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A study of ghost–gluon vertices in MAG: Feynman rules

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In the continuum regime the running coupling constant of QCD, obtained through the ghost-gluon vertex in maximally Abelian gauge (MAG), depends only on the renormalization factor of the diagonal gluon propagator, due to cancellation of the other renormalization factors. This fact is a clear manifestation of Abelian dominance. In this work we present a tree-level calculation (Feynman rules) of the ghost-gluon vertices in MAG on the lattice. These vertices can be useful for numerical and/or perturbative studies of the QCD running coupling and of the gluon propagator in the lattice formulation of QCD.

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1. Introduction

In the low-energy regime the maximally Abelian gauge (MAG) is suitable for the study of the dual-superconductivity mechanism for color confinement. According to this mechanism, QCD could be described by an effective Abelian theory with monopoles. The condensation of the magnetic charges might give rise to quark confinement (see for instance [1, 2] and references therein). Other important features of MAG are that, in the continuum, it is a renormalizable gauge (see for example [3, 4] and references therein) and that it has a simple formulation on the lattice (see for example [5, 6]). These features allow one to compare results obtained in the continuum to results from lattice calculations.

The infrared behavior of the running coupling constant in MAG can be studied through the vertex with two off–diagonal ghosts and one diagonal gluon. Then, the running coupling is related to the renormalization constants by the following expression

$$\alpha(p) = \alpha_0 \, Z_D(p) \left[\frac{Z_G(p)}{Z_V(p)}\right]^2 , \tag{1.1}$$

where Z_D is the renormalization factor of the diagonal gluon propagator, Z_G is the renormalization factor of the off–diagonal ghost propagator and Z_V is the renormalization factor of the vertex [7]. On the other hand, from perturbative studies in the continuum [7] one knows that $Z_G = Z_V$ and therefore the running coupling depends only on the renormalization factor of the diagonal gluon propagator

$$\alpha(p) = \alpha_0 \, Z_D(p) \, . \tag{1.2}$$

This fact can be seen as a clear manifestation of Abelian dominance.

In this paper we present a tree–level calculation of the ghost–gluon vertices on the lattice in MAG with the determination of the Feynman rules for these vertices. In Section 2 we present a brief review of the Yang–Mills theory in MAG, both in the continuum and in the lattice case. In Section 3 we expand the ghost action on the lattice, obtaining the Feynman rules for the ghost–gluon vertices in MAG. In the same section, we conclude the paper with some remarks.

2. Yang–Mills theory in MAG

In this section we set our notation and definitions for the $SU(2)$ case.

2.1 Continuum case

The Abelian configurations of the gauge field are identified with the diagonal components $\mathscr{A}_\mu^3$ corresponding to the $U(1)$ subgroup of the $SU(2)$ group, i.e.

$$\mathscr{A}_\mu = \mathscr{A}_\mu^a T^a + \mathscr{A}_\mu^3 T^3 \, , \; a = 1,2 \, . \tag{2.1}$$

The MAG gauge-fixing condition is given by

$$\mathscr{D}_\mu^{ab} \mathscr{A}_\mu^b = 0 \tag{2.2}$$

with

$$\mathscr{D}_\mu^{ab} = \delta^{ab} \partial_\mu - g \varepsilon^{ab3} \mathscr{A}_\mu^3 \, . \tag{2.3}$$

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This can be obtained by minimizing the norm of the off–diagonal components with respect to gauge transformations. Indeed the stationarity condition

$$\delta \int d^4x\, \mathscr{A}_\mu^a \mathscr{A}_\mu^a = 0 \tag{2.4}$$

implies

$$\mathscr{D}_\mu^{ab} \mathscr{A}_\mu^b = 0 \ , \tag{2.5}$$

where δ corresponds to a gauge transformation for the off–diagonal fields, i.e.

$$\delta \mathscr{A}_\mu^a = -\mathscr{D}_\mu^{ab} \Lambda^b + g \varepsilon^{3ab} \mathscr{A}_\mu^b \Lambda^3 \ . \tag{2.6}$$

The remaining local $U(1)$ invariance can be fixed by imposing the additional condition (for the diagonal component)

$$\partial_\mu \mathscr{A}_\mu^3 = 0 \ . \tag{2.7}$$

In MAG the Yang–Mills partition function is written as

$$Z = \int [D\mathscr{A}] \ \det \left[\mathscr{M}^{ab} \right] \ \delta \left(\mathscr{D}_\mu^{ab} \mathscr{A}_\mu^b \right) \ \delta \left(\partial_\mu \mathscr{A}_\mu^3 \right) \ \exp\left[-S_{YM}\left(\mathscr{A} \right) \right] \ , \tag{2.8}$$

where $\mathscr{M}^{ab}$ is the Faddeev-Popov matrix

$$\mathscr{M}^{ab} = -\mathscr{D}_\mu^{ad} \mathscr{D}_\mu^{db} - g^2 \varepsilon^{3ac} \varepsilon^{3bd} \mathscr{A}_\mu^c \mathscr{A}_\mu^d \ . \tag{2.9}$$

2.2 Lattice case

On the lattice, the Yang–Mills theory can be defined by the Wilson action [8], which in turn is written in terms of link variables $U_\mu(x) \in SU(2)$ group. The gauge fields on the lattice are related to the link variables through the relation

$$A_\mu(x) = \frac{1}{2i} \left[U_\mu(x) - U_\mu^\dagger(x) \right] \ . \tag{2.10}$$

At the same time, the link variables can be written as

$$U_\mu(x) = U_\mu^0(x) \, \mathbb{1} + i A_\mu^B(x) \, \sigma^B \ , \ \ B = 1,2,3 \ , \tag{2.11}$$

where $\mathbb{1}$ is the 2×2 unit matrix and σ^B are the Pauli matrices with

$$\left[U_\mu^0(x) \right]^2 + \sum_B \left[A_\mu^B(x) \right]^2 = 1 \, . \tag{2.12}$$

The gauge field in the continuum $\mathscr{A}_\mu(x)$ is related to the link variable by the following expression

$$U_\mu(x) = \exp\left[i \bar{a} g_0 \mathscr{A}_\mu(x) \right] \ , \tag{2.13}$$

where g_0 is the bare coupling constant, $\bar{a}$ is the lattice spacing and

$$\mathscr{A}_\mu(x) \equiv \frac{\sigma^B}{2} \mathscr{A}_\mu^B(x) \ . \tag{2.14}$$

For small $\overline{a}$ one finds

$$A_\mu^B(x) \;\approx\; \frac{\overline{a}\,g_0}{2}\,\mathscr{A}_\mu^B(x) \;+\; \mathscr{O}(\overline{a}^3)\,. \tag{2.15}$$

In order to fix the MAG on the lattice one should minimize the functional

$$\mathscr{E}\,[U] = -\,\frac{1}{8V}\sum_{x,\mu}\mathrm{Tr}\left[\sigma_3\,U_\mu(x)\,\sigma_3\,U_\mu^\dagger(x)\right]\;. \tag{2.16}$$

Indeed, using the properties of the Pauli matrices one can verify that minimizing the functional above is equivalent to minimize the norm of the off–diagonal gauge fields [as in Eq. (2.4)].

3. Ghost–Gluon Vertices on the Lattice

Using the notation of Ref. [9], finite differences on the lattice are written as

$$\hat{\partial}_\mu^L\phi(x) \;=\; \phi(x)-\phi(x-\overline{a}e_\mu) \tag{3.1}$$

$$\hat{\partial}_\mu^R\phi(x) \;=\; \phi(x+\overline{a}e_\mu)-\phi(x)\;. \tag{3.2}$$

By considering a gauge transformation $\gamma(x)=\exp[i\omega^A(x)T^A]$, with $A=1,2,3$, an infinitesimal gauge transformation on the lattice can be written as

$$\delta A_\mu^B = -\left[\Gamma^{AB}\hat{\partial}_\mu^R + 2\varepsilon^{ABC}A_\mu^C\right]\,\omega^B(x)\;, \tag{3.3}$$

where Γ^{AB} is related to the Haar measure and is given by

$$\Gamma^{AB} = \delta^{AB} + \varepsilon^{ABC}A_\mu^C - \frac{1}{6}\{T^C,T^D\}^{AB}A_\mu^C A_\mu^D + \ldots \tag{3.4}$$

with $T^A = \sigma^A/2$. One can notice that lattice artifacts are present in (3.3) due to Eq. (2.15).

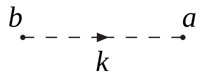
After using the Faddeev–Popov quantization method, the ghost action can be expanded in the series

$$\begin{aligned}
S_{FP} \;=\; & \sum_x \overline{c}^a(x)\,\Big\{-\Box\,\delta^{ab} \\
& -\; g_0\,\varepsilon^{ab}\left[\partial_\mu^L\,\mathscr{A}_\mu^3\left(1+\frac{\overline{a}}{2}\partial_\mu^R\right)+\left(1-\frac{\overline{a}}{2}\partial_\mu^L\right)\mathscr{A}_\mu^3\,\partial_\mu^R\right] \\
& +\; g_0^2\,\delta^{ab}\left(1-\frac{\overline{a}}{2}\partial_\mu^L\right)\mathscr{A}_\mu^3\mathscr{A}_\mu^3\left(1+\frac{\overline{a}}{2}\partial_\mu^R\right) \\
& -\; g_0^2\,\varepsilon^{ac}\varepsilon^{bd}\left(1-\frac{\overline{a}}{2}\partial_\mu^L\right)\mathscr{A}_\mu^c\mathscr{A}_\mu^d\left(1+\frac{\overline{a}}{2}\partial_\mu^R\right)+\mathscr{O}(\overline{a})+\ldots\Big\}\;c^b(x)\;,
\end{aligned} \tag{3.5}$$

where $\partial^{L,R} = \frac{1}{\overline{a}}\hat{\partial}^{L,R}$ and $\Box = \sum_\mu \partial_\mu^L\partial_\mu^R$. One should observe that the lattice (discretized) version of the Faddeev–Popov matrix has an infinite number of vertices. One should also note that, for the numerical evaluation of the (off-diagonal) ghost propagator [6, 10], one uses the Faddeev–Popov matrix obtained from the second variation of the functional (2.16).

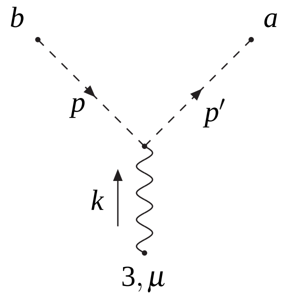
3.1 Feynman Rules for the Ghost Sector

One can now perform the Fourier decomposition (see for instance Ref. [9]) of Eq. (3.5) in order to obtain the Feynman rules for the ghost sector in momentum space. The first term on the rhs of (3.5) is the off–diagonal ghost propagator:



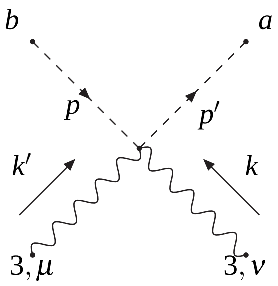
$$= \quad \frac{1}{k^2}\,\delta_{ab} \; . \tag{3.6}$$

The second term is a vertex involving one diagonal gluon and two off–diagonal ghosts



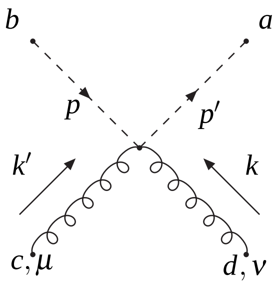
$$= \quad (2\pi)^4\;\delta^4(k+p-p')\;\times$$
$$i\,g_0\,\varepsilon^{ab}.\left[p'_\mu\,.\cos\left(\frac{p_\mu \overline{a}}{2}\right) + p_\mu\,.\cos\left(\frac{p'_\mu \overline{a}}{2}\right)\right] \; . \tag{3.7}$$

The third term is a vertex with 2 diagonal gluons and 2 off–diagonal ghosts



$$= \quad (2\pi)^4\;\delta^4(k'+k+p-p')\;\times$$
$$(-2g_0^2)\;\delta^{ab}\;\delta^{\mu\nu}.\;\cos\left(\frac{p_\mu \overline{a}}{2}\right).\cos\left(\frac{p'_\mu \overline{a}}{2}\right) \; . \tag{3.8}$$

The forth term is a vertex with 2 off–diagonal gluons and 2 off–diagonal ghosts



$$= \quad (2\pi)^4\;\delta^4(k'+k+p-p')(-g_0^2)$$
$$\left(\varepsilon^{ac}\varepsilon^{db}+\varepsilon^{ad}\varepsilon^{cb}\right)\;\delta^{\mu\nu}.\;\cos\left(\frac{p_\mu \overline{a}}{2}\right).\cos\left(\frac{p'_\mu \overline{a}}{2}\right) \; . \tag{3.9}$$

The vertices above are the ones that survive in the limit $\overline{a} \rightarrow 0$ and should be compared with the vertices obtained in the continuum [11]. A preliminary numerical study of the vertex with one diagonal gluon and two off-diagonal ghosts has been presented in [12].

The study presented here is preliminary, since a more careful analysis should be performed in order to check the renormalizability of the theory in this context. In particular, one would expect the inclusion of a quartic ghost term in the action, as occurs in the continuum [3].

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References

[1] Z. F. Ezawa and A. Iwazaki, Phys. Rev. **D 25** (1982) 2681.

[2] T. Suzuki and I. Yotsuyanagi, Phys. Rev. **D 42** (1990) 4257.

[3] H. Min, T. Lee and P. Y. Pac, Phys. Rev. **D 32** (1985) 440.

[4] J. A. Gracey, JHEP **0504** (2005) 012.

[5] V. G. Bornyakov, M. N. Chernodub, F. V. Gubarev, S. M. Morozov and M. I. Polikarpov, Phys. Lett. **B 559** (2003) 214.

[6] T. Mendes, A. Cucchieri and A. Mihara, AIP Conf. Proc. **892** (2007) 203.

[7] M. Quandt and H. Reinhardt, Int. J. Mod. Phys. **A 13** (1998) 4049.

[8] K. G. Wilson, Phys. Rev. **D 10** (1974) 2445.

[9] H. J. Rothe, *Lattice Gauge Theories. An Introduction*, 3rd. edition, World Scientific Publishing Co. Pte. Ltd. , 2005.

[10] T. Mendes, A. Cucchieri, A. Maas and A. Mihara, arXiv:0809.3741 [hep-lat].

[11] T. Shinohara, T. Imai and K. -I. Kondo, Int. J. Mod. Phys. **A 18** (2003) 5733.

[12] A. Mihara, A. Cucchieri and T. Mendes, Int. J. Mod. Phys. **E 16** (2007) 2935.