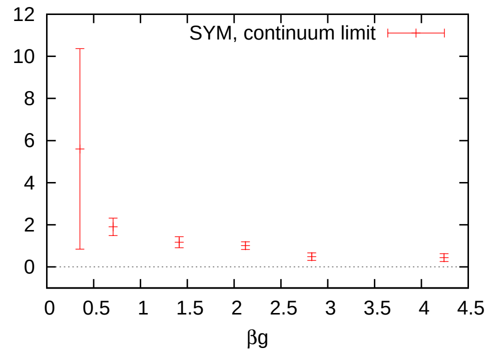

Figure 2: Expectation value of the Hamiltonian density for super Yang-Mills versus inverse temperature β. All quantities are measured in unit of dimensionful gauge coupling g.

As a check of the method, we first investigate supersymmetric quantum mechanics. The known fact is that the form of potential decides whether SUSY is broken or not. We use a lattice model given in [25], which has nilpotent Q and Q-exact action. The details of the measurement and forms of the potential are found in [21]. Here we only show Figure 1, from which we can easily distinguish SUSY broken case and not-broken case. Our method actually works as expected.

Next let us investigate the super Yang-Mills case. Figure 2 shows the result. We put the statistical errors only in the plot. Although we can not deny a possibility of non-zero small ground state energy that means breaking of the SUSY, the plot shows the value of the ground state energy is small and close to zero. We need to take a limit of the inverse temperature $\beta \to \infty$ but the plot implies that the β we use is enough large since the expectation value of the Hamiltonian density is almost saturated. Note that all quantities are measured in the dimensionful gauge coupling g. Some details of the measurement is in order. We fix the physical spacial size $L_S = 1.41/g$. We discard first 20,000–30,000 trajectories as thermalization. We calculate the Hamiltonian every 10 trajectories. In order to reduce the errors, we take an average over the lattice. The errors are obtained using a jackknife analysis with binning. The bin size with which the autocorrelation disappears is typically 10–20. We list the number of the configurations after the binning in Table 1.

A potential danger comes from the non-compact flat direction of scalar fields. The current lattice model as well as the target theory in the continuum has classical flat directions. Figure 3 shows in fact the scalar fields are not stabilized at the origin of the potential. It rather goes far away over the cut off scale.[7] We regard this fact as an evidence that we have actually integrated over the non-compact configuration space of the scalar fields. In fact the quantity of interest, the Hamiltonian density, does not depends on the norm of the scalars (Fig. 4).

5. Conclusion and Discussion

We carried out the RHMC simulation for two-dimensional $\mathcal{N} = (2,2)$ super Yang-Mills based on Sugino model, which exactly keeps one scalar supercharge. Using the simulation, we observed

[7] Effects of the large scalar fields will be discussed in [26].

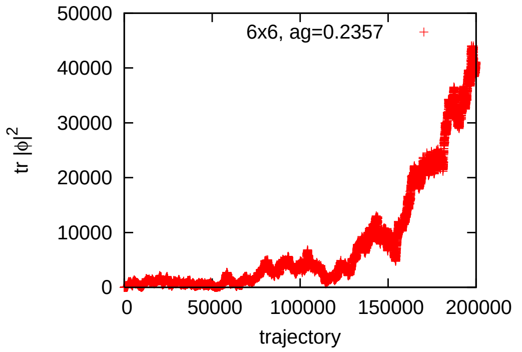

Figure 3: Evolution of the scalar norm over the trajectories in the anti-periodic case. The scalar norm is measured by $1/g^2$. It tends to be larger as the simulation runs.

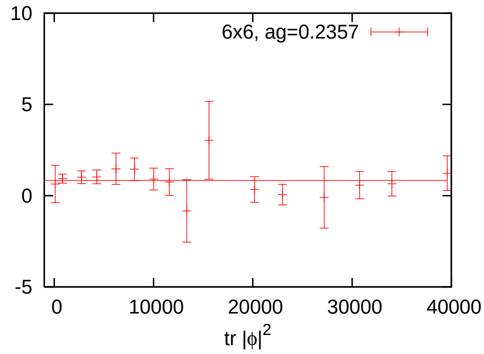

Figure 4: Scalar norm dependence of the hamiltonian density at fixed lattice spacing. No dependence can be found.

		N_T/N_S					
N_S	ag	0.25	0.5	1	1.5	2	3
6	0.2357	—	500	1,700	1,300	1,000	1,100
8	0.1768	—	500	1,100	1,100	280	700
10	0.1414	—	—	—	—	—	175
12	0.1179	20	600	110	450	500	—
16	0.08839	10	—	—	—	—	—
20	0.07071	20	—	—	—	—	—

Table 1: Numbers of independent configurations after binning for $N_T \times N_S$ lattice, N_T refers temporal direction N_S refers spacial direction.

the ground state energy which is useful to check the dynamical SUSY breaking. Compared with the result in [20, 21], which did not utilize the dynamical fermion but the fermion effects were reweighted, the current simulation drastically reduced the error. An extension to couple the matter multiplet based on [27] will be an interesting application.

Before giving the conclusive result with respect to SUSY breaking using this simulation, we should check whether the current lattice model actually describes the target continuum theory. We should check the restoration of the other three supercharges explicitly. It is no longer an assumption based on the perturbative discussion but the current simulation with dynamical fermion allows us to give an explicit numerical check [26].

Acknowledgments

The author would like to thank F. Sugino and H. Suzuki for useful discussion. He also thanks H. Kawai, M. Hanada, H. Matsufuru and K. Murakami. This work is supported by the Special Postdoctoral Researchers Program in RIKEN. The numerical simulation was done using Riken Super Combined Cluster(RSCC). The simulation code in C++ was developed based on [28].

References

[1] D. B. Kaplan, E. Katz and M. Unsal, JHEP **0305** (2003) 037 [hep-lat/0206019].

[2] F. Sugino, JHEP **0401** (2004) 015 [hep-lat/0311021].

[3] S. Catterall, JHEP **0411** (2004) 006 [hep-lat/0410052].

[4] H. Suzuki and Y. Taniguchi, JHEP **0510** (2005) 082 [hep-lat/0507019].

[5] A. D'Adda, I. Kanamori, N. Kawamoto and K. Nagata, Phys. Lett. B **633** (2006) 645 [hep-lat/0507029].

[6] T. Takimi, JHEP **0707** (2007) 010 [arXiv:0705.3831].

[7] P. H. Damgaard and S. Matsuura, JHEP **0708** (2007) 087 [arXiv:0706.3007].

[8] P. H. Damgaard and S. Matsuura, JHEP **0709** (2007) 097 [arXiv:0708.4129].

[9] J. Giedt, PoS **LAT2006** (2006) 008 [hep-lat/0701006].

[10] K. Nagata, arXiv:0805.4235.

[11] S. Arianos, A. D'Adda, A. Feo, N. Kawamoto and J. Saito, arXiv:0806.0686.

[12] M. Kato, M. Sakamoto and H. So, JHEP **0805** (2008) 057 [arXiv:0803.3121].

[13] S. Catterall, JHEP **0603** (2006) 032 [hep-lat/0602004].

[14] H. Suzuki, JHEP **0709** (2007) 052 [arXiv:0706.1392].

[15] H. Fukaya, I. Kanamori, H. Suzuki and T. Takimi, PoS **LAT2007** (2007) 264 [arXiv:0709.4076].

[16] J. W. Elliott and G. D. Moore, PoS **LAT2005** 245 (2006) [JHEP **0511** 010 (2005)] [hep-lat/0509032].

[17] J. W. Elliott and G. D. Moore, JHEP **0711**, (2007) 067 [arXiv:0708.3214].

[18] J. W. Elliott, J. Giedt and G. D. Moore, arXiv:0806.0013.

[19] M. A. Clark, A. D. Kennedy and Z. Sroczynski, Nucl. Phys. Proc. Suppl. **140** (2005) 835 [hep-lat/0409133].

[20] I. Kanamori, H. Suzuki and F. Sugino, Phys. Rev. D **77** (2008) 091502 [arXiv:0711.2099].

[21] I. Kanamori, F. Sugino and H. Suzuki, Prog. Theor. Phys. **119** (2008) 797 [arXiv:0711.2132].

[22] M.A. Clark and A.D. Kennedy, `http://www.ph.ed.ac.uk/mike/remez~mike/remez`, 2005.

[23] J. C. Sexton and D. H. Weingarten, Nucl. Phys. B **380** (1992) 665.

[24] K. Hori and D. Tong, JHEP **0705** (2007) 079 [hep-th/0609032].

[25] S. Catterall and E. Gregory, Phys. Lett. B **487** (2000) 349 [hep-lat/0006013].

[26] I. Kanamori and H. Suzuki, in preparation.

[27] F. Sugino, arXiv:0807.2683 [hep-lat].

[28] M. Di Pierro, *Comput. Phys. Commun.* **141** (2001) 98 [hep-lat/0004007].