K-meson vector and tensor decay constants and B_K-parameter from $N_f = 2$ tmQCD

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We present work in progress on the computation of the *K*-meson vector and tensor decay constants, as well as the *B*-parameter in Kaon oscillations. Our simulations are performed in a partially quenched setup, with two dynamical (sea) Wilson quark flavours, having a maximally twisted mass term. Valence quarks are either of the standard or the Osterwalder-Seiler maximally twisted variety. These two regularizations can be suitably combined in order to obtain a B_K parameter which is both multiplicatively renormalizable and $O(a)$ improved.

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1. Introduction: theory and computational setup

In the last few years, the ETM Collaboration (ETMC) has been carrying out state-of-the-art lattice QCD simulations with $N_f = 2$ dynamical flavours (sea quarks) and 'lightish" pseudoscalar meson masses (300 MeV $< m_{PS} < 550$ MeV). Strangeness clearly enters the game in a partially quenched context. Several physical quantities are currently being analyzed for a few lattice spacings. In the present work we will present *preliminary* results on: (i) the vector meson mass m_{K^*}, (ii) its decay constant f_{K^*}, (iii) the ratio of tensor to vector decay constants f_T/f_V, computed at the K* physical mass and (iv) the B_K parameter for neutral Kaon oscillations. Other subgroups of the collaboration are working on decay constants in the light and strange quark sector [1, 2].

ETMC simulations are performed with the tree-level Symanzik improved gauge action. The $N_f = 2$ sea quark flavours are regularized by the standard Wilson fermion action with a maximally twisted mass term (referred to as "standard tmQCD case") [3] . This means that the the two light flavours are organized in a flavour doublet $\bar{\psi}^T = (\bar{u}\ \bar{d})$ and the fermion lattice action is given by

$$\mathscr{L}_{tm} = \bar{\psi}\left[D_W + i\mu_q\tau^3\gamma_5\right]\psi \qquad (1.1)$$

with D_W denoting the critical Wilson-Dirac operator. This formulation has well known advantages, amongst which: (i) Renormalization properties are much simpler than with standard Wilson quarks, in many cases of interest (e.g. pseudoscalar decay constants, chiral condensate, B_K...); (ii) improvement is automatic at maximal twist [4]. One should note, however that for Weak Matrix Elements (WMEs) of 4-fermion operators (e.g. B_K), it is not possible to retain both automatic improvement (through the standard tmQCD formalism, with all flavours at maximal twist) and multiplicative renormalization for the relevant operator; see refs. [5, 6]. One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD, in which valence quarks enter with a distinct action for each flavour and a fully twisted mass term [7]:

$$\mathscr{L}_{OS} = \bar{\psi}_f\left[D_W + i\mu_f\gamma_5\right]\psi_f \qquad\qquad f = u, d, s\cdots \qquad (1.2)$$

Suitable combinations of μ_f signs for each flavour in the above action ensure automatic improvement and multiplicative renormalization for say, B_K. The OS option is a compromise, since unitarity issues arise at finite lattice spacing when sea and valence flavours are treated differently. Nevertheless, in our partially quenched setup ($N_f = 2$ sea quark flavours and a valence strange quark) compromising unitarity is anyway unavoidable in any regularization.

The ETMC runs are performed at three gauge couplings β. Here we report work in progress, confining ourselves to the "master run" at $\beta = 3.90$, corresponding to a lattice spacing of $a \simeq 0.086(1)$fm (i.e. $a^{-1} \simeq 2.3$GeV). The lattice volume is $V = 24^3 \times 48$. Our ensemble consists of 240 gauge field configurations for the K^*-meson mass and decay constants and 200 configurations for B_K. The sea quark mass is set at five values $a\mu = 0.0040, 0.0064, 0.0085, 0.0100, 0.0150$, corresponding to pseudoscalar meson masses in the range 300MeV $< m_{PS} < 550$MeV. The valence quark masses are set at seven values; those of the sea quarks plus the values $a\mu = 0.0220, 0.0270$, which are meant to facilitate the interpolation to the physical strange quark mass. We use the calibration results of previous ETMC work [8, 9]; e.g. the physical light quark mass is at $a\mu_d = a\mu(m_\pi) = 0.00079$, whereas the physical strange quark mass is at $a\mu_s = a\mu(m_K) = 0.0217(10)$.

For $B_{\mathrm K}$ only, we checked for finite volume effects by running also at $V = 32^3 \times 64$ at the lowest quark mass ($a\mu = 0.0040$).

For the computation of correlation functions, we used stochastic sources of the extended "one-end trick" of refs. [10, 11]. For a concise exposition of the method see also ref. [12].

2. Vector meson masses and decay constants

The vector and tensor decay constants are defined by the formulae:

$$\langle 0|\mathscr{V}_k|V;\lambda\rangle = f_V \varepsilon_k^\lambda m_V \qquad\qquad \langle 0|\mathscr{T}_{0k}|V;\lambda\rangle = -if_T \varepsilon_k^\lambda m_V \qquad (2.1)$$

Vector meson masses and the decay constants f_V, f_T are computed from the correlators (in continuum notation) $C_{V_k V_k}(t) = \sum_{\vec{x},k} < \mathscr{V}_k(x)\,\mathscr{V}^\dagger{}_k(0) >$ and $C_{T_{0k}T_{0k}}(t) = \sum_{\vec{x},k} < \mathscr{T}_{0k}(x)\,\mathscr{T}^\dagger{}_{0k}(0) >$. The best estimate of the vector meson effective mass is provided by the correlator $C_{V_k V_k}(t)$. The estimate from the correlator $C_{T_k T_k}(t)$ is much noisier. We compute $f^T/f^V|_{K^*}$ from the large time asymptotic behaviour of the ratio $[C_{T_{0k}T_{0k}}(t)/C_{V_k V_k}(t)]^{1/2}$.

As discussed in ref. [13], the correctly normalized vector current in the tmQCD fermion regularization is $\mathscr{V}_\mu = Z_A A_\mu^{\mathrm{tm}}$, while in the OS setup we have $\mathscr{V}_\mu = Z_V V_\mu^{\mathrm{OS}}$. With $\tilde{T}_{\mu\nu} \equiv \varepsilon_{\mu\nu\rho\sigma} T_{\rho\sigma}$, we similarly have for the tensor density $\mathscr{T}_{\mu\nu} = Z_T T_{\mu\nu}^{\mathrm{tm}}$ and $\mathscr{T}_{\mu\nu} = Z_T \tilde{T}_{\mu\nu}^{\mathrm{OS}}$. We use the RI/MOM [14] estimates for the (re)normalisation constants, computed in a tmQCD framework in ref. [13], obtaining $Z_T(a^{-1}; \beta = 3.9) = 0.769(4)$, $Z_A(\beta = 3.9) = 0.771(4)$ and $Z_V(\beta = 3.9) = 0.6104(2)$. The slight differences between the above values and the ones of ref. [13] are due to a revised analysis.

We wish to highlight straightaway the two problems we have encountered: (i) For all sea quark masses, when the valence quark masses are in the lightest range (say, around $\mu_{\mathrm{val}} = 0.0040$), the vector meson effective mass has a poor plateau. The situation already improves at the next mass value of our simulation, $\mu_{\mathrm{val}} = 0.0064$. Nevertheless, since the signal-to-noise ratio behaves as expected (i.e. it drops like $\exp[-(m_{\mathrm V} - m_{\mathrm{PS}})t]$), the ρ-meson mass and decay constant can still be extracted (see results presented in ref. [1]). (ii) A poor quality vector meson effective mass is also seen when $\mu_{val} < \mu_{sea}$. This problem is absent in the pseudoscalar channel.

We thus proceed as follows: the strange vector meson consists of two valence quarks (μ_l, μ_h). We compute the necessary observables (vector meson mass, vector decay constant and the ratio of the tensor to vector decay constant) for all combinations of $a\mu_l = a\mu_{sea}$ and $a\mu_h = 0.0150, 0.0220, 0.0270$. In this way unitarity holds in the light quark sector, while the heavy valence quark mass, in a partially quenched rationale, spans a range around the physical value μ_s. Examples of the quality of our signal (for the lightest $a\mu_l = a\mu_{sea}$ masses) are given in Fig. 1.

Subsequently, keeping $a\mu_h$ fixed, we extrapolate these results linearly to the physical down quark value $a\mu_d = 0.00079$ (alternative fits are currently under investigation). This is repeated for all three $a\mu_h$ values, before the results are linearly interpolated to the physical strange quark mass $a\mu_s = 0.0217(10)$. Examples of the quality of such extrapolations and interpolations are displayed in Fig. 2. Our final results are (tmQCD regularization on the lhs and OS regularization on the rhs)

$$\begin{aligned}
aM_V|_{K^*} &= 0.422(10)(03) & aM_V|_{K^*} &= 0.437(08)(04) \\
af_V|_{K^*} &= 0.106(05)(02) & af_V|_{K^*} &= 0.117(03)(01) \\
f_T/f_V|_{K^*} &= 0.770(20)(09) & f_T/f_V|_{K^*} &= 0.759(19)(07)
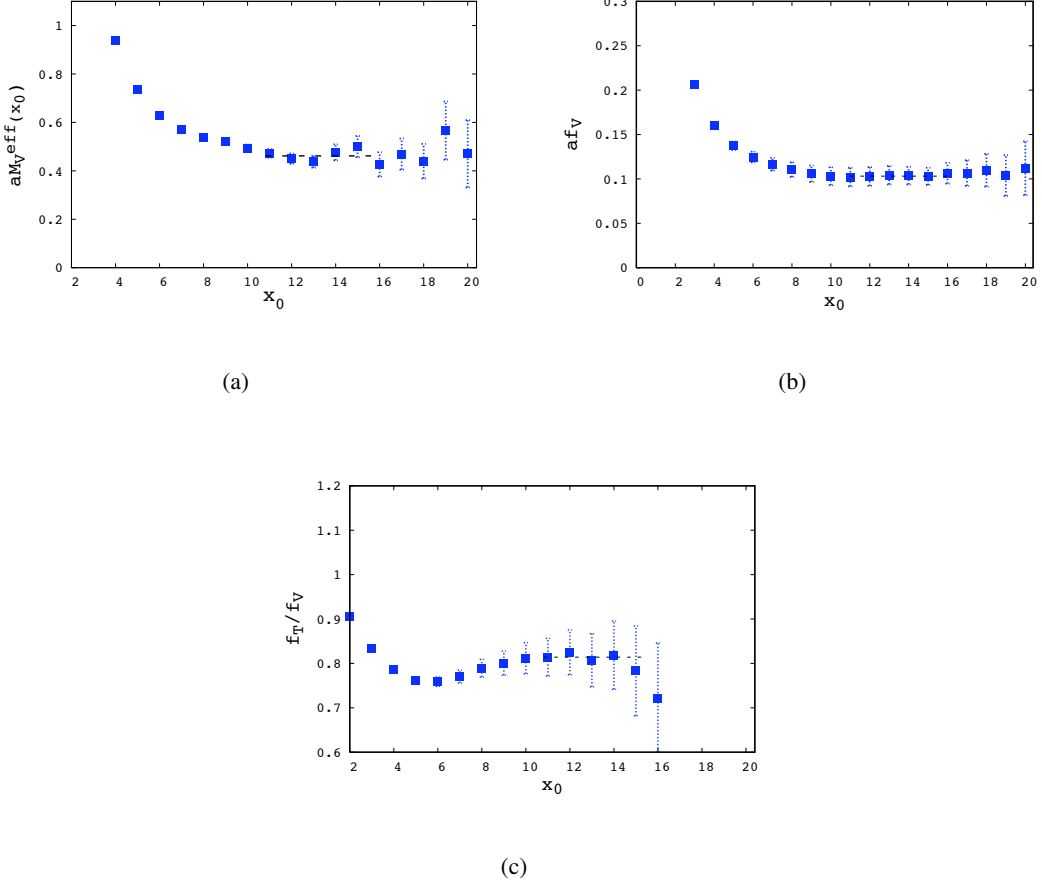\end{aligned} \qquad (2.2)$$

(a)

(b)

(c)

Figure 1: Vector meson quantities plotted as a function of the operators' time separation, computed at $a\mu_l = a\mu_{sea} = 0.0040$ and $a\mu_h = 0.0150$. (a) effective mass; (b) decay constant; (c) ratio f_T/f_V. The quoted results are obtained by taking data over the time plateau $11 \leq x_0 \leq 16$.

The first error is statistical. The second is systematic and includes the uncertainty of the $a\mu_s$ estimate plus (for the decay constants) that of the renormalization factors Z_V, Z_A, Z_T. The fairly good agreement between tmQCD and OS estimates suggests that cutoff effects in the vector channel are small, in accordance with the expectations of ref. [15]. This result must clearly be confirmed by repeating the analysis at other β values. The experimental values of the vector meson mass and decay constants, expressed in lattice units (at $\beta = 3.9$, we use $a \simeq 0.086$ fm) are $am_{K^*} = 0.381$ and $af_{K^*} = 0.0927$, which are reasonably close to our lattice estimates. Finally, the tensor to vector decay constant ratio is RG-run from the scale $a^{-1} \sim 2.3$GeV to the usual reference scale of 2GeV, obtaining $f_T/f_V|_{K^*} = 0.764(19)(03)$. This is pretty close to the continuum limit quenched result $f_T/f_V|_{K^*} = 0.74(2)$ of ref. [16].

3. The K-meson bag parameter

We now apply the proposal of ref. [7] in an $O(a)$-improved calculation of B_K. Recall that,
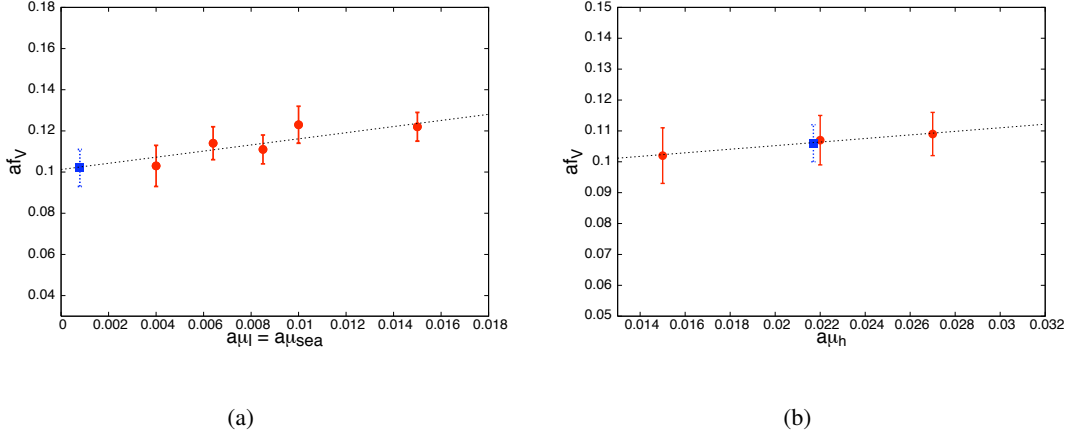
Figure 2: (a) Extrapolation of the vector meson decay constant f_V to the physical down quark mass $a\mu_d$, at fixed heavy quark mass $a\mu_h = 0.0150$. (b) Interpolation of $f_V(a\mu_d)$ to the physical strange quark mass $a\mu_s$ of the vector meson decay constant f_V. The red circles correspond to data points, the blue squares to the extrapolation and interpolation results.

since we require both automatic improvement and multiplicative renormalization of this quantity, the setup is "mixed": the two Kaon valence quarks are maximally twisted in standard tmQCD fashion, while the two anti-Kaon ones are OS valence quarks (or vice versa). In this way, the continuum (renormalized) four-fermion operator is related to the lattice bare one as follows (the notation should be obvious)

$$\mathcal{V}_\mu \mathcal{V}_\mu + \mathcal{A}_\mu \mathcal{A}_\mu = Z_{VA+AV} \left[V_\mu^{\text{tm}} A_\mu^{\text{OS}} + A_\mu^{\text{tm}} V_\mu^{\text{OS}} \right] \tag{3.1}$$

With $x_0 = w$ an arbitrary reference time-slice, the relevant 3-point correlation function consists of two "K-meson walls" with noise sources at fixed times (w and $w + T/2$) and a 4-fermion operator living at different time-slices x_0, with $w < x_0 < w + T/2$. The signal quality is improved in various ways. For example, the reference time w is varied from configuration to configuration. Moreover, as already stated previously, the valence quarks emanating from w are of the OS variety, while those emanating from $w + T/2$ are standard tmQCD ones. This situation is also reversed and the corresponding correlation functions are suitably averaged.

The B_K-parameter is measured from correlation functions with the light valence quark mass kept equal to the sea quark mass ($a\mu_l = a\mu_{\text{sea}}$). At fixed heavy valence quark mass $a\mu_h$, we fit the light mass behaviour in $a\mu_l$, using the $SU(2)$ Partially Quenched Chiral Perturbation Theory (PQ-χPT) result of refs. [17, 18]:

$$B(\mu_h) = B_\chi(\mu_h) \left[1 + b(\mu_h) \frac{2B_0}{f^2} \mu_l - \frac{2B_0}{32\pi^2 f^2} \mu_l \ln\left(\frac{2B_0\mu_l}{\Lambda_\chi^2}\right) \right] \tag{3.2}$$

With the coefficient $2B_0/f^2$ known from earlier chiral fits of the light quark sector [8, 9], the above relation requires a two-parameter fit (B_χ and $b(\mu_h)$) in the chiral region. In this way we extrapolate $B(\mu_h)$ at the physical down quark mass $a\mu_d$. The result is shown in Fig. 3(a). Alternative polynomial fits are currently under study. The final step is to linearly interpolate the $B(\mu_h)$ estimates to

the physical strange quark mass value $a\mu_s$. This is shown in Fig. 3(b). Note that the result of the highest $a\mu_h$ has not been included, as it lies rather far from $a\mu_s$.

Finite size effects appear to be under control, since at $\mu_{sea} = 0.0040$ and $\mu_l = \mu_h = 0.0100$, we find $B_K^{\mathrm{bare}} = 0.591(5)$ for the $L = 24$ lattice and $B_K^{\mathrm{bare}} = 0.598(8)$ for the $L = 32$ one.

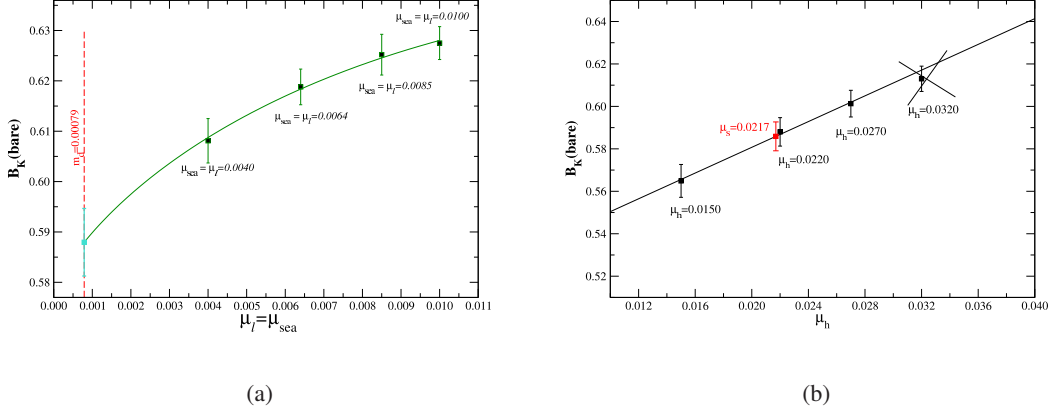|	
(a)	(b)

Figure 3: (a) Extrapolation of B_K to the physical down quark mass $a\mu_d$, at fixed heavy quark mass $a\mu_h = 0.0220$. The B_K datapoints are computed at degenerate light and sea quark masses, $\mu_l = \mu_{\mathrm{sea}}$ and fitted to eq. (3.2). (b) Linear interpolation of B_K, as a function of the heavy valence quark mass $a\mu_h$, to the physical strange quark mass $a\mu_s$.

Renormalization is again performed in the RI/MOM scheme [19]. The quality of our preliminary results for the multiplicative renormalization factor Z_{VA+AV} is shown in Fig. 4(a). The absense of mixing with "wrong chirality" operators is explicitly demonstrated in Fig. 4(b), where all mixing coefficients are shown to be vanishing. We estimate $Z_{VA+AV}(\beta = 3.9; 2\mathrm{GeV}) = 0.454(18)$.
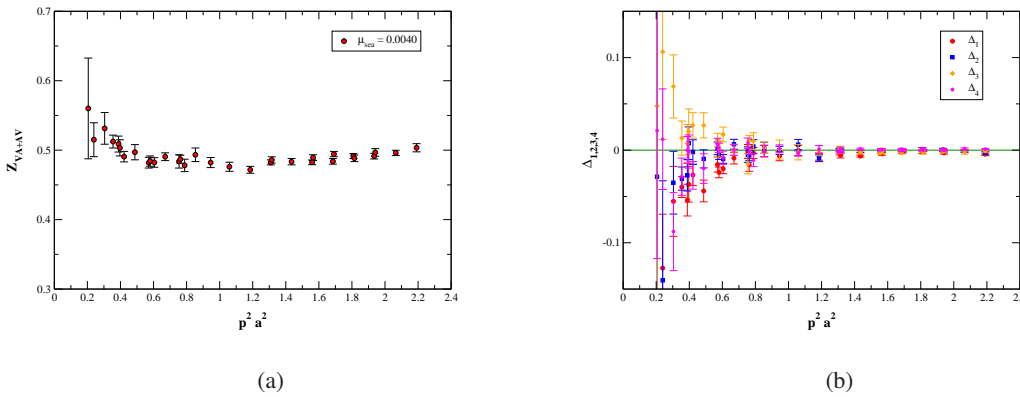

(a)	(b)

Figure 4: (a) RI/MOM computation of the multiplicative renormalization factor Z_{VA+AV}. (b) Mixing coefficients Δ_k ($k = 1, \cdots, 4$) with other four-fermion operators with "wrong chirality".

The results reported in this work are very encouraging, but a word of caution is in place here.

At fixed β, the two Kaon states, obtained with different regularizations (i.e. tmQCD and OS) are not degenerate, differing by $O(a^2)$ discretization terms. The two different exponential decays, as well as the factors m_K^{tm} and m_K^{OS} of the matrix element $< \bar{K}^0(m_K^{tm})|O_{VA+AV}|K^0(m_K^{OS}) > \propto m_K^{tm}m_K^{OS}$, cancel out in the ratio of B_K. The Kaon mass splitting is sometimes quite significant; for the $\beta = 3.90$ case in hand, with $a\mu_l = a\mu_{sea} = 0.0040$ and $a\mu_h = 0.0220$ we have $m_K^{tm} = 0.2391(07)$ and $m_K^{OS} = 0.2923(16)$. It is important to monitor the size of this splitting with increasing β, in order to confirm that it vanishes like a^2.

Our preliminary result at a single lattice spacing is $B_K(2\text{GeV};\text{RI}/\text{MOM}) = 0.56(2)$, corresponding to $B_K^{RGI} = 0.77(3)$.

Acknowledgements

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